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ROUTE SURVEYING

ROUTE SURVEYING

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SECOND EDITION

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PREFACE TO THE SECOND EDITION

This new volume retains the form and essential characteristics of the previous edition. The major changes are to be found as extensive revisions of Chapters 4 and 5 on Circular Curves and Spirals, respectively, and in the addition of an entirely new chapter (Chapter 6) on the string-lining method of realining existing railroad curves. Important revisions and additions of lesser extent have been made in many parts of the book for the purpose of presenting the material more clearly or completely and to make it conform to the latest practice.

In Chapter 4 on Circular Curves an arc definition of the degree of curve has been adopted in place of the chord definition previously used. An arc definition provides a constant relation between the radius and the degree of curve which is exact for all curves. Consequently, it is possible to use a table of the functions of a 1° curve without approximations or corrections even for sharp curves. The arc definition here used departs from past custom in that it is based on a unit arc length of 100.007 ft., corresponding to a radius of 5730.00 ft. for a 1° curve. This plan has the distinct advantage, in addition to ease of calculation, that curves may be staked out with 100-ft., 50-ft., 25-ft. or other convenient chord lengths with a minimum of cumulative field error without applying chord corrections. Table 10 gives the radii and their logarithms for curves up to 20° in conformity with this definition.

The spiral is considered by many engineers, who apparently do not thoroughly understand its characteristics, to be complex and difficult to use. In reality it is but slightly more difficult of application than a simple curve if approached from the standpoint of rate-of-curvature rather than from that of length. New material, therefore, has been added in Chapter 5 to demonstrate this simplicity and to emphasize the advantages of making the rate-of-curvature the controlling factor and of subordinating length to the curvature characteristics.

The portion of Chapter 5 relating to highway spirals has been rewritten and expanded to include treatment of the needs for spiraling highway curves and of the relationship between safe speeds, superelevation, and the selection of suitable spirals. It is believed that this is the simplest as well as the most complete treatment of highway

spirals yet presented. Although highway spirals are actually more essential to safe operation of traffic than are railroad spirals, few highway departments have adopted spiraling as a definite policy while many highway engineers do not, as yet, appreciate the importance of spiraling. It is hoped, therefore, that the treatment of highway spirals here given may aid in rectifying this situation and in promoting the more general use of spirals in the interests of highway safety.

The diagram for railroad spirals, showing the relation between spiral curvature, spiral length, degree of curve, speed, and superelevation, has been revised to include speeds up to 100 miles per hour. A similar diagram for highway spirals has been added.

The string-lining method of realining existing railroad curves has been used by the leading railways for many years but the subject has not been treated in college textbooks. It is believed that the method is of sufficient importance to include a special chapter (Chapter 6) on the subject.

In Chapter 8 on Earthwork the treatment of the mass curve and distribution diagram has been revised and expanded to make the subject matter clearer and the method of more universal application.

In addition to the changes in the text the problems and answers at the ends of certain chapters have been revised to agree with the new material presented.

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PREFACE TO THE FIRST EDITION

INSTRUCTION in surveying in most engineering schools consists of courses in plane and topographic surveying, followed by a course in railroad surveying. For many years this procedure was entirely satisfactory, since railroad location and construction formed the largest part of the engineering work requiring special surveys.

During recent years, however, the location, construction, and operation of highways, and also of pipe lines, transmission lines, etc., have attained such importance that a knowledge of the surveys required has become a necessary part of the student's training. Very few schools give special courses in highway surveying and therefore, in general, such treatment of the subject as is given is merely incidental to other courses in surveying or in highway engineering, and consequently is meager and incomplete. Since the basic methods of highway or other route surveying are essentially the same as those of railroad surveying, it seems entirely logical, and now highly desirable, that the old courses in railroad surveying be extended to include highway and other route surveys, which properly should be taught as a single subject.

With this thought in mind, the authors in revising their "Textbook on Railroad Surveying" have enlarged its scope and character to include surveys for highways, transmission lines, pipe lines, canals, etc., and have changed its title to "Route Surveying." In so doing the text has been almost entirely rewritten and its contents increased nearly 100 per cent. This new book therefore presents in a single volume a complete and comprehensive treatment of railroad, highway, and other similar route surveys.

The aim throughout has been to present the essentials of the subject in simple, concise form and to give modern, practical methods only. To this end, detailed mathematical solutions are omitted, obsolete methods abandoned, and only the more common problems outlined; consequently, the subject-matter occupies considerably less space than that in the ordinary handbook, but at the same time all phases of the subject are thoroughly treated.

From the standpoint of the instructor, some of the essential characteristics of the book are:

1. A coordination of the entire subject, indicating the general order and the solution of the various steps in the work.

2. The explanation of reasons for or conditions under which the various problems arise.

3. The assumption of a knowledge of elementary surveying and plane trigonometry on the part of the reader.

4. The consequent omission of detailed mathematics. Outlined solutions only are given, and hence the student must follow the solution of a problem through step by step instead of substituting blindly in some formula which has the appearance of being suitable. The student is therefore more thoroughly trained in the plan of attack, and consequently can more readily handle the multitude of problems which he can not find "all worked out" in this or any other book.

Although designed as a book for student use, it is believed that it will appeal to the practicing engineer as well, and the latter's attention is respectfully called to the following features:

1. The simplified curve nomenclature, as recommended by the American Railway Engineering Association, has been adopted.

2. The spiral here given is of a general form and is equally applicable with chords of any desired length, as a so-called "Six-chord" or "Ten-chord" spiral. Table 12 gives spiral data for curves up to 25° and for values of k from $\frac{1}{4}$ to 10.

3. Parabolic curves have been allotted an entire chapter, and all phases of the vertical curve have been thoroughly treated.

4. Turnouts are presented in practical form as actually used on steam roads in America, that is, with *straight* frogs and switches.

5. Diverging track and connection problems are solved so as to bring the frogs at an existing rail joint. The authors do not know of any other book which considers this phase of track work.

6. The coordinate method of computing end areas is used, and a very efficient method of computing end areas of uncompleted and irregular sections is presented. This method is especially valuable for making monthly estimates.

Acknowledgment is hereby made of the courtesy of Prof. J. C. Nagle in permitting the use of Tables 10, 13, 15, and 16, which are taken from his "Field Manual for Railroad Engineers"; to Professor Carhart for Table 14, which is taken from his "Field Book for Civil

Engineers"; and to Prof. Walter Loring Webb for Tables 9 and 10 from his work on "Railroad Construction," which appear as Tables 17 and 18 in this volume.

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ROUTE SURVEYING

CHAPTER 1

ROUTE SURVEYS

1. Introduction. Route surveying is that branch of general surveying which treats of the field work and calculations required in the location, construction, and maintenance of routes of transportation and communication such as railroads, highways, canals, pipe lines, and electric transmission lines.

Although such routes may extend over large distances, the width of way in every case is very narrow, and hence the shape of the earth affects only the absolute direction, not the relative direction, of the lines which compose the route. Consequently, the methods of plane surveying are entirely adequate for route surveys and are universally used.

2. Classification of Terrain. The practices in route surveying differ greatly in different parts of a country due partly to the nature of the particular project, partly to local preference, but principally to differences in topography.

All country can be divided broadly into three classes:

1. Level, prairie country which offers few or no obstacles in the way of hills, valleys, etc., and consequently allows the locating engineer much latitude in the placing of the line to meet the established needs.

2. Rolling, hilly country through which several lines are possible, none of them departing to any great extent from the direct line between controlling points, but which require careful investigation by the locating engineer to choose the best one.

3. Mountainous country where the judgment, skill, and ingenuity of the locating engineer are taxed to find a satisfactory route, or in some cases even a possible route.

Obviously there is no sharp division between these various classes

of country, and one class may merge into another almost imperceptibly. Likewise any or all of them may be found on any given project, and hence such a project may call for familiarity with the methods applicable to each.

3. Kinds of Survey. The different surveys may be classified by the character of the route to be established, as Railroad Surveys, Highway Surveys, Pipe-line Surveys, etc. These surveys, however, do not differ greatly in their general characteristics. Such differences as do occur are due principally to the limitations of alinement and grade permissible for each kind of work, and to a less degree to the topographic features, especially existing improvements, or established routes of similar character. In general, however, each of these basic projects calls for a sequence of specific surveys as follows:

1. A reconnoissance survey, or a rapid general survey for the purpose of selecting practicable routes.
2. A preliminary survey, with the usual surveying instruments, for selecting the particular route and for designating the alinement, grades, and structures.
3. A location survey for establishing the alinement on the ground.
4. Construction surveys for reestablishing the alinement where necessary and for laying out the various details of the project.
5. Maintenance surveys for the upkeep and improvement of the project after it is once built.

Railroad location and construction show these various surveys in their most complete and comprehensive form, hence railroad surveys will be treated first, and in detail, and the surveys for other kinds of routes will be given by suggesting the application or modification of the methods of railroads surveys to these other kinds of surveys.

RAILROAD SURVEYS

Railroad Reconnoissance Surveys

The terminals and intermediate points connected by a railroad are determined by its promoters with a view to the amount of traffic that can be expected from them. The nature and the amount of the traffic and the direction of heaviest haul determine

the maximum gradients and curvature advisable. The first thing to be determined by the locating engineer is the directions of the lines joining controlling points. This information can be obtained with considerable accuracy from the United States Geological Survey maps of the territory, or similar maps if such are available. Otherwise, such maps as can be obtained locally will have to be used.

4. Reconnoissance. A reconnoissance is then made of the strip of country through which the road is to pass, as the result of which some of the routes are eliminated as impracticable and one or more are chosen for a more detailed survey.

If the country is of the first class, a reconnoissance is rarely necessary, and the first survey is in the nature of a preliminary.

For country of the second class, the reconnoissance should be made across-country, following, as closely as possible, the direction of the line determined from the map. Usually the trip is made on foot or on horseback, but occasionally an automobile may be used. If in following the direct line between controlling points, obstacles are met with which can not be surmounted, such as high hills, the engineer should explore on both sides of the obstacle and decide, if possible, which route offers the least resistance. Before deflecting from a straight line, the engineer must be sure that his reasons for doing so are justified from an economic standpoint. In country of the second class it is seldom that grades cause much trouble, and the main lookout of the engineer is to keep the amount of curvature as low as possible. The result of the reconnoissance through country of the second class is that one or more routes are selected for a more detailed survey.

It is in mountainous country—third class—that all the skill of the engineer is brought into play. The drainage of the country should be carefully studied, as it plays a very important part in the location of a railroad. If the controlling points are in the same valley, the main problem is solved, and the conforming of the alinement to the topography is merely a matter of detail. But when the controlling points are in different valleys, the ridges between them have to be crossed, and the principal object of the reconnoissance is to discover the most favorable crossing places in the valleys and on the ridges. The saddles in the ridges and the most favorable river crossings become secondary controlling points. The locations of all such points are platted on the map, and their elevations and distances apart are recorded. The most important and useful

"instruments" used on reconnoissance are the judgment and experience of the locating engineer, as upon these depend the extent and the cost of more detailed surveys and the cost of construction and of operation.

Recently aerial survey methods have been adapted to railroad reconnoissance and preliminary surveys. Observation flights, aided by aerial photographs of the more important portions of the line, serve many, if not all, of the needs of reconnoissance, and can be made with the minimum expenditure of time and effort, and usually at a smaller cost.

Railroad Preliminary Surveys

5. Preliminary. In country of the first class, a preliminary line is run for direction. The magnetic bearing of the direct line between controlling points is measured on the map, and a line having this direction is initiated from the first controlling point and produced to the second controlling point. Since the direction of the line as obtained from the map and this direction as laid off in the field are subject to considerable error, the first line run will probably pass to one side of the second controlling point. The distance by which this transit line misses its mark is noted, and the angular correction that must be applied to it is computed. The next line is run on the corrected direction and becomes the location line.

In running the preliminary line, stakes should be placed only at transit points. A straight line is the most difficult one to run, and particular care must be taken to avoid errors. Where changes of direction are necessary, the deflection angles should be carefully measured. It must be remembered that the preliminary is a reference traverse, and if the lines are not straight and the angles are not correct, the purpose for which it is run is defeated. The distances between hubs can be determined with sufficient accuracy by means of the stadia.

In country of the second class, preliminary lines are run over each of the routes chosen by the locating engineer on the reconnoissance survey. The data taken are such that the several routes can be compared with respect to distance, grades, and curvature. Usually lines run with the transit and stadia will give sufficient data as regards distance and curvature; and the elevations of enough commanding points can be taken with the stadia to indicate the grades that will be required by the several routes. From these

data, one of the routes will usually appear superior to the others, and what is generally known as a preliminary survey is then made over the selected route.

A preliminary survey, as generally understood, is a topographic survey of a narrow strip of country within which the road will pass. The purpose of this survey is to secure data from which a topographic map can be platted upon which the *paper location* is projected. The transit and stadia traverse already run over the chosen route is used as the base line from which the topography is taken. Although this line is usually measured with the tape, time used for this purpose is wasted, as it is impossible to plat the traverse distances to a consistent degree of accuracy. Spirit levels are then run over the line to determine the elevations of the transit points, and bench marks are established at half-mile intervals.

In country of the third class, several preliminary lines are required, each following one of the routes chosen by the locating engineer. The approximate grades of the lines between secondary controlling points are obtained from the data taken on the reconnoissance survey, and the preliminary line should be chosen so that the grade line thus determined will conform as nearly as possible to the surface of the ground. If this is done, topography will not have to be taken as far on each side of the line as would otherwise be necessary. In mountainous country, topography can be taken more accurately with the hand level than with the stadia, and line stakes are placed every 100 ft. and their elevations determined by spirit leveling, so that the hand level can be used. In other respects the methods of surveying in mountainous country are the same as those in country of the second class.

Railroad Location Survey

6. Location. The line is run very carefully on the location survey, stakes are driven every 100 ft., the plusses and angles of all property lines are taken, and curves are run in where indicated on the map. The distances from intersections with property lines to the nearest Government section monument, or other legal monument, are carefully chained, so that accurate descriptions of the right of way can be drawn up. All buildings near the line which will be damaged by it must be located and an estimate of the damage must be made. A level party follows the transit party, taking profile levels and establishing bench marks, unless these have been previously established. From these data, alinement maps, right-of-way maps, and

profiles are made; and after the grade line is established on the profile, construction can begin.

Railroad Construction Surveys

7. Reference Stakes. After the location has been made and accepted, and just before construction begins, the beginning and end of all curves and intermediate points on long tangents are "tied-in" by reference stakes so that after the construction work is completed these points can be relocated in their correct positions. There are several methods of referencing a point; the one shown in Fig. 1 is very satisfactory. A is the point to be referenced. First choose

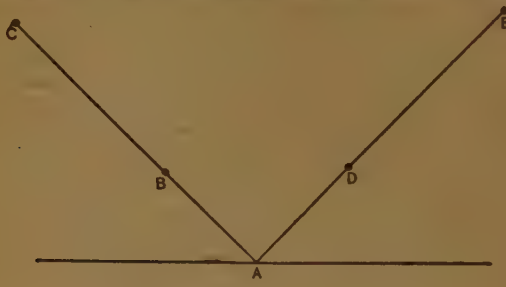

FIG. 1.

permanent points at C and E at least 300 ft. from A, making the angle CAE as nearly a right angle as possible. A distant windmill or house-chimney makes an ideal point. If these are not to be had, a nail driven into the trunk of a tree about 5 ft. from the ground is good. If hubs are used they should be made as permanent as possible, and located so that they will not be disturbed. After C and E have been determined, set a hub at point B on the line AC, and another one at D on the line AE. B and D should be placed far enough from the center line of the road so that they will not be disturbed during construction. Point A is relocated after construction by the intersection of the lines CB and ED, the transit being set up at B and D. Care and judgment should be used in locating reference points, as they *should be* used after the roadbed has settled for locating permanent monuments.

8. Slope Stakes. Slope stakes are then set at each station on each side of the center line at the points where the side slopes of the

cut or fill will intersect the ground surface. These stakes are for the guidance of the contractor, and have marked on them the vertical distance from the ground at the stake to the level of the roadbed. (For method of setting slope stakes, see section 239.)

9. Distribution Stakes. Distribution stakes are set to show the contractor the desired movement of the material from the cuts into the fills. The proper location of these stakes is explained in Section 252.

10. Borrow-pit Stakes. These stakes are set to indicate to the contractor the limits within which he may borrow earth for making the fills.

11. Culverts and Trestles. The location of all culverts and trestles must also be staked out. If possible, these structures should be constructed in advance of the grading so that openings will not have to be left in the fill to be completed later.

12. Time. The time to set stakes is just before they are needed. If they are set too long a time ahead of construction, some of them will invariably be knocked out and will require resetting.

13. Monthly Estimate Surveys. During the last few days of every month, surveys are made to determine the amount of work which the contractor has completed during the month, so that he may be paid.

14. Finishing Stakes. From the slope stakes the contractor can construct the roadbed to within a few inches of the correct grade. At this stage of the work it is customary to give finishing stakes, which are stakes driven to grade at the edges of the roadbed at each station. From these stakes the contractor is able to finish the roadbed to the correct grade and width.

15. Center Stakes. After the earthwork is completed, the important points on the center line are relocated from the reference stakes; and center stakes, usually untacked, are driven from which the track is laid.

16. Grade Stakes. After the track is laid, grade stakes are driven at every station and at those points where the grade changes, with their tops to the grade of the final top or base of rail. Grade stakes are placed on the inside of curves, and the inner rail placed at grade. The outer rail is then superelevated the required amount above the inner rail.

17. Right-of-way Stakes. The right-of-way fences are usually built as soon as the materials for construction can be hauled over the line. For the guidance of the fence foreman, stakes are placed

on each right-of-way line (1) opposite the beginning and the end of all curves, (2) opposite each station on curves, (3) from 300 to 400 ft. apart on tangents, and (4) at all jogs in the right of way. These stakes should be long enough to be seen above weeds, wheat, oats, etc. Laths are excellent for this purpose.

18. Property Surveys. An exceedingly important part of the surveying operations is the determination of right-of-way and property lines and the legal description of the lands involved. This may be done as part of the preliminary and location surveys, but frequently special surveys are necessary.

Where the United States rectangular system of land surveys is employed, all lines should be tied to the Government monuments, and the descriptions should be made with respect to the legal subdivisions of land. Metes and bounds descriptions should be avoided whenever possible. Where the general land survey is by metes and bounds, this method must be used, and care should be taken that the surveys and descriptions are accurate.

19. Special Surveys. After the track is laid, special structures, such as station buildings, water tanks, cattle pens, etc., will require staking out. In addition, the parts of the right of way which are leased to coal, lumber, and grain companies must be staked.

Railroad Maintenance Surveys

Due to the fact that it takes two or three years for the roadbed to settle and for the track to become thoroughly embedded in the ballast, it will be necessary to reset center and grade stakes frequently during this period. All center stakes which are set after the track is laid should be tacked.

20. Monuments. After the track and roadbed have settled thoroughly, permanent monuments should be placed at the beginning and the end of all spirals and circular curves, between the branches of compound curves, and at intermediate points on long tangents. A section of steel rail about 3 or 4 ft. in length, embedded in the subgrade in the center of the track, with its top level with the ties and with a small drill hole to represent the point, makes an excellent monument. The monument should be free from contact with the ties and ballast to prevent its being moved by any shifting of the track.

21. Additional Tracks. Side tracks, business tracks, branch-line tracks, yard tracks of various kinds, cross-overs, etc., must be

staked out from time to time, as the need for such tracks arises. There is no end to surveys of this kind, and all the large railroads employ maintenance parties who do nothing else.

Organization of Parties

The field corps is usually divided into (1) a transit party, (2) a level party, (3) a topography party, and sometimes (4) a land-line party.

22. TRANSIT PARTY.—The members of the transit party and their duties are as follows:

The Locating Engineer is the chief of the entire surveying corps, and receives his instructions from and reports to the chief engineer of the railroad company. His duties are to direct all of the surveying operations from reconnoissance to location, to prepare the paper location, and to make all decisions relative to location, subject to review by the chief engineer. Frequently the chief engineer performs the duties of locating engineer.

The Chief of Party is next in rank to the locating engineer. His duties are to direct all the surveys, to provide accommodation for his party, to pay all general expenses, and in case a camp is necessary to purchase all supplies, and to manage the camp. On construction the chief of party usually becomes *Resident Engineer* and has charge of from 8 to 15 miles of construction.

The Transitman is next in rank to the chief of party, and in the latter's absence is in charge of the party. His duties are: to do the transit work, which consists of lining in the chainmen, measuring the angles between successive tangents, noting the bearings of the tangents, measuring the angles which the line makes with all railways, highways, streams, and property lines, and recording the plusses at which they cross the line; and to keep the notes of the transit party. The transitman frequently acts as chief of party.

The Head Chainman * ranks next to the transitman *in the transit party*, and is directly in charge of the rear chainman, stakeman, and axmen. His duties are: to see that the distances are chained correctly; to see that the stakes are driven on line, that they are driven straight, and that they are marked correctly; to direct the axmen where to cut in opening up the line; to set new transit points; and to direct the taking of plusses. The head chainman has a very impor-

* The term *tapeman* is frequently used since the steel tape has now replaced the old time chain.

tant position, as he regulates the speed of the entire party. In open country frequently the locating engineer takes this position. The head chainman carries the *zero end* of the tape.

The Rear Chainman's duties are: to hold his end of the tape on the last stake driven while the head chainman gets the distance; and to take and record all plusses which he turns over to the transitman at frequent intervals.

The Rear Flagman's duties are: to give the transitman a sight on the backsight station whenever he signals for it; and to carry excess baggage. The rear flagman *should be* a wide-awake man with good eyesight.

The Stakeman's duties are: to carry the stakes; to mark the station numbers on the stakes; and to drive the stakes as directed by the head chainman.

The Axmen do all the necessary clearing in order that the transit and level parties may have a clear path. They are sometimes required to make the stakes.

23. THE LEVEL PARTY.—The members of the level party and their duties are as follows:

The Levelman is chief of the level party and ranks next to the transitman in the surveying corps, when there is no regular topography party. His duties are: to run profile levels over the line and to establish bench marks; and to keep the level notes.

The Rodman's duties are: to hold the rod vertically upon the ground at each station, and at those intermediate points where the longitudinal slope of the ground changes; and to keep " peg notes " as a check on the levelman's computations.

24. THE TOPOGRAPHY PARTY.—The members of the topography party and their duties are as follows:

The Topographer usually holds equal or superior rank to the levelman in the surveying corps. This position is a very important one and should be filled by an experienced man. Frequently either the transitman or the levelman performs the duties of topographer. The duties of the topographer are: to take all necessary data for making an accurate contour map of a strip of country sufficiently wide to enable the engineer to make an intelligent projected location; and to record these data in such a way that they will be readily understood by the draftsman. The topographer is assisted in his work by one or more rodmen and tapemen.

25. THE LAND-LINE PARTY.—The duties of this party are: to measure the angles which the line makes with all railways, highways, streams, and property lines; to tie-in the line to the nearest Government monuments so that legal descriptions may be prepared of the required right of way; and to secure the names of the property owners. This work is highly important and should be done by an experienced man. Often, either the transit party or the topography party does the work of the land-line party.

26. Drafting. In addition to the above field parties there is the field draftsman, who does his work in camp. His duties are to plat the notes taken by all the parties the previous day, which necessitates the use of two sets of field note-books or else loose-leaf note-books. In some cases the draftsman, with the help of the locating engineer and the transitman, plats the notes each night; and the levelman plats the profile of the line over which he ran levels that day. Thus the map is kept up to date, and the locating engineer can project his location as the line advances.

Methods of Making Railroad Preliminary Surveys

The preliminary survey is the most expensive survey, and is of primary importance, since the location depends directly upon it. Hence the method of making it should be given considerable thought in order that it may be done with accuracy and economy. There are at present three general methods used: (1) the transit and tape method, (2) the transit and stadia method, and (3) the plane table method.

27. The Transit and Tape Method. This method is by far the most common. The transit party runs the line with transit and tape, the level party follows taking profile levels, and the topography party follows the level party; the land-line party may come in anywhere after the transit party. This kind of a survey will require from six to fifteen men, depending on whether the several parties have a separate personnel and on the number of axmen required to open the line.

28. The Transit and Stadia Method. This method consists in running the line with the transit and stadia. Stakes are placed only at transit stations, and the elevations of these points are determined by transit and trigonometric leveling. The location and elevations of important intermediate points along the line are determined in a similar manner. While the transit is at each station, the topography

around that point is taken with the stadia. Thus all the needed data are taken as the line advances. These data may be recorded in the note-book and worked up later by the party draftsman; but it is much better if the draftsman plats the notes as the transitman takes them and draws in the contours while the landscape is before him. The draftsman holds a very important position in this party and should be an expert in that line of work. A survey by this method is very accurate as regards the contours, which are the most important item on a preliminary map; and, if good men are employed, it is more efficient than the first method. This method requires a transitman, a draftsman, two rodmen, and as many axmen as the nature of the country may require.

29. The Plane Table Method. This method is very similar to the one just described and differs from it mainly in the use of instruments. The plane table takes the place of the transit, and the plane-tableman does the drafting. Owing to the difficulty in handling and setting up the plane table, it is doubtful whether this method is as efficient as the transit and stadia method; and although the services of the draftsman are dispensed with, yet the progress is possibly not so rapid.

30. Airplane Surveys. The development of the technique of making topographic maps from aerial photographs has made airplane surveys available for railroad reconnoissance and preliminary surveys. For the former, simple observation flights may suffice for selecting the feasible route or routes. For the latter, a regular series of photographs combined with some instrument work on the ground to establish points of control will often furnish sufficient data for the selection of the specific route, and even the preparation of the paper location.

The airplane survey is most valuable in rough, rugged country where the difficulty of making ground surveys is great. In such cases the airplane survey is vastly more rapid, and hence more economical.

31. Remarks. For long lines that justify the employment of a large number of men, the transit and tape method is probably the most efficient. In other cases, however, the transit and stadia method will prove the more economical. The latter method has not been used to any considerable extent, due to the fact that few engineers fully appreciate the advantages of the stadia method; but in the few cases in which it has been tried it has fully demonstrated its superiority, particularly for open country.

32. Bench Marks. When spirit-levels are run over the line, bench marks should be established at half-mile intervals, approximately, and should be placed far enough from the center line so that they will not be disturbed during construction. After construction, permanent bench marks should be established on all permanent structures, such as concrete bridges, and at every station building along the line. It is extremely poor practice to use spikes driven into telegraph poles, mile posts, etc., as these are frequently moved and reset. If the road has few concrete or steel structures, then bench marks may be established on trestles. When these are renewed, the elevations of the several parts will rarely be changed more than an inch; whereas a bench mark on a telegraph pole may be changed several feet.

Note.—For further information on the subject of Railway Surveys and the Economics of Railway Location, see A. M. Wellington's "Economic Theory of the Location of Railways"; F. Lavis' "Railway Location Surveys and Estimates"; C. C. Williams' "Design of Railway Location," or W. L. Webb's "Economics of Railroad Construction."

HIGHWAY SURVEYS

Since a highway is a route of vehicular transportation very similar in operation to a railway, the general features of highway surveys are identical with those of railroad surveys. Such variations as do occur are due primarily to the difference in limitations placed on alinement and grades and to the fact that the majority of highway surveys are made for the purpose of improving existing roads.

In the days of horse-drawn traffic very sharp curves were permissible, but with the increased use of the motor car longer radii are demanded, and the practice in this respect is approaching that used on the railroads. Owing to the fact that each motor vehicle contains its own power plant, and also that the adhesion factor of the wheels to the road is much higher than for the locomotive, the permissible grades are much greater on the highways than on the railways. These factors give a somewhat greater flexibility in the location of the highway, especially in rough country.

One other factor must be kept in mind by the locating engineer of the highway, and that is the importance of appearance. On railroads only the enginemen see the various features as they approach them. Such of the passengers as see them get only a

glimpse from the observation platform as they recede. On highway, however, every traveler is looking forward, and every defect as well as every virtue is distinctly visible. Furthermore, the driver of a motor vehicle is dependent on the visibility of the roadbed and its structures for the safe handling of the vehicle, whereas the engine driver is relieved from the hazard of steering; hence in addition to good appearance, there must be safe sight distances.

Highway Reconnoissance Surveys

33. On New Location. Where a new road is to be laid out in new country, the reconnoissance survey is exactly like that for a railroad. The same methods are employed and the effect of the different classes of terrain is the same. Such roads may be entirely new routes or comparatively short stretches in existing highways where relocation is contemplated.

34. On Existing Roads. When the project is the improvement of an existing road, the object is to determine which of perhaps several possible routes is the most feasible. Each of the possible roads must be investigated. Such a reconnoissance is best made in an automobile equipped with an accurate odometer and a gradometer. One man drives the car and reads the instruments, while a second man makes sketches and keeps notes. A record is made of all such features as railroad crossings, stream crossings, steep grades, flood levels, entries into and routes through municipalities, availability of materials, etc., which may affect the cost of construction or the safe and convenient use of the highway after improvement. From these data the most satisfactory route can be determined.

Highway Preliminary Surveys

35. On New Location. When a highway is to be laid out on new ground, either entirely or as a relocation of part of an existing road, the preliminary survey is made in essentially the same way as the railroad preliminary. The same data are required and the same methods are employed. In only one respect is there any marked difference. The topography is usually more carefully taken, especially the cross-sections which are generally made with level, rod, and tape. This is because the information is employed not only for establishing the alinement but also for platting the cross-sections and for computing the amount of earthwork.

36. On Existing Roads. When an existing road is to be improved, a preliminary traverse is run to which all of the essential features of the existing road are tied. Care is taken to secure all information which may affect the nature or cost of the project and especially its relationship to existing improvements such as buildings, farm entrances, etc. Cross-sections are taken with the level and tape so that they can be platted and the earthwork determined.

Frequently old curves are to be abandoned and new ones of longer radii substituted. Each of these is essentially a short relocation and should be so treated. Especial attention should be given to the possibility of the elimination of jogs and slight deflections and other irregularities. The matter of appearance, as well as the serviceability of the improved road, should be kept constantly in mind.

Other Highway Surveys

37. Location and Construction Surveys. For highways these surveys are essentially the same as for railroads. Certain modifications may be required to suit the particular kind of road surfacing or the character of auxiliary structures employed.

38. Land-line Surveys. Since most of our highway rights of way consist merely of easements to the public for road purposes, the true title to the rights of way resting in the land owner, it is essential that the land lines be accurately determined, carefully described, and faithfully recorded as required by law. In so doing, and during construction, all Government monuments and other land-marks should be carefully referenced, reestablished, and preserved, and a complete record made of any changes made in the position or nature of the monuments.

39. Maintenance Surveys. As with the railroad, there is a perpetuity of miscellaneous surveys after the road is built for the purpose of maintenance, improvement, and additions. These are made at the time and in the manner suited to the particular needs.

Street Surveys

In a physical way, city streets bear much the same relation to the general highway system that yards and terminals do to a railroad system. From the administrative standpoint, however, the two are far different. On the railroad, both the main line and the yards

are under the same management, whereas the streets are under the jurisdiction of administrative bodies entirely separate and distinct from those controlling the highways, and hence the process of laying out and improving city streets is entirely different from the methods on rural roads.

40. Subdivision Surveys. Streets are laid out by the subdividers of the property. With the increasing adoption of city and regional plans and the extension of the powers of municipalities to control the subdivision of the land adjacent to them, it is highly important that subdivisions be properly made.

In flat country a simple boundary survey may be sufficient, including of course such things as streams and railroads which may affect the street plan or the use of the property. In more rolling territory a complete topographic survey should be made with care, since the streets must be adapted to the topography. The topographic map should be carefully preserved as it will prove of inestimable value in later improvements of the territory.

On the map of the territory, the paper layout or plat is made. This should be made after all features have been carefully studied. In some cases, especially on rough land, it is advisable to make the field layout before finally filing the plat. In any case the approval of the proper authorities must be obtained and the plat ultimately filed as required by law in the particular locality.

The ground layout should be carefully made. Permanent monuments should be placed at all principal corners and at such secondary corners as will make it possible to locate any lot or street line quickly and accurately. In general this means that all block corners should have permanent monuments, but it is not necessary to monument all lot corners, at least in the original layout. The descriptions of all monuments should be placed on the map and filed as part of the official plat.

The methods used on such surveys are transit and stadia or plane table and stadia for the topography, and transit and tape for the boundaries, with spirit-leveling where necessary.

41. Pavement Preliminary Surveys. No reconnoissance survey is made for street improvements. The street to be improved is chosen by the promoters, the city authorities, or by the property owners. Its position has already been established in the making of the subdivision. If the land is undeveloped, it may be necessary to retrace the street lines, but if the property is more or less built up, the position of the street is usually clearly enough defined for

preliminary purposes without running a center line. This is especially true if sidewalks are in, as they are supposed to conform to the property lines.

If sidewalks exist, the measurements may be made on the walks, usually on one only. If sidewalks are not in, chaining pins or stakes may be used. If the streets are curved, some engineers prefer to run the center line, but property-line measurements are just as good, if correctly used. If the proposed pavement connects with an existing one, the measurement should begin at that end, otherwise there is no choice. Plusses to the nearest 0.1 ft. should be taken at all intersecting street and alley property lines, at all railroad and stream crossings, and at all other points where the natural topography or existing improvements may affect the design of the pavement. Cross-sections are then taken at intervals of 50 or 100 ft. as conditions suggest. In addition, the elevations of all features that may affect the grade of the pavement are determined. Bench marks should be established preferably in each block. The level circuit should be closed back over the bench marks to the starting point. The datum to be used is usually a city datum, and careful tie must be made to existing bench marks.

A convenient party for pavement preliminaries consists of four men, a chief of party, a levelman, and two rodmen. The chief of party and one rodman can chain the line, while the levelman and the other rodman pick up an existing bench mark and bring the levels to the street and establish the first bench mark on it. The chief of party should head the chaining, as his judgment should guide in tying-in the desired points. The party can then unite. The chief should keep notes, make sketches, and direct the work, while the levelman takes the cross-sections, using two rods, as this greatly expedites the work. Usually it is not necessary to use a tape on the cross-sections, because the distances are short and the rodmen can pace them. A reading is normally taken on the center line, at the gutter lines of the proposed pavement, and at each property line. This gives essentially five profiles of the street on the lines mentioned, and furnishes the data necessary to establish a grade line and to compute the amount of excavation.

Included in the preliminary should be a complete record of all drainage facilities, including manholes, inlets, tile lines, etc., with the depths to the flow line and the elevations of the tops, unless this information is available on other maps or plans. A record should also be taken of all other improvements, such as telephone and light

poles, sewer and other manholes, etc., which may affect the design or construction of the pavement, or which may require moving.

42. Pavement Location Surveys. From the data taken on the preliminary survey, a complete set of plans is made, including a well selected grade line. This grade line should be used in the field, unless some obvious change is required. It is poor practice to attempt to lay a grade line in the field.

The principal requirement for construction is a set of stakes, locating the curbs (or edges of pavement) and giving the grades. The best practice is to drive a set of "rough grading" stakes on each side of the pavement at intervals of about 50 ft. These stakes should be set back from the curb line from 5 to 8 ft. so as to be clear of grading operations. Level readings are taken on the tops of the stakes, and cuts to subgrade are computed and furnished the contractor. These stakes should be located with fair accuracy from the street lines.

After the rough grading is completed, exact line and grade stakes should be run for each edge of pavement or for each curb. If plain curb is used, these stakes may be about 2 ft. *inside* of either the back or the face of curb, as the contractor prefers, and driven to grade or some fixed distance above or below grade. If a combination curb and gutter is used, the stakes are best placed about 1 ft. *inside* the edge of gutter and to the elevation of edge of gutter. If no curb is used, the stakes should be about 1 to 2 ft. *outside* the edge of the slab and driven to grade.

Stakes should be placed at intervals of not more than 50 ft. on tangents and 25 ft. on horizontal and vertical curves. It is poor practice to set a few stakes and trust to the contractor to fill in.

43. Sidewalk Surveys. Frequently sidewalks are built before the pavements. In so doing it should be remembered that this fixes the pavement grade to a considerable extent, and consequently the sidewalk grades should receive careful attention. In flat country and on residence streets there is considerable flexibility between curb and sidewalk grades. On business streets, however, the sidewalks must conform both to the property and to the curb; consequently the entire design of both sidewalks and pavement should be worked out together, even if only one is to be built. In rolling country, where there may be considerable cross slope in the streets, the design of the grade lines for both walks and for the pavement should be made at the same time. The preliminary surveys should therefore be made accordingly.

44. Other Street Surveys. Lighting systems, storm sewers, sanitary sewers, water and gas pipes, etc., are frequently placed in the streets. These should receive consideration both in making the subdivisions and in fixing the street widths so that there may be ample room. The individual survey for each of these improvements follows the ordinary methods for such work.

CANAL AND DRAINAGE SURVEYS

45. Canals. A canal is an artificial waterway for the purpose of providing a means of transportation, or for the purpose of conducting the water itself to desired points. A drainage ditch is a canal for conducting away excess water.

A drainage ditch or canal must be kept within such grade limits that adequate flow is maintained without developing excess velocities which may cause erosion. The curves also must be sufficiently flat so that the banks will not be seriously eroded. A canal for navigation must have an alinement such that the type of vessel using it can be easily and safely navigated. Usually the flow of water is small, and frequently intermittent, due to the operation of the locks necessary to overcome differences of elevation.

It is thus seen that any canal is quite restricted as to grade, and this has a serious effect on the location, although there may be wide flexibility in alinement. The surveys for canals must therefore take cognizance of these limitations, and this may modify to some extent the surveys as outlined for railroads.

The reconnoissance may be exactly the same as for a railroad. A drainage canal may follow a stream, and a study must be made of the stream as to possible relocations and cut-offs. A transportation canal must be supplied with water for operation, and the determination of possible sources is part of the reconnoissance.

Preliminary surveys are very similar to those for railroads. A line is run in the same way, and topography is taken in the same way, together with such other data as may affect the location or design of the canal.

Location surveys are essentially the same, except that offset lines must normally be used, because, unlike on the railroad, the center line can not be used for the final work, since the canal will be filled with water. The same is true of construction surveys and maintenance surveys. In none of these is there anything of unusual note, if due consideration is given the characteristics and purposes of the particular project in hand.

PIPE-LINE SURVEYS

46. Character and Purpose. Pipe lines as here considered are pipe, extending through open country, often for long distances, for the purpose of transporting liquids, usually petroleum or water.

Oil pipe lines are usually of comparatively small diameter, from 8 to 12 in., and extend long distances. They depend on pumps for forcing the oil through the pipe, and consequently they are very flexible as to alinement and grade. Water pipes on the other hand may reach several feet in diameter and often depend on gravity for flow. With large sizes, the curves must be comparatively easy, and for gravity flow the grades must be suitable, usually all in one direction, but not necessarily so, because an effective hydraulic grade can be maintained even if the actual grade dips below it and rises to it again.

Pipe lines are usually placed under-ground. This often permits the continued use of the ground above for ordinary purposes, especially for agriculture. The right of way, therefore, consists merely of an easement for the construction, maintenance, and repair of the line. Frequently the easement is leased on an annual rental basis.

These factors are of importance as they have an effect on the character and extent of the surveys.

47. Surveys Required. A reconnoissance is necessary to choose the practicable routes. This is similar to the railroad reconnoissance. A preliminary survey then follows, which may be very similar to the railroad survey in case of a large water line on gravity flow, but may be a simple line for a small size oil line. Sometimes the preliminary and the location survey for an oil line are combined. In both cases great care must be taken with the land-line surveys, so as to make it possible to prepare adequate descriptions and contracts for the right-of-way easements, and to locate the line accurately so as to remain on these easements. Line and grade stakes are required for construction, and occasionally where deep cuts are made, slope stakes may be needed.

Pipe lines may require special surveys, where they cross large streams or where tunnels may be employed. These are, however, similar to railroad surveys for the same purpose, keeping in mind the essential characteristics of the project.

TRANSMISSION-LINE SURVEYS

48. Character and Purpose. Transmission lines are built for the purpose of transmitting electric power or for conducting communication by telephone or telegraph. Power lines may differ considerably from telegraph and telephone lines in their details of construction, but all are essentially the same in the surveys required.

In most cases transmission lines are carried overhead on poles or towers. This permits the use of the ground for other purposes, except for the area immediately around the towers or poles. Here again rights of way are easements similar to those for pipe lines. The poles and towers, however, permanently occupy the land, whereas the space between is needed only during construction and when repairs are made.

Transmission lines are very flexible in line and grade. Right-angle turns may be made at a single pole or tower, or curves may be used if necessary. In general, changes of direction are desirable at a minimum number of poles or towers on account of lateral bracing. A single tower at an angle would, therefore, be preferable to several towers, making a curve. The grades can have nearly any value, depending on the clearance between the wires and other objects in the spans.

Sometimes transmission lines are placed under-ground. In these cases, they can be considered as pipe lines so far as the surveys are concerned.

Accessibility is an important factor in the location of transmission lines. For this reason telephone and telegraph lines are frequently located along highways, and the poles are placed on the highway right of way. Power lines may parallel highways, but they should be far enough from them so as not to induce electric currents in existing telephone and telegraph lines.

As far as possible, woods and forest should be avoided, because of the expense of clearing and of maintaining. Also, power lines should be a safe distance from all houses.

49. Surveys Required. The surveys for a transmission line are almost identical with those for pipe lines. A reconnoissance survey is made to choose the practicable routes. More detailed surveys are made to establish tower locations, take care of stream crossings or steep inclines, and provide for other special features. Suitable sites for substations or repeater stations must be considered.

Location surveys and land-line surveys are in general the same as for pipe lines.

CHAPTER 2

MAPS, PLANS, AND PROFILES

THE location, construction, and maintenance of any route of transportation requires the preparation of a large number of drawings which may be broadly classified as follows:

1. *Maps*, which are horizontal projections of a considerable portion of the route, showing the essential features.

2. *Plans*, or comparatively large scale drawings of special features of the route, or more often of specific structures such as bridges and buildings. The term *plans*, however, is frequently applied to the detailed horizontal projection of a highway or street (the paper location) and the term *details* used for the drawings of specific structures.

3. *Profiles*, which are essentially vertical projections corresponding to the maps. *Cross-sections* are short profiles transverse to the main direction of the route.

Railroad maps, plans, and profiles are typical of all route drawings and therefore will be considered first.

Railroad Reconnoissance Maps

50. Reconnoissance Map. This map consists of a general sketch of the country which the locating engineer has investigated, and shows the several routes that are possible. Only controlling points, such as towns to be passed through, available stream crossings, saddles in the ridges, etc., are shown. If an existing map of the country is available, it is best to draw in the routes directly upon it. The notes taken on the reconnoissance survey regarding the geological formations, the cultural features of the country, the width, depth, and current of streams, etc., are considered part of the reconnoissance map, and frequently have great weight in the choice of route.

51. Reconnoissance Profile. This profile is made from the elevations of the controlling points and the distances between them

Profiles are made for each of the several routes and are frequently placed on the same sheet, so that a more intimate comparison can be made. The purpose of the reconnoissance maps is to eliminate the impracticable routes and to determine which ones will bear a more detailed investigation.

Railroad Preliminary Maps

52. First Preliminary Maps. These maps show the transit and stadia lines run, and all railroads, highways, and streams that cross the lines. Profiles corresponding to these maps are made from the elevations of the traverse stations. From a study of these maps and profiles, one route—occasionally two—will appear superior to the others, and the preliminary survey proper is then made over this route.

53. Preliminary Map. The preliminary map proper is generally made to a scale of from 200 to 400 ft. to the inch, and is a complete topographic map of a strip of country from 100 to 1000 ft. in width. All highways, railroads, streams, buildings, and other features that may affect the location and construction of the line are shown. Frequently it is desirable to show the property lines, the names of land owners, and Government or other land monuments. Contours are an essential part of this map. The usual contour interval is 5 ft., but in very rough country 10 ft., and in very flat country 2 ft. may be used. All geological formations and other similar features that might affect the location of the line are also indicated. Occasionally, special conditions may require an auxiliary map to a scale of 50 or 100 ft. to the inch over short stretches of the line.

54. Projected Location. The projected location, often termed the *paper location*, is made on the preliminary map. The line which will give the best alinement and grades from the standpoint of operation is first projected, and this is the one to be used, unless the cost of construction is prohibitive. In case the cost is excessive, a line whose construction will come within the allowable cost, and at the same time keep within the allowable limits of curvature and grade, is laid out. This can be done only by trial and requires a large amount of skill and judgment. After the tangents have been projected, they are connected by curves which most nearly conform to the contour of the ground, and at the same time keep within the maximum curvature. The entire alinement is then reviewed, and

changes made in tangents or curves as may appear desirable, until a satisfactory alinement is obtained. If the curves are to be spiraled, allowance must be made at this time.

In order to determine the grades and the amount of earthwork for the several projected lines, it will be necessary to construct a profile for each of them from the contour map. In locating grade lines on these profiles, it must be remembered (1) that intersecting railroads must be crossed either at grade or at a clearance distance above or below grade, (2) that highways can be raised or lowered within certain limits, and (3) that streams must be crossed a safe distance above high-water marks.

The projected location is platted in detail on the preliminary map and a profile of the projected line is constructed from the contours, and a tentative grade line is established on the profile.

55. Location Notes. Notes are then made from the accepted projected location. The bearings of tangents, the plusses of the beginnings and the ends of all curves, the central angles of all curves, and the degrees of curve are scaled off the map and recorded. These condensed notes are used in making the final location.

Railroad Location Map

56. Location Map. The location map is usually made to the same scale as the preliminary map, and shows all railroads, highways, streams, and property lines that cross the line, together with their plusses; the names of the property owners and the amount of right of way required from each; the Government subdivision lines and the numbers of sections, etc.; the distances from the line to Government monuments; the boundaries of each field through which the line passes; the location and size of all stream openings; and any other data that will be of use to the construction engineer or to the right-of-way agent. If the line is located in territory not under the United States rectangular land-survey system, the line must be tied to property lines and to such land monuments as exist.

The location map can often be made by adding the necessary data to the preliminary map, and then making a new tracing, omitting the contours and such other features as are not desired on the final map.

57. Location Profile. A profile is made from the levels which are run over the located line, and shows: the ground line; the grade line,

including the per cent of grade of the different portions and the elevations of all points where the grade changes and of all stations on vertical curves; the location and dimensions of all openings in the embankment; the plusses of railroads, highways, streams, and openings in the embankment; the elevations of the top of rail of all intersecting railroads; the crown of roadway of highways or streets that pass overhead or underneath, and the high-water marks of streams; the descriptions and elevations of all bench marks; a rectified alinement map at the bottom of the profile sheet, containing practically all the data that are shown on the location map; the earthwork distribution diagram, which is drawn between the profile and the alinement; on the profile, the economical movement of the earth from the cuts into the fills; and the amount of excavation, overhaul, and borrow at all points along the line. In fact, the location profile contains practically all the information needed by the resident and construction engineers. It is used in staking out all the construction work, with the exception of buildings, bridges, trestles, and other structures for which separate plans are made.

Additional profiles are made for any change in the elevations of highways and railroads crossed by the line. These profiles are short and are sometimes placed on the location profile opposite the points where the changes are made. Cross-sections for embankments, cuts, borrow pits, or spoil banks are made as the need arises.

Railroad Right-of-Way Maps

58. Right-of-Way Maps. These maps are of two kinds, legal maps and maintenance-of-way maps.

Legal right-of-way maps are made on sheets which are the same size as the sheets on which the conveyance deeds are drawn up, usually $8\frac{1}{2}$ by 13 in. A separate map is made for each description, and a blueprint is attached to the deed and becomes a part of it and is recorded along with the deed. The railroad company usually preserves its copies of the deeds along with the right-of-way maps in the form of a book, which is known as the right-of-way book.

Maintenance-of-way right-of-way maps are made from the location map; and a separate sheet, about $8\frac{1}{2}$ by 10 in., is used for each section (that is, Government section) or for each mile of track. These sheets are bound into a book and are used by the maintenance-of-way department.

Railroad Construction Maps

59. Station Maps. These maps are usually made to a scale of 50 to 100 ft. to the inch and show the proposed buildings and tracks at each station. Any changes made in the plans during construction are recorded on the map. Some railroads have standard plans for station layouts, and these are followed as closely as conditions will permit. The station maps are accompanied by the plans and details of the buildings and other structures required in the station layout.

60. Progress Profiles. Each month, after the monthly estimate surveys have been made, the amount of grading done during the month is shown graphically on the location profile in colored pencil. Each month a different color is used, so that the progress of the construction work can be seen at a glance, and the amount done each month compared with that of preceding months.

61. Progress Photographs. For the same reasons, photographs should be taken each month, or oftener, of the various structures under construction, such as arches, bridges, trestles, station buildings, etc. Photographs are excellent auxiliaries to a written report.

Highway Reconnoissance Maps

62. Reconnoissance Maps. When the highway is to be located through new territory, the reconnoissance map is practically identical with the railroad reconnoissance map.

When the highway consists principally of the improvement of an existing road, the reconnoissance map frequently takes the form of the *strip map*. This is a map on a scale of about one inch to the mile, and shows a strip of land along the proposed route from 4 to 6 miles in width, or enough to show the several alternate routes under consideration. On this map are indicated the various routes, the one which seems most desirable being most prominently shown. Notations of the items recorded on the reconnoissance are made on the map. Profiles may or may not be platted from the reconnoissance notes as the needs appear to indicate.

Highway Preliminary Maps

63. Preliminary Maps. When the proposed highway is through new territory, the preliminary map becomes the exact counterpart of the railroad preliminary map. The same data are platted, and

the line is projected in the same manner, taking into consideration the different limits placed on grades and curvature. One difference, however, is that the cross-sections are platted to scale and are used in conjunction with the profile in establishing the grade line.

When the road is essentially the improvement of an existing highway; the preliminary map becomes a study sheet for the working out of the construction plans. The data taken on the preliminary survey are platted to the scale of the finished plans, which is usually 100 ft. to the inch. A profile with the same horizontal scale and a suitable vertical scale for the given territory is also drawn. The cross-sections are platted on separate sheets to suitable scales, usually 10 ft. to the inch both ways.

The horizontal alinement and the grade line are then worked out as before, and the necessary changes in right of way, etc., are determined and shown on the map.

Highway Construction Plans

64. Construction Plans. A location map is rarely made for a highway, unless the location is for a considerable distance through new territory. In general, the location and construction drawings are combined on a series of plan-profile sheets more nearly corresponding to the location profile of the railroad.

To facilitate the administration of the Federal Aid Law, the Bureau of Public Roads devised the so-called Federal Aid Sheets and required their use on all federal-aid projects. These sheets have been almost universally adopted for highway construction drawings. The sheets are about 24 by 36 in. in outside dimensions and are ruled complete on each sheet. Spaces are provided for identifications, while the main ruling is made in three styles as follows:

1. Half-plan-half-profile sheets, on which the upper half is blank to receive the plan and the lower half is ruled for profiles. The horizontal divisions are one-half inch and the vertical divisions are one-tenth inch.

2. Quarter-plan-quarter-profile sheets, on which the upper quarter is plain, the next quarter below is profile-ruled, the next quarter is plain, and the lowest quarter is profile-ruled. This sheet is used for straight alinement where grades are light, as it permits twice as much line to be shown on a single sheet.

3. Cross-section sheets, which are ruled to one-tenth inch in both directions.

The detailed horizontal alinement, together with all of the features needed on a general construction plan, are shown in the plan areas. The profile shows the grade line, earthwork distribution and balance points, and the other features as described for the railroad location profile, except that the distribution diagram is not shown. If this is used, it is made on separate sheets.

In addition to the plan-profile sheets, cross-section sheets, details of structures, etc., must be made. A complete set of road plans will thus consist essentially of the following:

1. A cover sheet showing the general location and title of the project and certain general data.
2. A series of plan-profile sheets as described.
3. Sheets showing the standard cross-sections of the completed roadway.
4. Detailed drawings of bridges, culverts, drainage appurtenances, and other structures.
5. The cross-section sheets used in determining the earthwork.

During construction, progress sheets showing the work completed each day, week, or month are made. These are similar to the railroad progress profiles, but may also include a progress plan. Progress photographs are as valuable on highway work as on railroad work.

Street Plans and Profiles

65. Plans and Profiles. Since no reconnoissance is made, there is no reconnoissance map on street work.

The preliminary map is merely a study sheet for the establishment of the horizontal plan and the grade line, and is rarely preserved. Since most pavement work is paid for by special assessment, it is usually necessary to file the plans for record along with the ordinance providing for the work. Only the finished drawings are filed.

The completed set of street pavement plans is therefore very similar to the highway plans, and consists of the plan-profile sheets, the details of drainage appurtenances, curbs, etc. Cross-section sheets are usually not included, and the cover sheet is often omitted, except on very large projects.

Where sewers and other underground work, sidewalks, etc., are included in the project, their positions are shown on the plan, and

their grades are shown on the profile. Separate projects for sewers, sidewalks, etc., are similar to the foregoing.

Highway and Street Right-of-Way Maps

66. Right-of-Way Maps. The title to street and highway right of way is vested in the public. The descriptions are therefore required to be filed for public record, as provided by law in each locality. Sometimes the records are mere descriptions, but more frequently plats of greater or less completeness are included. Since a public record is required, dependence is made upon it, and local road and street authorities are often very lax in keeping records of their own. Moreover, many of the records filed are very poor, consequently highway and street rights of way are frequently in doubt.

Existing rights of way have been provided by dedication, purchase, or condemnation. New roads and streets are opened, or additional right of way is provided in the same way. The records should be full and complete, and each local authority having jurisdiction would do well to make records similar to the right-of-way maps of the railroads. These should be in duplicate, one set filed as required by law and the other set kept for local record.

Subdivision Plats

67. Subdivision Plats. When a new subdivision or addition to a city is laid out, a map of the tract of land showing the lots, blocks, street, alleys, etc., must be prepared and filed with the proper authorities. It then becomes the official record and the basis of the legal descriptions of the various subdivisions of land contained in it.

Extreme care must therefore be taken to make the plat complete and accurate. All dimensions should be carefully checked to see that no errors creep in and that there are no ambiguities in the layout or in the dimensions. It is a rule of land surveying that distances take precedence over angles. Distances should, therefore, be provided wherever possible, but, in case angles are desired to govern, distances which might supplant them should be omitted.

All Government or other existing land lines should be shown, and any monuments on them which control the subdivision should be carefully noted and, if possible, referenced so as to be relocated

easily, if they should happen to be disturbed. New monuments, established as part of the subdivision, should be accurately located, described, and referenced.

In general, contours are not placed on the record plat. A separate map, made on the legal plat as a base, should be used for showing the topography. A copy, however, should be filed with the city engineer, as it will prove of great service in planning pavements, sewers, etc.

Canal, Pipe-line, and Transmission-line Maps

68. Miscellaneous Maps. Since the general scheme of the location, construction, and maintenance of canals, pipe lines, transmission lines, etc., is very similar to that of a railroad, the maps and other drawings required are the counterpart of those required by the railroad, keeping in mind the peculiarities of the particular type of route under consideration.

Reconnoissance, preliminary, location, construction, progress, and maintenance maps and profiles will usually be required as well as plans and details of special structures. These drawings should be clear and well executed and made to scales suitable for the particular work.

CHAPTER 3

DISTANCE, CURVATURE, AND GRADES

THERE are three principal factors which the engineer must consider in the location of any route of transportation, if the project is to be an economic success. These factors are *distance*, *curvature*, and *grades*, and their importance is due to their influence on (1) the cost of construction, (2) the cost of maintenance, and (3) the cost of operation. It must be remembered that the first cost is a fixed amount, whereas the costs of maintenance and operation continue as long as the route exists, and consequently the additional cost of operating and maintaining a poorly located line will in time offset any saving that may have been effected in the cost of construction.

These factors have certain normal characteristics which are briefly discussed in the following sections. It must be remembered, however, that special conditions may modify or displace the basic characteristics for the time being. Such special conditions must receive careful consideration, but the fundamental ideas must not be overlooked.

Distance

Other things being equal, the ideal alinement would be straight between controlling points. Such an alinement rarely exists, because in some cases it may be actually impossible to build a straight line, and in most cases the first cost will probably be prohibitive, due mainly to excessive earthwork, unfavorable bridge sites, expensive right of way, etc. It is normally necessary, therefore, to deflect from the direct line and by so doing introduce additional distance. Again, the grade of a direct line may be too steep for economical operation, and this can be corrected only by the process of *development*, or the introduction of extra distance in order to reduce the grade. Furthermore, under certain conditions, the amount of grading may be so reduced by increasing the distance that the longer line will actually be cheaper to build than the shorter line. The problem, therefore, is to find a line on which the saving in the cost of construction, due to less grading or more favorable special

features or the saving in operating cost on a flatter grade, more than compensates for the increased cost of construction, maintenance, and operation due to the greater length of route.

69. Construction. Exclusive of special features, such as large bridges, tunnels, etc., the cost of construction of any route is practically proportional to its length. Normally, therefore, any additional distance adds proportionally to the cost of construction. Of course any unusual item introduced by the additional distance must receive due consideration.

70. Maintenance. In general, the maintenance costs of a route are also directly proportional to the length, and this view is sufficiently accurate for the locating engineer when considering the advisability of introducing additional distance.

71. Operation. Operating costs also increase with the length, but the increase may not be directly proportional to the increased distance. This is especially true on railroads where the cost of operating small additional distances is only about one-third as much per train-mile as the average train-mile cost over an entire engine division. The cost of the additional distance then gradually increases until it reaches the average cost, which occurs when the additional distance is still well under the length of an engine division. The present tendency to increase the length of engine divisions also tends to decrease the average train-mile costs and at the same time tends to reduce the distance in which the cost of additional distance equals the average cost. Commercial canals show similar characteristics in the relation between operating costs and distance.

It has been found that the average cost of operating individual motor vehicles on highways is practically proportional to the distance run. Although this fact may not be exactly true for traffic taken collectively, it is sufficiently accurate for ordinary use and is generally so taken. The same fact is essentially true for pipe and transmission lines.

The increased cost of operation due to added distance should not be confused with, or neglected on account of, a very real saving in operating costs which may result from decreased grades obtained by development. In fact the economic limit of development is reached when the added costs due to increased distance are just equal to the decreased costs due to flatter grades.

72. Revenue. Distance is sometimes closely related to the revenue to be obtained from commercial projects such as railroads, or to the service to be rendered by public projects such as highways. For

example, it may be found that, by deflecting from the direct line, additional traffic and hence additional revenue can be obtained by the railroad, or that additional population can be served by the highway. The locating engineer must then balance the increased cost of construction, plus the perpetual costs of maintaining and operating the additional distance, against the probable increase in revenue or the improved public service. This often involves the question as to whether the main line should be deflected through the new point, or whether the main line should be made direct and the desired point reached by a branch. This must be studied as a local problem. In general, trunk highways are finding it more economical to keep the main line direct and to use a branch, even for short distances, rather than to deflect the main route. This may be somewhat less true of railroads.

Curvature

Aside from the fact that the presence of curvature indicates deviation from a direct line, with the introduction of additional distance, curvature itself affects the three major items of cost.

73. Construction. Curvature adds to the cost of construction on practically all routes. Due to superelevation and to drainage requirements, both railroads and highways are more difficult to construct on curves than on tangents. On railroads, too, it may be necessary to compensate grades for curvature as explained in section 77, and this may add indirectly to the cost of construction by adding to the earthwork. Transmission lines must make provision to balance side pull on curves and this adds to the cost. Canals and pipe lines may, however, show little or no increased cost of construction on curves.

74. Maintenance. Railroad track is more difficult to maintain to line and surface on curves than on tangents, and there is also increased wear on rails, ties, and ballast, all adding to the costs of maintenance. Records seem to show that highway curves are only slightly more expensive to maintain than tangents when the surface is paved, but when the surface is of gravel or stone the maintenance cost on curves is high. Erosion of the banks on curves adds to the maintenance costs of drainage canals. Barge canals may show no increase, because of the low velocity of flow. Pipe lines also show little increase in maintenance due to curvature, whereas transmission lines may have considerable increase, because of the special provisions required by side pull. This may be reduced

in transmission lines by making abrupt angles in the alinement instead of regular curves.

75. Operation. Curvature appears to have little or no effect on the cost of operating canals and transmission lines, nor on pipe lines when the changes of direction are made by gradual curves and not by abrupt bends.

So far it has not been possible to determine whether curves add directly to the cost of operating highways or not, and therefore it is usual to assume that they do not. Curvature, especially if sharp, may so reduce speed as to limit the capacity of the road, cause traffic congestion or add to the hazards of traveling. Although these conditions can not be evaluated in money they are important items and must receive due consideration by the locating engineer.

On railroads, however, it has been demonstrated that curvature adds materially to the cost of operation, due especially to increased resistance to train movements. The effect of this will be briefly discussed in the following sections.

76. Railroad Curve Resistance. The wheels on engines and cars are rigidly attached to the axles. Consequently, in rounding curves both wheels must make the same number of revolutions, although they travel different distances, because of the difference in length of the inner and the outer rails. This results in a certain amount of slippage of the wheels, which adds to the tractive resistance. Apparently this resistance does not increase directly with the degree of curve, but the rate of increase becomes less as the curves become sharper. In addition to the foregoing, if the train does not run at the speed for which the curve is superelevated, there is an additional pressure of the flanges on the rails, and this adds to the tractive resistance. The variation in resistance with speed is not large and is generally neglected.

Values of the curve resistance have been determined experimentally and the following practices have been adopted. For curves up to about 8°, the curve resistance is taken as about 0.8 lb. per ton, or the equivalent of the resistance of a 0.04% grade, for each degree of curvature. For curves between 8° and 15°, the resistance is taken as about 0.6 lb. per ton, or 0.03% grade, for each degree of curve. For curves above 15°, the values are 0.4 lb. per ton, or 0.02% grade, for each degree of curve. The resistance at starting is about twice these values, and this should be considered, if stops are to be required on curves.

77. Compensated Grades. From the standpoint of operation, the effect of a curve is to increase an ascending grade on which it occurs

by the amount of the curvature resistance. Thus a 6° curve on a level grade would be equivalent to a grade of $6 \times 0.04 = +0.24\%$ on straight track; and on a $+1.00\%$ grade it would be equivalent to a $+1.24\%$ grade on straight track.

It is therefore evident that if a maximum operating grade has been chosen, the actual grade must be reduced by the amount of the curve resistance. This is termed *compensating for curvature*, and the grade is spoken of as a *compensated grade*.

For example, if a maximum operating grade of 2.00% is chosen and a 10° curve occurs on it, the actual grade must be $2.00 - (10 \times 0.03) = +1.70\%$, and this is the grade to be established on the profile over the length of the curve. This grade would be termed a "2% compensated grade."

On descents, the effect of curvature is to reduce the slope numerically. Thus for descents, the operating grade in the above example would be $-1.70 + 0.30 = -1.40\%$. This condition, however, is normally favorable to traffic and is usually not considered.

78. Choice of Curvature. Curves may limit the speed of traffic on railroads and highways. Formerly only passenger trains were considered as being affected by this limitation of speed. At present, however, freight trains are being operated at express speeds and freight service is therefore demanding as easy curves as passenger service. On highways the increasing speed of motor vehicles, both passenger and freight, with the resulting necessity of greater sight distances and easier control of the vehicle, is also demanding flat curves.

The total resistance to train movement is directly proportional to the total change of direction and varies only slightly, as previously shown, with the degree of curve. Hence the important thing on railroad location is to keep the total curvature (sum of the central angles, I) as low as possible. In the large amount of railroad realinement work which has been done, decreased total curvature has always been a primary object, with increased radius a secondary consideration.

The total central angle is of little importance on highways as far as cost of operation is concerned. But, because of the need for ample sight distances and ease and safety in driving, a minimum of curvature, combined with flat curves, is highly desirable. Consequently, on both new highways and the relocation of old ones, increasing attention is being given to reducing the total curvature to a minimum.

Total central angle appears to be of little moment on canals, pipe lines, and transmission lines. Flat curves are desirable on all of

these, except transmission lines, where often the entire change of direction can be made more economically at one or two special towers or poles, which are designed to take the side pull resulting from the tension in the wires.

Railroad Grades

In flat, prairie country grades are so small that they play little or no part in the choice of location, but in rough country grade is the most important of the three factors which affect the choice of route for a proposed railroad, and is the one which requires the most study by the locating engineer. The effect of grade upon the costs of construction, maintenance, and operation will be discussed separately, as in the case of the preceding factors.

79. Construction. The cost of construction is directly affected by the grades adopted. If steep grades are used, the cost of construction may be no more than would be the case in more level country with flatter grades. On the other hand, flat grades in rough country mean high fills and deep cuts, which increase the cost of construction tremendously. The question before the locating engineer in comparing two routes, or in comparing two sets of grades on the same alinement, is whether or not the greater cost of grading on the line with the flatter grades is justified by the saving in maintenance and operation expense. This is purely a question of economics. The difference in the cost of grading is easily computed, and the saving in maintenance and operating costs can be estimated with a resonable degree of accuracy, as explained in subsequent sections.

80. Maintenance. The maintenance items affected by grade are renewal of ties and rails and increased depreciation of locomotives and cars. It is found that the life of ties is somewhat less on steep grades than on level grades; also, that the wear on rails is greater, due to the slipping of wheels, to the use of sand to obtain traction, and to the greater number of engine-miles required because of the larger number of trains necessary to haul the same total tonnage. The wear on the drivers, car wheels, brake shoes, and drawbars is also greater on grades than on level track. The increased cost of maintenance, however, is not large. Unless the grades are steep enough to require the use of brakes in descending, the additional cost is practically negligible. Where brakes are used, Wellington estimated the additional cost on the items mentioned to be about 5%.

81. Operation. The cost of operating a railroad with steep grades is very great. The subject of locomotive performance is so

TABLE 1 *

TRAIN RESISTANCE—POUNDS PER TON

Speed, miles per hour	Column Headings Indicate the Average Weights Per Car												
	15 tons	20 tons	25 tons	30 tons	35 tons	40 tons	45 tons	50 tons	55 tons	60 tons	65 tons	70 tons	75 tons
6	7.7	6.9	6.1	5.5	4.9	4.4	4.1	3.8	3.5	3.3	3.2	3.1	3.0
8	8.0	7.1	6.3	5.6	5.0	4.6	4.2	3.9	3.6	3.4	3.3	3.2	3.1
10	8.2	7.3	6.5	5.8	5.2	4.7	4.3	4.0	3.7	3.5	3.3	3.2	3.2
12	8.4	7.5	6.7	6.0	5.4	4.8	4.4	4.0	3.8	3.6	3.4	3.3	3.3
14	8.7	7.8	6.9	6.2	5.5	5.0	4.5	4.2	3.9	3.7	3.5	3.4	3.4
16	9.0	8.0	7.1	6.4	5.7	5.1	4.7	4.3	4.0	3.8	3.6	3.5	3.5
18	9.3	8.3	7.4	6.6	5.9	5.3	4.8	4.5	4.1	3.9	3.7	3.7	3.6
20	9.6	8.5	7.6	6.8	6.1	5.5	5.0	4.6	4.3	4.0	3.9	3.8	3.7
22	9.9	8.8	7.9	7.0	6.3	5.7	5.2	4.8	4.4	4.2	4.0	3.9	3.8
24	10.2	9.1	8.1	7.3	6.6	5.9	5.4	4.9	4.6	4.3	4.2	4.1	4.0
26	10.5	9.4	8.4	7.5	6.8	6.1	5.6	5.1	4.8	4.5	4.3	4.2	4.1
28	10.9	9.7	8.7	7.8	7.0	6.3	5.8	5.3	4.9	4.7	4.5	4.4	4.3
30	11.3	10.0	9.0	8.0	7.3	6.6	6.0	5.5	5.1	4.9	4.7	4.5	4.5
32	11.6	10.4	9.3	8.3	7.5	6.8	6.2	5.8	5.3	5.0	4.9	4.7	4.6
34	12.0	10.7	9.6	8.6	7.8	7.1	6.5	6.0	5.5	5.3	5.1	4.9	4.8
36	12.5	11.1	9.9	8.9	8.0	7.4	6.7	6.2	5.8	5.5	5.3	5.1	5.0
38	12.9	11.4	10.2	9.2	8.3	7.6	7.0	6.5	6.0	5.7	5.5	5.3	5.2
40	13.4	11.8	10.6	9.5	8.6	7.9	7.3	6.8	6.3	6.0	5.7	5.6	5.5

* From Bulletin 43, Engineering Experiment Station, University of Illinois; Freight Train Resistance, Its relation to Car Weights, by Edward C. Schmidt.

closely related to the cost of operating grades that it must be considered before the cost of operation can be discussed further. The resistances to be overcome by the power of the locomotive are tractive resistance, grade resistance, and curvature resistance.

82. Tractive Resistance. The resistances to be overcome on straight level track are (1) inertia resistance, or the resistance to be overcome in starting or in accelerating a train; (2) rolling resistance, caused by the friction of the wheels against the rails; (3) journal friction, caused by the friction of the axles in the journals; (4) atmospheric resistance, caused by movement through a still atmosphere; and (5) oscillation resistance, due to uneven track. It is not considered necessary to discuss each of these forms of resistance separately, but all of them can be combined into a single item which may be termed *tractive resistance.* In Table 1 are given the total tractive resistances on level track for speeds from 6 to 40 miles an hour and for trains of loaded cars weighing from 15 to 75 tons each.

The resistance caused by head winds and wet, slippery rails are not included in the values given.

83. Grade Resistance. The grade resistance can be determined with mathematical exactness and is 20 lb. per ton for each 1% of grade. Thus it is seen that the resistance offered by a 1% grade is about five times the resistance on a level track, and the importance of grade resistance is obvious.

84. Curve Resistance. If a curve comes on a grade, there is an additional resistance to overcome, unless the grade is compensated for curvature. The subject of compensation for curvature and curve resistance have already been considered in sections 76 and 77, and therefore a discussion here is unnecessary, further than to say that, if the grade is not compensated, the curvature resistance must be added to the tractive resistance and the grade resistance to obtain the total resistance to be overcome by the locomotive in hauling a train up the grade.

85. Locomotive Performance.—The tractive effort of a locomotive depends on the weight on the drivers and the coefficient of friction between the driving wheels and the rails. The coefficient of friction varies from about 0.20 to 0.35. For example, suppose that the weight on the driving wheels is 216,000 lb. and that the coefficient of friction is 0.25; then the tractive effort of the locomotive is 54,000 lb. If the total resistance of the train, including the engine and tender, is more than 54,000 lb., the drivers will spin on the rails and the train will be stalled. This is assuming of course that the cylinder power of the locomotive is greater than the tractive effort, and in a properly designed locomotive this is always the case when the train is moving at speeds below 10 miles an hour. At higher speeds a locomotive can not make steam fast enough to supply the cylinders with steam at full boiler pressure, and the tractive effort therefore decreases. As the speed increases, the tractive effort continues to decrease until it is just sufficient to overcome the tractive resistance, and the maximum speed of the train is reached. Thus at speeds over 10 miles an hour, speed also becomes a factor in tractive effort. When climbing the steepest grade on a division, however, the maximum train speed will normally be about 10 miles an hour and at this speed the full tractive effort is available. Hence, in studying the effect of grade upon operation, the full tractive effort is used. Table 2 gives the engine data for five types of engines used by the Illinois Central Railroad. The values in this table can be used by the student in any grade problem assigned by the instructor.

TABLE 2

LOCOMOTIVE DATA, ILLINOIS CENTRAL RAILROAD

Class	Type	Service	Weight, Thousands of Pounds				Coal tons	Water gals.
			On drivers	On engine	On tender	On total		
1200	4–6–2	Pass.	169.5	280.0	182.2	462.2	15	9,000
1700	2–8–2	Frt.	218.3	282.7	167.5	450.2	15	9,000
2400	4–8–2	Pass.	243.5	362.5	204.0	566.5	18	10,000
3000	2–10–2	Frt.	296.5	382.0	208.6	590.6	16	12,000
7000	2–8–4*	Frt.	248.0	388.0	286.0	674.0	20	15,000

Class	Type name	Areas, Square Feet			Length, Feet		Fired	Steam pressure lb./sq.in.
		Grate	Heating	Superheating	Over couplers	Wheel base		
1200	Pacific	55.0	3625	826	.....	.....	Hand	190
1700	Mikado	70.4	4102	1099	77.12	65.79	Hand	175
2400	Mountain	75.4	4643	1227	88.85	77.71	Stoker	200
3000	Central	88.3	5165	1285	93.90	82.83	Stoker	190
7000	Lima	100.0	5164	2111	82.48	.....	Stoker	240

Class	Cylinders, in.	Diameter of drivers, in	Friction Coef. calculated	Tractive Effort, Miles per Hour				
				0–10	20	30	40	70
1200	26 × 28	75	0.24	40,760	32,000	25,000	20,000	11,000
1700	27 × 30	63	0.24	51,600	38,600	28,000	21,000	
2400	28 × 28	73	0.21	51,100	42,800	33,000	26,000	15,000
3000	30 × 32	63	0.25	73,900	52,000	38,000	28,000	
7000	28 × 30	63	0.28	69,400†	52,000	38,000	28,500	16,000

* Booster engine on trailing truck which is cut out at about 12 miles per hour.
† 81,400 with booster.

86. Weight of Train. The practical application of the foregoing discussion on the relation of grade, tractive, and curve resistance to locomotive performance is to determine the weight of train which a given type of locomotive can haul up a given grade at a given speed.

Since the engine must move itself against grade, tractive, and rolling resistance, the net effort available to move the train, called

the *drawbar pull*, is equal to the tractive effort of the engine minus the total resistance of the engine and tender. The drawbar pull, divided by the total unit resistance of the train, will give the weight of train. This may be reduced to the form of an equation as follows:

$$W = \frac{T - (R_e + 20G + C)w}{R_t + 20G + C} \tag{1}$$

in which, W = weight of train in tons, exclusive of engine and tender.

w = weight of engine and tender in tons.

T = tractive effort of engine in pounds at the given speed.

R_e = tractive resistance of engine and tender in pounds per ton at the given speed.

R_t = tractive resistance of given type of train in pounds per ton at the given speed.

C = curve resistance in pounds per ton, taken as the same for both engine and train.

G = grade in per cent.

If it is desired to find the grade up which a given locomotive can haul a train of given weight at a given speed, Eq. 1 can be changed to the form,

$$G = \frac{T - [R_e w + R_t W + C(W + w)]}{20(W + w)} \tag{2}$$

Sometimes it is desired to know the tractive effort required to haul a given train up a given grade at a given speed. Since the type, and hence the weight, of engine is unknown, the first step is to determine the drawbar pull which will be required. This can be obtained from Eq. 1, since the numerator of the right-hand member of that equation is the drawbar pull. From this value an engine may be selected from those available, or this requirement may be made the basis of the design of a locomotive with sufficient tractive effort to supply the required drawbar pull under the given conditions.

87. Illustrative Examples. *Problem* 1. What weight of train of 60-ton cars can be hauled up a straight 0.8% grade at a speed of 10 miles an hour by the Mikado locomotive whose data are given in Table 2?

Substituting in Eq. 1,

$$W = \frac{51{,}600 - (3.5 + 20 \times 0.8)225.1}{3.5 + 20 \times 0.8} = 2421 \text{ tons}$$

Problem 2. How many trains will it take to haul a gross daily tonnage each way of 24,000 tons over a division which has maximum grades of 0.8%, using the Mikado locomotive of Problem 1? How

much must the grade be reduced to haul the same tonnage with one less train a day?

It was found in Problem 1 that the Mikado locomotive could haul in one train a load of 2421 tons. It will require, therefore, ten trains each way to handle the traffic.

If nine trains are used instead of ten, the weight of each train must be $24{,}000 \div 9 = 2667$ tons. Substituting this value in Eq. 2, we have,

$$G = \frac{51{,}600 - 3.5(225.1 + 2667)}{20(225.1 + 2667)} = 0.717\%$$

Therefore, if the 0.8% grade were reduced to 0.717%, the traffic could be handled with one less train each way.

88. Electric Locomotives. Electric traction has been developing rapidly since about 1915, especially where water power was available and fuel scarce, on heavy mountain grades, or in large city terminals and long tunnels where smoke abatement was imperative. Suburban and local interurban service is usually provided by the familiar electric motor car, but heavy electric locomotives have been designed for freight service and for hauling standard passenger trains over electrified divisions.

Electric locomotives at present are far from standardized, as are steam locomotives. New types and weights are constantly being developed. In general, however, all of them belong to one of the following types:

1. *Locomotives equipped with variable speed, direct current motors.* The current supply may be direct current, or single-phase, alternating current may be used on the trolley with rotary converters on the engines to supply direct current to the motors. The maximum tractive effort can be developed up to speeds of 20 to 25 miles an hour, which permits rapid acceleration on starting and also high sustained speeds on heavy grades. At higher speeds the tractive effort may decrease rapidly, and the maximum speed of the electric locomotive is generally less than that of the steam locomotive.

2. *Locomotives equipped with single-phase, alternating current, variable speed motors.* The trolley usually carries a relatively high voltage with transformers on the locomotives to supply suitable voltage to the motors. The operating characteristics of this type are similar to the first type.

3. *Locomotives equipped with three-phase, alternating current, induction motors.* The motors are supplied with suitable current by phase-converters on the locomotives, which in turn are supplied with high voltage, single-phase current from the trolley. These

are essentially constant speed motors and, aside from suitable provisions for starting, normally operate at one or two predetermined running speeds. The maximum speed is relatively low, but the tractive effort is high and well sustained at the running speeds.

Any of these types may be fitted for "regenerative braking," which consists of arrangements whereby the motors act as generators on descents, or when the train is decelerating, and feed power back to the line to assist in moving other trains. This feature is especially of value on mountain divisions where one train on a descent may be kept to a safe speed and actually aid another train to ascend the same or another grade. The economic saving in power by this device may be quite high.

The train which one engine can handle on a given grade, or the grade up which a given train can be hauled, is computed in exactly the same way for the electric locomotive as for the steam locomotive when the characteristics of the engine are known.

4. *Diesel-Electric Locomotives.* During recent years the Diesel-electric locomotive has been developed rapidly, especially for fast passenger service as typified by the so-called "streamliners." The essential characteristic of such locomotives is an oil engine, generally of the Diesel type, connected directly to an electric generator. Power from the generator is then used to drive the engine through motors of the usual type. The controls are so arranged that as power is required the throttle of the oil engine is automatically regulated to the load. The operating characteristics of such locomotives are essentially the same as those of the regular electric locomotive.

Engines of this type in small sizes have been used for several years in switching and other local service. Recently, however, large locomotives, comparable to the largest steam passenger engines, and capable of handling a 15-car train at speeds above 100 miles per hour, have been put into service. An innovation in this field is the steam-turbine-electric locomotive of the Union Pacific Railroad in which the oil engine is replaced by a steam turbine with very high-pressure boiler, fired with oil, and the used steam condensed in air-cooled condensers. This is a heavy, fast engine for passenger service.

Experiments are also being made with a "fluid transmission" which would enable the drive wheels to be driven directly from the oil engine. If this device proves successful, it will probably open a field for light, fast, local passenger service.

89. Examples. The first type described in the preceding section is well represented by the heavy passenger locomotives used on the C. M. St. P. & P. Ry. in the Cascade Mountains. These engines

have twelve driving axles with 1000-volt, gearless, direct current motors of about 3400 horsepower. The trolley line carries 3000-volt, direct current. The total weight is 521,000 lb., of which 457,600 lb. are on the drivers. The maximum tractive effort is about 137,000 lb. At 20 miles an hour it is 105,000 lb., at 30 miles an hour it is about 42,000 lb., but at the designed maximum speed of 65 miles an hour it drops to about 10,000 lb. Regenerative braking is used.

The second type is represented by the Pennsylvania Railroad's passenger locomotive having eight driving wheels. The trolley current is 11,000-volt, single-phase, alternating current which is stepped down to 600 volts for the motors. The total weight is 408,000 lb. with 308,000 lb. on the drivers. The maximum tractive effort is 92,400 lb., and the maximum speed is 70 miles an hour.

The third type is best represented by the monster engines of the Virginian Railway, a coal-hauling road with heavy grades and no passenger service. These engines are over 150 ft. in length and weigh 1,284,000 lb. with 922,600 lb. on the twenty-four drivers. The trolley line carries 11,000-volt, single-phase, alternating current. The motors are three-phase induction motors arranged for normal operating speeds of 14 and 28 miles an hour, and have a continuous rating of 6000 hp. The maximum tractive effort is 277,500 lb., dropping to 162,000 lb. on one-hour rating at 14 miles an hour and to 78,800 lb. on continuous rating at 28 miles an hour. Regenerative braking is provided.

90. Economics of Grade Reduction. From the preceding discussion, the importance of grades upon operation is quite evident. The problem of the locating engineer is to estimate the saving in operating expense which will result from a reduction in grade. Suppose that the ruling grade on one route is such that ten daily trains each way are required to handle the traffic; whereas on the other route being compared with the first only nine trains will be required. What is the saving in operating cost caused by the elimination of one train, if the engine division is 100 miles in length? Assume that the average cost of one train-mile is $3.50. Then, the cost of operating one train for one day over this division would be $350. The elimination of one train, however, does not result in a daily saving of $350, since the overhead expenses are not reduced and the cost of operating the remaining nine trains will be increased, due to the heavier trains. Railroad economists have estimated the saving at about 40%. The daily saving, therefore, will be

$350 \times 0.40 = $140 for one train. For one train each way for 365 days, the saving would then be $140 \times 2 \times 365 = $102,200. This annual saving capitalized at 5% amounts to $2,044,000; and this is the maximum amount which could be justifiably spent in grade reduction. If it is desired to pay off this indebtedness in a series of years, then the amount which can be expended is the present value of an annuity equal to the annual saving at the given rate of interest for the desired term of years. This is given by the equation,

$$P = S\left(\frac{1-(1+r)^{-N}}{r}\right) \tag{3}$$

in which P is the present value of the annuity, S, for N years and r is the current rate of interest.

91. Minor Grades. Minor grades are those which do not limit the weight of train which can be hauled over the line by one locomotive. Such grades are of three kinds, according to their effect on the performance of the locomotive and on the consequent cost of operation.

First, there are those grades whose drop (vertical height) is sufficiently small that a locomotive can operate them without shutting off steam, or, in other words, the locomotive will not reach a dangerous speed due to the down-grade while exerting a continuous pull on the train. The effect of these grades on train operation is negligible, both as regards the effort required of the locomotive and the time required for the train to traverse a given distance, since the kinetic energy acquired in descending one grade is utilized in ascending the opposite grade.

The second kind of minor grades consists of those whose drop is so great that steam must be shut off to prevent the train from acquiring a dangerous velocity, and hence the effort of the locomotive must be increased on the opposite up-grade. Therefore, there is a loss of power, since the locomotive is working intermittently and not at its greatest efficiency.

The third kind of minor grades consists of those which require the use of brakes in descending them. This causes an enormous loss of energy because part of the kinetic energy gained by the descent is absorbed by the brakes, and hence is not available on the next up-grade. Furthermore, the locomotive is working at a disadvantage, and there is considerable wear on equipment due to the action of the brakes.

92. Ruling Grades. Ruling grades are those which limit the weight of the train which can be hauled by one locomotive. Cars can not be picked up or dropped off along the line to make up a train in accordance with the grades met with, and hence a train must run through from one terminal of a division to the other. Therefore, excluding pusher grades, and such short steep grades as can always be operated by momentum, the maximum grade is the ruling grade on that division.

93. Momentum Grades. Grades which are steeper than the ruling grade, but which are so short that all of the momentum acquired on the preceding down-grade is not dissipated before the top of the grade is reached, are called momentum grades. Railroad engineers are not in accord as to the advisability of using such grades. Some maintain that momentum grades should not be used, because should a train stop on the grade the locomotive would not have sufficient power to start the train and continue. Others hold the view that momentum grades are justified by experience, since such grades are being operated over daily throughout the country with satisfactory results.

Whether or not a proposed grade, steeper than the ruling grade, can be operated satisfactorily with the aid of momentum depends on the speed acquired at the foot of the grade, the minimum desirable speed at the top of the grade, and the length of the grade.

A train moving at a velocity of v feet per second has stored up enough kinetic energy to carry it up a grade whose total height in feet is $h = 1.05\frac{v^2}{2g}$, if friction is ignored. The equation, $h = \frac{v^2}{2g}$, is the familiar equation for velocity head. Since the rotating wheels have stored up an additional 5% of energy, the coefficient 1.05 is used for determining the velocity head of a moving train. Changing the velocity from feet per second, v, to speed, V, in miles per hour, the velocity head becomes equal to $0.0351\,V^2$. Thus if a train has a velocity of 30 miles an hour, and the locomotive is exerting just enough pull to overcome friction, it has sufficient kinetic energy to carry it through a vertical height of 31.6 ft. before it will come to a stop. If a minimum speed of V_t miles an hour is desired at the top of the grade, the total rise of the grade must not exceed $h = 0.0351(V^2_b - V^2_t)$, where V_b and V_t are the speeds at the bottom and at the top of the grade, respectively. The locomotive, however, will normally be exerting a tractive effort greater than that

necessary merely to overcome friction. This effort will vary with the speed. The train resistance also varies somewhat with the speed.

It is evident that the length of the proposed grade (steeper than the ruling grade) is the essential factor in determining whether or not the grade can be operated with the aid of momentum. This length in feet can be computed approximately from the following equation:

$$L = \frac{71(V_b^2 - V_t^2)}{R + 20G - \dfrac{T_a}{(W + w)}} \tag{4}$$

in which T_a is the average tractive effort of the locomotive, and the other symbols have the same significance as in Eq. 1. For example, let $V_b = 30$ miles an hour, $V_t = 10$ miles an hour, the gross tonnage, $W + w$, $= 3000$ tons, and $G = 1.00\%$. What is the maximum length of this grade which can be operated by the Mikado locomotive in Table 2? The tractive effort of this locomotive at 10 miles an hour is 51,600 lb., and at 30 miles an hour it is 28,000 lb. The average tractive effort T_a is, therefore, 39,800 lb. The average tractive resistance from Table 1 is 4.2 lb. per ton. The length of grade is then,

$$L = \frac{71(30^2 - 10^2)}{4.2 + 20 \times 1.0 - \dfrac{39{,}800}{3{,}000}} = 5211 \text{ ft.}$$

94. Virtual Grades and Virtual Profiles. A virtual grade is the effective value of an actual grade when operated partly by momentum. A virtual profile is one on which the virtual grades are platted instead of the actual grades. The characteristics of a virtual profile are:

1. When a train is standing, the actual profile and the virtual profile coincide.

2. When a train is accelerating, the virtual profile diverges upward from the actual profile.

3. When a train is decelerating, the two profiles approach each other.

4. When a train is moving at a uniform speed, the two profiles are parallel, and the vertical distance between them in feet, to scale, is the velocity head corresponding to the speed.

5. The vertical distance between the two profiles at any point is the velocity head corresponding to the speed at that point.

95. Pusher Grades. A pusher grade is one which is so steep that one or more extra engines are required to haul the train which one engine can handle on the remainder of the division. It will nearly always happen that some grades on a division will be considerably greater than the majority of the grades, and unless these can be reduced to the general average at a reasonable cost, or a pusher engine is used, they limit the weight of train over the division and become the ruling grades.

A pusher grade adds enormously to the cost of operation, since the auxiliary engines must be maintained, and, furthermore, they pass twice over the line for each train hauled and therefore do not operate at maximum efficiency. Obviously, therefore, a pusher grade must be considerably in excess of the ruling grade on the remainder of the division before its extra cost of operation would be justified.

96. Minimum Grades. The minimum grade which should be used depends upon the drainage of the roadbed. Across fills, there is no objection to a level grade, but through cuts there must be sufficient grade to enable the water which falls or runs into the cut to drain away quickly. If the cut is short, the roadbed grade may be level, and drainage effected by constructing the side ditches on a grade. For long cuts, however, sufficient fall can not be obtained for the ditches unless the roadbed itself is on a grade. Many railroads use a minimum grade through cuts of 0.10%, and this seems to be satisfactory.

97. Choice of Grades. The first step in projecting a grade line on a profile is to determine approximately the ruling grade. The lowest value of the ruling grade is that of a uniform slope between terminals. The maximum value depends on the type and weight of locomotives, the weight of trains, etc., and can never be exactly determined. The general practice is simply to choose a maximum ruling grade which will probably fit the territory through which the line runs. In assuming this maximum value, due consideration should be given to the direction of heaviest traffic, and easier grades secured if possible for trains in this direction.

A grade line is then laid out on the profile, keeping below this maximum, if possible, in such a way that the fills will balance the cuts and the total amount of earthwork will be kept as low as possible in order to reduce the cost of construction. The determination of the most economical grade line will require a number of trials, and each trial should be carefully studied as to its effect on opera-

tion as well as on first cost. After such a line has been laid out, it may be found that the assumed maximum grade has been exceeded at some point, and study must be given as to whether it is possible to reduce this grade, or whether it is justifiable perhaps to increase it sufficiently to be operated as a pusher grade. Or it may be found that only one grade approaches the chosen maximum, and again the line must be studied to determine whether it is possible economically to reduce this grade to the next lower. For example, the maximum grade on a division is found to be 1.0%, and there are no others over 0.8%, and possibly only two of these, the next lower being 0.5%. The problem then is to decide the advisability of reducing the 1.0% grade to 0.8%, and then possibly reducing the three 0.8% grades to 0.5%. This process of reducing the ruling grade should be continued until the additional cost of construction equals the sum which can profitably be spent for the purpose of reducing the ruling grade. In these studies consideration must be given to the question of suitable operating speeds in order that a satisfactory schedule can be maintained over the division.

On a given alinement it may be found impossible to reduce the grade to a suitable value at a reasonable cost. In this case it will be desirable to consider a change in the alinement, perhaps to the extent of adding distance to secure a flatter grade. The increased cost of the longer line and the added cost of operating the additional distance must be balanced against the decreased operating costs of the flatter grade.

Highway and Street Grades

In a general way the effect of grades on highway traffic is the same as on railroad traffic. Some differences, however, are found, due principally to (1) the difference in type of motive power and the fact that each vehicle has its own power plant, (2) the difference in the coefficient of friction between the wheels and the roadway, (3) the difference in tractive resistance on the roadway, (4) the difference in the character and cross-section of the roadway, and (5) to a less extent by the fact that the automobile is steered by the driver instead of by rails.

98. Minimum Grades. Where the cross-section is such that surface water can be drained off laterally, the grade may be level for an indefinite distance, as on long embankments. Where the water is collected by side ditches, the grade may also be level, but for a limited distance, depending on how much the ditch can be deepened in order to give it sufficient grade. Where the surface water is col-

lected by gutters they must have adequate slope. When the pavement grade is sufficient for good drainage the gutters are given the same slope. When the street grade is too flat to drain well inlets are placed at short intervals and the gutters sloped each way from them on suitable grades. This results in variable height of curb since the top of curb should follow the general street grade.

Concrete gutters can be satisfactorily built on a minimum grade of about 0.2%. Other types of gutters should have about 0.3%. Open side ditches may have minimum grades of about 0.1%, provided the width can be made adequate.

99. Maximum Grades. Each motor vehicle contains its own power plant, and by means of the gear shift the tractive effort can be greatly increased at the sacrifice of speed. It is possible, therefore, for the normal motor vehicle to climb grades greatly in excess of those desirable for use. Passenger cars on dry roads can negotiate grades up to about 25%, but loaded trucks are limited to about 15%.

Due to the fact that road surfaces may become slippery in bad weather, and to the difficulty of controlling the vehicle on descents, the conditions of safe descent, rather than the ability to climb, fix the maximum safe grades. On main highways where the traffic volume is large and the movement is fast, the maximum grade should not exceed 5%. On less important roads where there are fewer vehicles and speeds can be reduced, grades may be increased up to about 8%.

Streets are sometimes laid out on grades so steep that they are little used by vehicles. Such streets may be paved but esthetic and sanitary benefits rather than convenience to traffic are the justification for the improvement.

100. Tractive Resistance. Owing to the character of the road surface and of the vehicle tires, the tractive resistance on highways is considerably higher than on railroads. It varies also with the kind and condition of the road surface, and with the characteristics of the vehicles, and to a less extent with speed. Since all types of vehicles use the roads, only broad averages of the tractive resistance need be considered, and the effect of speed can be neglected. Table 3 gives ordinary values of the tractive resistance.

101. Grade Resistance. Grade resistance on highways is the same as on railroads, 20 lb. per ton for each per cent of grade.

102. Minor Grades. Minor grades are those which do not affect the normal movement of traffic. Satisfactory running speeds are

TABLE 3

AVERAGE TRACTIVE RESISTANCE OF DIFFERENT ROAD SURFACES TO PNEUMATIC TIRES AT NORMAL SPEEDS

Kind of Surface	Tractive Resistances Pounds per Ton		
	Poor condition	Good condition	Average condition
Untreated Earth	150–300	70	90
Oiled Earth	100	50	70
Gravel and Macadam	100	40	60
Bituminous Surfaces	75	35	45
Brick and Concrete	75	30	40

obtained at summits, excess speeds are avoided on descents, and brakes are not used to control the speed. Owing to the characteristics of the gasoline motor, such roadways may actually require less fuel than level grades, but the difference is not great and the fact does not justify the designing of undulating grades in place of level grades.

103. Momentum Grades. Momentum grades are those on which the momentum of the vehicle is used to aid in the ascent. This in fact occurs on all minor grades, but may be specifically considered on steeper grades. The maximum economic grade for motor vehicles is that which will permit them to ascend without shifting gears and within a reasonable speed range.

From mechanics it can be shown that,

$$G_a = \frac{3.51(V_b^2 - V_t^2)}{L} + \frac{T - R}{20} \qquad (5)$$

where G_a = the ascending grade in per cent.

L = the length of the grade in feet.

V_b and V_t = the speeds at the bottom and top of the grade, respectively.

T = the average tractive effort at the drive wheels in pounds per ton.

R = the tractive resistance in pounds per ton.

The constant 3.51 makes an allowance of about 5% for the inertia of the rotating parts of the vehicle in addition to the inertia of direct movement.

By assuming permissible values of V_b and V_t and of T and R, the permissible grade of a given length, or the maximum length of a given grade, can be determined. T will usually lie between 175 and 200, and values of R can be taken from Table 3.

If the grade is long, the effect of momentum is practically lost before the top is reached; or, if the vehicle is started on the grade, momentum does not act and Eq. 5 reduces to

$$G_a = \frac{T - R}{20} \tag{6}$$

This equation should be used for all grades more than about 2000 ft. long, and also where conditions do not permit the utilization of momentum.

104. Safe Descending Grades. The safe descending grade for economical operation is that which does not require the use of brakes and on which the vehicle will not reach unsafe speeds. It is given by the equation,

$$G_d = \frac{3.51}{L}(V^2_b - V^2_t) + \frac{R}{20} \tag{7}$$

in which G_d is the descending grade in per cent and the other terms are as in Eq. 5.

If the relation between G and L is greater than given by Eq. 7, brakes must be applied on the descent, and this is an economic loss.

105. Limiting Grade. Ruling grades as defined for railroads do not exist on highways, due to the characteristics of the vehicles. If, however, a grade becomes so long or so steep that it limits the number of vehicles which can traverse the road in a given time, it is analogous to the ruling grade, but more properly should be termed the limiting grade. Pusher grades, of course, do not exist on highways.

106. Establishing a Grade Line. On a rural road the laying of a grade line is exactly similar to the same process on a railroad, taking into consideration the limiting values. Tangents should be first projected and then connected with vertical curves. The curves should be checked to see that they provide ample sight distance. The line should then be reviewed and modified to give the best

grades and vertical curves and minimum earthwork. Long, easy vertical curves should be the rule, and the appearance should be carefully considered. Short adverse grades on a general ascent should be avoided. The desire to save a few yards of earthwork should not be permitted to introduce undesirable features into the grade line.

Street grades are established in much the same way, but the limiting factors of cross streets, existing improvements along the street, drainage, etc., are often more exacting. Adequate drainage and good appearance are essential factors, whereas the amount of earthwork is a negligible consideration.

Canal Grades

107. Grades. Drainage channels must have sufficient grade to carry the required amount of water and at the same time must not be steep enough to cause excessive velocities and the resulting erosion of the banks. These factors are limited by the character of the soil and by the amount of water to be carried, and are worked out from the hydraulics of open channels.

Transportation canals usually are required to use as little water as possible, and difference of elevation is secured by means of locks. The lock sites must be selected so as to give practically level grades between them.

Pipe-line Grades

108. Grades. Pipe lines for oil and water normally flow full and for a given size of pipe and quantity of flow, the velocity is constant. The resistance to flow is, therefore, that of pipe friction plus the difference in elevation of the two ends, since intermediate rise and fall is self-compensating. The length and steepness of pipe-line grades are, therefore, limited by the mechanical features of installing and maintaining the pipe. Water mains must be buried deep enough to prevent freezing, and this must be considered in establishing the grade line.

Sewer Grades

109. Grades. Sewers are essentially pipe lines operated by gravity. They are designed for maximum capacity when flowing full and for adequate velocity when the flow is the expected minimum. The grade line is, therefore, established with grades between the limits set by the hydraulic characteristics of sewers, and so as to provide the required amount of cover over the pipe.

CHAPTER 4

CIRCULAR CURVES

ALL route alinement consists of

1. Tangents, or straight sections, and
2. Curves, which unite the tangents.

Curves are of three kinds: (1) arcs of circles, (2) arcs of spirals, and (3) arcs of parabolas. The circle is employed generally for the body of horizontal curves, since it is more easily located with the transit and tape than any other form of curve. Spirals, also called easement curves or transition curves, are essentially cubic parabolas and are used at the ends of circular curves and between the branches of compound curves. Parabolas are used principally for vertical curves on grade lines and pavement crowns, but they are used occasionally for horizontal curves.

Circular curves are classified as Simple, Compound, and Reversed.

Simple Curves

110. FUNCTIONS.

A simple curve is an arc of a circle which unites two tangents differing in direction. The functions of a simple curve are shown in Fig. 2, and are defined in the following paragraphs:

Point of Intersection—*P.I.*—is the point where the two tangents intersect.

Tangent to Curve—*T.C.*—is the end of the tangent and the beginning of the curve. This point is also termed the Point of Curve, or *P.C.*

Curve to Tangent—*C.T.*—is the end of the curve and the beginning of the tangent. This point is also termed the Point of Tangent, or *P.T.*

Intersection Angle—*I*—is the *deflection angle* between the two tangents, and is equal to the angle at the center from the *T.C.* to the *C.T.*

The Radius of the curve is denoted by *R.*

Tangent Distance—T—is the distance from the $T.C.$ or the $C.T.$ to the $P.I.$ From Fig. 2,

$$T = R \tan \tfrac{1}{2}I \tag{8}$$

Long Chord—$L.C.$—is the chord from the $T.C.$ to the $C.T.$ From Fig. 2,

$$L.C. = 2R \sin \tfrac{1}{2}I \tag{9}$$

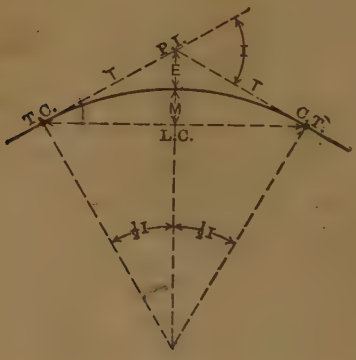

FIG. 2.

Middle Ordinate—M—is the ordinate to the curve from the middle of the Long Chord. From Fig. 2,

$$M = R - R \cos \tfrac{1}{2}I = R \operatorname{vers} \tfrac{1}{2}I \tag{10}$$

External Distance—E—is the distance from the middle of the curve to the $P.I.$ From Fig. 2,

$$E = \frac{R}{\cos \tfrac{1}{2}I} - R = R \operatorname{exsec} \tfrac{1}{2}I \tag{11}$$

Degree of Curve—D—is here defined as the angle at the center subtended by an arc of 100.007 ft., which corresponds to a radius of 5730 ft. for a central angle of 1°. Tables 10 and 13 and the methods given in this volume are based on this definition.

The definitions of degree of curve, as given by various authorities, vary slightly. Some define it as the angle at the center subtended by a 100-ft. arc, others define it as the angle at the center subtended

by a 100-ft. chord, and still others as the angle at the center subtended by a series of equal chords, totaling 100 ft., chosen so as to approximate the arc. A comparison of the results obtained by the use of these definitions is given in Section 111,

Length of Curve—*L*—is the distance along the curve between *T.C.* and *C.T.*

$$L \text{ (in stations)} = \frac{I}{D} \qquad (12)$$

Deflection Angle—$\frac{1}{2}D$—From geometry the angle between two chords, or a chord and a tangent, intersecting on the circumference of a circle is measured by one-half the intercepted arc. Therefore, for each station of 100 ft. on the curve, the angle between the chords is $\frac{1}{2}D$ and is called the *deflection angle.*

Subdeflection Angle—*d*—For a distance of less than 100 ft. on the curve, the angle between the chords is called a *subdeflection angle.*

If *C* is the distance in feet,

$$d \text{ (in degrees)} = \frac{D}{2} \times \frac{C}{100} \qquad (13)$$

and

$$d \text{ (in minutes)} = 0.3CD \qquad (14)$$

Equations 13 and 14 are exact only for the arc definition of *D*, but they are sufficiently accurate and are invariably used irrespective of how *D* is defined.

Total Deflection Angle is the angle at the *T.C.* between the tangent and a chord to any point on the curve.

Total deflection angles should be computed and recorded to the nearest 0.1 *minute, and should be turned off as accurately as the given transit will permit.*

111. Relation between R and D. If an arc definition of degree of curve is used, an exact relation between *R* and *D* can be found as follows:

The circumference of a circle in terms of its radius is $2\pi R$, and in terms of its degree of curve is length of arc times 360 divided by *D*; hence

$$R = \frac{\text{length of arc} \times 360}{2\pi D}$$

If the length of arc is taken as 100.007 ft.,

$$R = \frac{5730}{D} \qquad (15)$$

If the length of arc is taken as 100 ft.,

$$R = \frac{5729.58}{D} \tag{16}$$

From either of these equations it is evident that R varies inversely as D. Therefore, knowing the radius of a 1° curve, the radius of any other degree of curve can be found by simple proportion. By substituting R in terms of D in Eqs. 8 to 12, it is seen that for any given value of I the various functions of a curve are exactly inversely proportional to D. Thus, if the functions are known for any values of I and D, the functions of any other degree of curve for the same value of I can be found by simple proportion. This is the principal advantage of an arc definition of degree of curve, since a table of functions of a 1° curve for all values of I can be prepared and used to determine the functions of any degree of curve more quickly than by the use of the equations.

The objection raised to the use of an arc definition of degree of curve is that it is impossible to measure distances along an arc. This is true; distances can be measured with a tape only along a straight line. In order to locate two points a given distance apart along the arc, a chord length of less than this distance must be used. It would be quite easy to deduct a few hundredths of a foot in measuring the chord, in order that the distance along the arc may be the desired distance, but this is never done in practice.

If the length of arc is taken as 100.007 ft., and 100-ft. chords are used in the field for curves up to 3°, two 50-ft. chords for curves between 3° and 7°, and four 25-ft. chords for curves between 7° and 14°, the theoretical error in each chord length is less than the error in measurement, and the theoretical accumulative error for a curve of average length is well within the permissible error in chaining. Column 4 of Table 4 shows these theoretical errors.

The values of the functions of a 1° curve in Table 13 are based on the 100.007-ft. arc definition of degree of curve. If it is desired to use the 100-ft. arc definition, all values in this table must be reduced by 0.0074%.

If the 100-ft. chord definition, or the multiple chord definition, is used, values of R can be computed from Eq. 9.

For 100-ft. chords,

$$R = \frac{100}{2 \sin \frac{1}{2}D} = \frac{50}{\sin \frac{1}{2}D} \tag{17}$$

For 50-ft. chords,

$$R = \frac{50}{2 \sin \frac{1}{4}D} = \frac{25}{\sin \frac{1}{4}D} \tag{18}$$

And for 25-ft. chords,

$$R = \frac{25}{2 \sin \frac{1}{8}D} = \frac{12.5}{\sin \frac{1}{8}D} \tag{19}$$

For purposes of comparison, Table 4 has been prepared, showing the radii, arc lengths, and chord lengths for the several definitions of degree of curve.

112. Choice of Curve. The principal factors which determine the degree of curve for railways and highways are the topography and the speed of train or automobile. That degree of curve is used which best conforms to the topography, thereby reducing the amount of grading to a minimum, and at the same time enables the traffic to move at the desired speed. Since time is an important factor on main lines of transportation, the curves on such lines are much flatter, i.e., degrees of curve are smaller, than on secondary lines of transportation. In rough, hilly country, topography is the controlling factor, whereas in flat country speed controls, and curves of low degree, around which traffic can operate safely at maximum speed, are used.

Curves up to 10° are found on main-line American railways, but comparatively few are below 1° or above 6°. Degrees of curve as high as 20° are found on many primary highways. Many of these sharper curves are being replaced by much flatter curves, and highway practice is rapidly approaching that of the railways, because motor vehicle speeds are now as high or higher than train speeds.

113. Field Work. The starting point on any route survey is called station 0, the stake 100 ft. ahead is station 1, the next 100-ft. stake is station 2, and so on. Each stake is marked with its number, and the numbering is continued unbroken around the curves. The *T.C.* and the *C.T.* are very unlikely to come at a full station, but will come at some intermediate point. If the *T.C.* of a curve comes 65.4 ft. beyond station 85, its stationing would be 85 + 65.4. The first stake on the curve is at station 86, which is 34.6 ft. from the *T.C.* Thus the first and the last chords of a curve are of odd lengths and the deflection angles for these sub-chords are computed from Eq. 14.

TABLE 4

VALUES OF THE RADIUS AND LENGTH OF ARC FOR DIFFERENT DEFINITIONS OF DEGREE OF CURVE

$D°$	Arc = 100.0074			100-ft. chord		2 50-ft. chords		4 25-ft. chords		100-ft. arc
	R	Arc	Diff.* per 100′	R	Arc	R	Arc	R	Arc	R
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	5730.00	100.0074	−0.002	5729.67	100.002	5729.61	100.001	5729.60	100.000	5729.58
2	2865.00	"	−0.001	2864.93	100.006	2864.84	100.001	2864.80	100.001	2864.79
3	1910.00	"	0.005	1910.08	100.012	1909.91	100.003	1909.87	100.001	1909.86
4	1432.50	"	−0.002	1432.69	100.021	1432.47	100.005	1432.42	100.002	1432.39
5	1146.00	"	0.000	1146.28	100.032	1146.01	100.007	1145.94	100.002	1145.92
6	955.00	"	0.004	955.37	100.046	955.04	100.011	954.96	100.003	954.93
7	818.57	"	0.008	819.02	100.062	818.64	100.015	818.54	100.004	818.51
8	716.25	"	−0.002	716.78	100.081	716.34	100.019	716.23	100.005	716.20
9	636.67	"	−0.001	637.28	100.103	636.78	100.025	636.66	100.006	636.62
10	573.00	"	0.001	573.69	100.127	573.14	100.031	573.00	100.008	572.96
11	520.91	"	0.003	521.67	100.154	521.07	100.038	520.92	100.010	520.87
12	477.50	"	0.004	478.34	100.183	477.68	100.045	477.52	100.011	477.46
13	440.77	"	0.006	441.68	100.215	440.97	100.053	440.79	100.013	440.73
14	409.29	"	0.008	410.28	100.249	409.51	100.062	409.32	100.015	409.26
15	382.00	"	0.011	383.07	100.286	382.24	100.071	382.04	100.018	381.97
16	358.12	"	0.013	359.27	100.326	358.39	100.081	358.17	100.020	358.10
17	337.06	"	0.016	338.27	100.368	337.34	100.092	337.11	100.023	337.03
18	318.33	"	0.019	319.62	100.413	318.64	100.103	318.39	100.026	318.31
19	301.58	"	0.022	302.94	100.460	301.90	100.115	301.64	100.029	301.56
20	286.50	"	0.025	287.94	100.509	286.84	100.128	286.57	100.032	286.48

* Difference per station between actual measurements and the desired arc of 100.007 ft. when 100-ft. chords are used for curves up to 3°, two 50-ft. chords used for curves between 3° and 7°, and four 25-ft. chords used for curves above 7°.

The word station is used also to indicate a 100-ft. distance. For example, the length of a curve may be expressed in stations.

In staking out a curve, successive points are located by a measurement from the preceding point and by a line of sight. The line is not the same for all points, but is determined by the total deflection angle computed for each point. Thus in Fig. 3, point 1 is located by the chord $T.C.$–1 and the total deflection angle A; point 2 by the

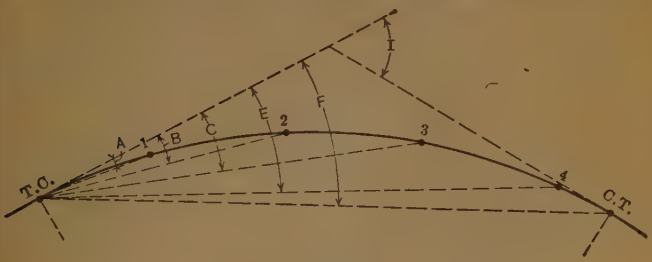

FIG. 3.

chord 1–2 and the total deflection angle B; and point 3 by the chord 2–3 and the angle C.

The first step in the field work is to determine the station numbers of the $T.C.$ and the $C.T.$ and to compute the total deflection angles for the entire curve. In Fig. 3, $T.C.$—1 and 4—$C.T.$ are odd distances, and 1–2, 2–3, and 3–4 are full stations. Then A is the subdeflection angle (d_1) for the chord $T.C.$–1, and is computed by Eq. 14.

$$A = d_1$$

Then $$B = A + \tfrac{1}{2}D = d_1 + \tfrac{1}{2}D$$

And $$C = B + \tfrac{1}{2}D = d_1 + \tfrac{1}{2}D + \tfrac{1}{2}D$$

$$E = C + \tfrac{1}{2}D = d_1 + \tfrac{1}{2}D + \tfrac{1}{2}D + \tfrac{1}{2}D$$

$$F = E + d_2 = d_1 + \tfrac{1}{2}D + \tfrac{1}{2}D + \tfrac{1}{2}D + d_2 = \tfrac{1}{2}I$$

The total deflection angles are thus computed by successive additions, and the entire series of computations is checked if the last value is $\frac{1}{2}I$. If stakes are placed 50 ft. apart the increments are $\frac{1}{4}D$, and if 25 ft. apart they are $\frac{1}{8}D$, instead of $\frac{1}{2}D$ as above. The form of notes is shown in Fig. 4.

On the ground the $T.C.$ and the $C.T.$ are located by measuring the tangent distance T from the $P.I.$ The curve can then be run in by means of angles and distances as explained above.

TRANSIT NOTES FOR LINE L

STATION	ALINEMENT	TOTAL DEFL. ANGLE	CALC. BEAR.	MAG. BEAR.	REMARKS
+ 55.8	⊙ C.T.	35°-41′	N00°-39′W	N00°-40′W	½ I = 35°41′
			Check		
62		34 -34			
61		32 -34			
10 560		30 -34			D = 4°00′
59		28 -34			I = 71°22′
58		26 -34			
57		24 -34			
56		22 -34			P.I. 10 555 + 00.3
55		20 -34			T = 1028.7
54	⊙	18 -34			
53		16 -34			
52	4° Curve Right	14 -34			
51		12 -34			
10 550		10 -34			
49		8 -34			
48		6 -34			
47		4 -34			
46		2 -34			
45		0 -34			
+ 71.6	⊙ T.C.	0°-00′	N72°-01′W	N72°-10′W	
44					
43					

FIG. 4.

J.E. Schmidt—Inst. 2

R.Carr—H.C. C.E.Agg—R.C. June 12, 1930

G.W. Pix—R.F. Clear—Warm

C. Wiles—Ax

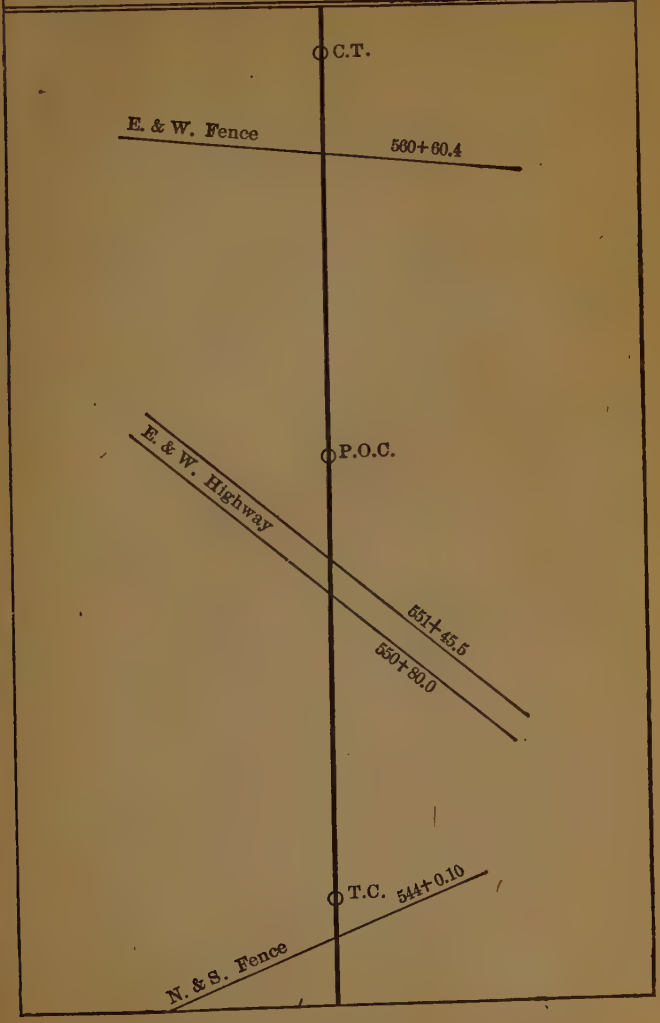

FIG. 4.

Since in turning off an angle with a transit, an error as large as 0.25′ may easily be made—which amounts to 0.07 ft. at 1000 ft. from the transit—the length of sight on a curve should never exceed 1000 ft.

It has been found by experience that if the angle between the tape and the line of sight is more than about 30°, the location of the point is inaccurate. Therefore the angle between the line of sight and the tangent at the transit station should never be more than 30°, that is, the product of the length of sight in stations and $\frac{1}{2}D$ should be less than 30°.

It is *good practice* in any case to run a portion of the curve from the *C.T.*, since the errors of surveying can be adjusted more satisfactorily on the curve than at the *C.T.* There should be no more error, however, in either line or distance than is permissible in good chaining under the particular conditions.

If the entire curve can not be run in from the *T.C.* and the *C.T.*, one or more intermediate set-ups on the curve will be necessary. There are, therefore, three possible positions of the transit in running in curves; the *T.C.*, any intermediate point, and the *C.T.* The curve notes as computed above are used in all three cases, as follows:

At the *T.C.* Orient the transit by a sight along the tangent with the plates set at zero. Turn off the total deflection angle of each station, successively, and chain the corresponding distances between them.

At Any Intermediate Point. Orient the transit by a backsight on the *last transit station with the telescope inverted* and the plates set at the total deflection angle of the station sighted at *as recorded in the notes.* To continue the curve, plunge the telescope [1] and set the plates at the total deflection angles of the succeeding stations and measure the corresponding distances between them.

At the *C.T.* Orient the transit by a sight along the tangent towards the *P.I.* with the telescope normal and the plates set at the total deflection angle of the *C.T.*, that is, $\frac{1}{2}I$. To run in the curve, set the plates at the total deflection angle of each station *as given in the notes*, and measure the corresponding distances.

114. The FUNDAMENTAL PRINCIPLES are:

When sighting at any station the plates must read the total deflection angle of the station sighted at. When on tangent at any station the

[1] The line of collimation must be in adjustment.

plates must read the total deflection angle of the station over which the transit is set.

If these principles are strictly observed a *mistake* in instrument work is impossible.

Example. The *P.I.* of two tangents is at Sta. 10555 + 00.3 and $I = 71° 22'$. It is desired to connect the tangents with a 4° curve.

$$T = \frac{T \text{ for a } 1° \text{ curve for } I = 71° 22'}{D}$$

$$= \frac{4114.9 \text{ (from Table 13)}}{4} = 1028.7 \text{ ft.}$$

Sta. *T.C.* = Sta. *P.I.* − *T* = (10555 + 00.3) − (10 + 28.7)

= 10544 + 71.6

$$L = \frac{I}{D} = \frac{71° 22'}{4} = \frac{71.367}{4} = 17.842 \text{ Stations.}$$

Sta. *C.T.* = Sta. *T.C.* + *L* = (10544 + 71.6) + (17 + 84.2)

= 10562 + 55.8

The distance from the *T.C.* to the first Station on the curve is 28.4 ft., hence

$$d_1 = 0.3\, CD = 0.3 \times 28.4 \times 4 = 34'$$

Deflection angle = $\frac{1}{2}D = 2° 00'$

The distance from the last Station to the *C.T.* is 55.8 ft., hence

$$d_2 = 0.3 \times 55.8 \times 4 = 67' = 1° 07'$$

Total Deflection Angle of				*T.C.*	=	0° 00′	
"	"	"	"	Sta. 10545	=	0° 00′ + 0° 34′	= 0° 34′
"	"	"	"	" 10546	=	0° 34′ + 2° 00′	= 2° 34′
"	"	"	"	" 10547	=	2° 34′ + 2° 00′	= 4° 34′
"	"	"	"	" 10562	=		= 34° 34′
"	"	"	"	*C.T.*	=	34° 34′ + 1° 07′	= 35° 41′
					=	$\frac{1}{2}I$ (check)	

These results are recorded in the form shown in Fig. 4.

115. Offsets from Chords Produced. In addition to his chaining duties the head chainman also sets the stakes on line as directed by the transitman. Obviously this work is much facilitated if the head chainman can quickly approximate the position of the stakes. This can be done by an offset from the line through the last two stations. For example, in Fig. 5, A and B are the last two stations set. Produce AB to E the distance C from B. F is located by the offset $2y$ from E and the distance C from B. y can be found by Eq. 20. If E is carefully lined in and C and $2y$ measured with the tape, the point can be located with considerable accuracy; and, if a *transit is not at hand*, this method can be employed in locating an entire curve. However, this method is most valuable as an aid to

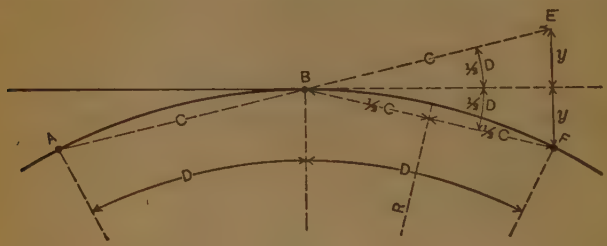

FIG. 5.

the head chainman in approximating the position of the stakes, and it should *always* be used. If E is only roughly lined in and $2y$ is paced, the points can be located within less than one foot, which is sufficient for the purpose.

From Fig. 5,

$$y : C :: \tfrac{1}{2}C : R$$

Whence $$y = \frac{C^2}{2R}$$ (C and R expressed in feet)

Expressing R in terms of D, we have

$$y = \frac{C^2 D}{11460}$$

If n represents C expressed in stations, then

$$y = 0.873n^2D \qquad (20)$$

y is the *tangent offset* to the point F.

For $C = 100$ ft., $n = 1$,
and $y = 0.873D$, and $2y = 1.746D = 1\frac{3}{4}D$ (approx.) (21)
For $C = 50$ ft., $n = \frac{1}{2}$,
and $y = 0.22D$, and $2y = 0.44D$

The method as given above is applicable only when the chords are of uniform length. Normally there is an odd length of chord at the beginning of the curve, and consequently the first two stations on the curve must be located somewhat differently. The first station is located by using a tangent offset y_1, computed from Eq. 20 for the proper chord length c. The offset Y_1 from this first chord, produced the normal chord distance C, to a tangent to the curve at the first station is found from the proportion $Y_1 : y_1 :: C : c$. The offset from this tangent to the second station on the curve is the distance y for the chord C as given above. Hence, the total offset for the second station from the first chord produced is $Y_1 + y$. From the second station the curve is continued as previously explained. At the end of the curve a similar problem arises and is solved in the same way.

116. Chord Offset Method. There are some cases in which the deflection angle method of running in curves can not be used to

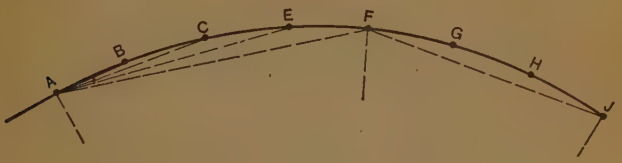

FIG. 6.

advantage. For instance, in heavily wooded country it may not be economical to locate every station by the deflection method on account of the large amount of clearing necessary for the lines of sight. Fig. 6 illustrates this condition. If the stakes B, C, E, and F were all set from A it would necessitate the clearing of a line to each or practically the entire area between the curve and the chord AF. If, on the other hand, F were set from A by its total deflection angle and the chord AF, and B, C, and E located by offsets from this chord, the only clearing required would be along AF

and the short offsets. The transit would then be moved to F and the process repeated to J, and so on.

The chord AF is the $L.C.$ of a curve whose value of I is the central angle of the curve $ABCEF$, and therefore can be determined from Eq. 9 or from Table 13. In chaining this chord, temporary stakes are set at the full station distances, although perpendicular offsets from these will not give the exact location of the curve stations since the chord AF is shorter than the line $ABCEF$. This error may be neglected for the following reasons: (1) in practice the error at any point will rarely exceed one foot; (2) the perpendicularity of the offsets is established by eye, hence is not exact; and (3) the approximate location of these stations is sufficient since this method would be used only on first location to determine the profile. The offsets are computed as follows:

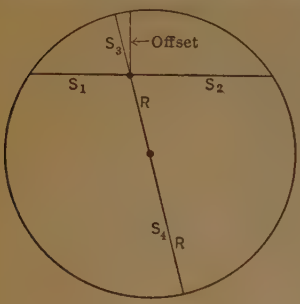

FIG. 7.

From geometry the products of the segments of two intersecting chords are equal. Then from Fig. 7,

$$S_1 \times S_2 = S_3 \times S_4$$

or

$$S_3 = \frac{S_1 \times S_2}{S_4} = \frac{S_1 \times S_2}{2R - S_3}$$

Since S_3 is small compared with $2R$ it may be neglected in the denominator, and since in practice the difference between S_3 and the offset will rarely reach 0.05 ft., we may write

$$\text{Offset} = \frac{S_1 \times S_2}{2R} = \frac{S_1 \times S_2 \times D}{2 \times 5730} \text{ (Approx.)}$$

Reducing S_1 and S_2 to stations, we have

$$\text{Offset (in feet)} = 0.873 S_1 S_2 D \text{ (Slide Rule)} \qquad (22)$$

117. Middle Ordinate Method. If the curve is to be staked out with the tape alone, the middle ordinate method may be more con-

venient than the offsets-from-chords-produced method, because the middle ordinates are shorter than the offsets. As shown in Fig. 8, point a is located by a tangent offset from the chord produced. This offset is $\frac{1}{4}y$, and is equal to $0.218\,n^2D$. If the length of chord used is 100 ft., n is 1 and the offset is $0.218D$. The middle ordinate at point a is m, which is also equal to $\frac{1}{4}y$, or $0.218D$. The middle ordinates at points b, c, d, and e are also equal to $0.218D$. Point b is located by swinging the tape about the $T.C.$ as a pivot, until the 50-ft. point on the tape is the distance m from point a. The tape must be stretched taut and m carefully measured. This operation can be carried out more accurately if a strong, light string, such as a silk fishline, be used instead of the tape. Points c, d, and e are located in the same manner. This method requires an extra man to read the middle ordinates.

The middle ordinate can be obtained trigonometrically. In triangle 1, Fig. 8, it is evident that $m = R \text{ vers } \frac{1}{2}\theta$.

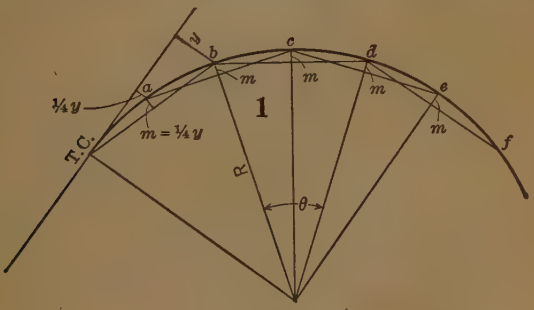

FIG. 8.

If the chord, C, is 62 ft. in length, the middle ordinate m, in inches, is equal approximately to the degree of curve, since

$$m \text{ (inches)} = 12 \times 0.218 \times 0.62^2 \times D = 1.006D \qquad (23)$$

This is the easiest and quickest method of determining the degree of an existing curve.

The middle ordinate for any constant length of chord varies directly as the degree of curve. The sum of all the middle ordinates of a curve varies directly as the central angle of the curve. As long as the central angle remains constant, the sum of the middle

ordinates remains constant, regardless of the degree of curve used to connect the tangents, provided the same length of chord is used. This is true even if the curve is irregular, and certain portions are sharper than other portions.

The middle ordinate method is the basis of the string-lining method of realining existing curves, which is treated in Chapter 6.

118. Curve through a Fixed Point. Sometimes the topographic conditions make it necessary to determine the radius of the curve which will pass through (or near) a fixed point and connect the two given tangents.

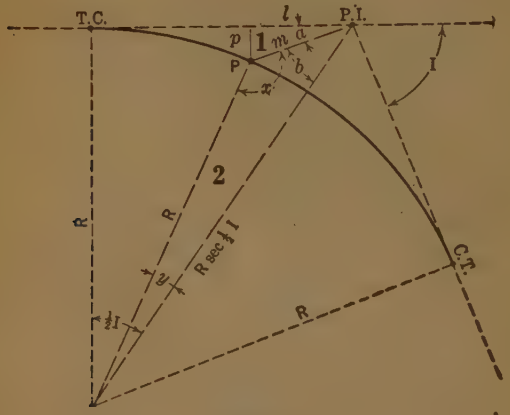

FIG. 9.

In Fig. 9 the intersection angle I and the location of point P are known. The point P is usually known through its coordinates l and p from the initial tangent.

In triangle 1, the altitude p and the base l are known, whence the angle a and the hypotenuse m can be computed. Or a and m can be measured in the field.

In triangle 2, the side m is known and the angle

$$b = 90 - \left(a + \frac{I}{2}\right);$$

By the sine proportion,

$$\sin x = \sin (180 - x) = \frac{R \sec \frac{1}{2}I}{R} \sin b = \sec \tfrac{1}{2}I \sin b = \frac{\sin b}{\cos \frac{1}{2}I}$$

and, $$y = 180 - (b + x)$$

The angles and the side m are now known, whence the radius R can be determined by the sine proportion. D is then obtained from R to the nearest 10′, unless the curve must pass exactly through P.

119. Broken-back Curves. When two curves in the same direction are separated by a short tangent, the layout is termed a broken-back curve. Such curves possess poor riding qualities and their appearance is bad, which is especially objectionable on highways.

The maximum length of tangent which can exist and still make the layout broken-backed is indefinite. If the intervening tangent is less than the sum of the two runoff distances normally placed on the tangents, the curve is undoubtedly broken-backed. Since it is undesirable to run out the superelevation and immediately run it in again, there should be a piece of tangent of indefinite length between the two points where the superelevation becomes zero. It would seem that this should at least equal the sum of the runoff distances. This means that any layout with a tangent less than about 500 ft. between two curves in the same direction should be considered a broken-back curve.

Broken-back curves can usually be avoided with a little care in projecting the location. Spirals can be used to replace all or part of the tangent, or the two curves can be compounded with a third. Spirals are considered in Chapter 5. Compounding can be used both on new work and to improve existing curves. The most common case of compounding is when the two degrees of curve and the length of the intervening tangent are known, and it is desired to compound with a chosen curve and find the position of the two points of compounding.

In Fig. 10, D_1, D_2, and l are known and D is chosen.

In triangle 1, the base l and the altitude $R_2 - R_1$ are known, whence the angles a and b and the hypotenuse O_1O_2 can be computed.

In triangle 2, the three sides are known, whence the three angles c, d, and e can be determined (see appendix).

Angles x and y are now found by arithmetic.

Since the central angle x and the degree of curve D_1 are now known, the distance back from the $C.T.$ to the $C.C._1$ can be readily

found. In the same way the distance from the $T.C.$ to the $C.C._2$ can be determined, and also the length of the D curve.

A small station equation will exist at the $C.C_2$, equal to the difference in distance along the old and the new alinements between the $C.C._1$ and the $C.C._2$.

Sometimes it is desired to know how much the center line will

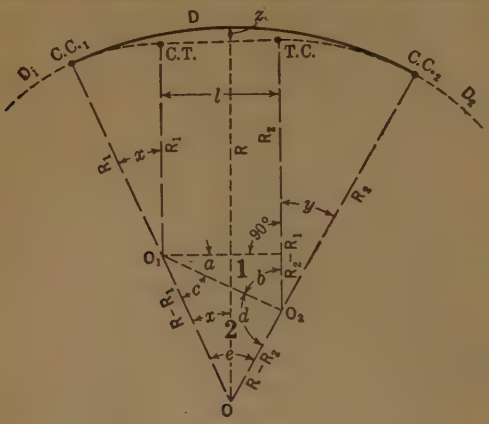

FIG. 10.

be moved laterally by such compounding. From the figure this distance is

$$z = R - R_1 - (R - R_1)\cos x = (R - R_1)\,\text{vers}\,x \text{ (approx.)}$$

This equation is exact when $R_1 = R_2$ and is sufficiently accurate unless there is a very large difference between R_1 and R_2.

Change of Location Problems

After the line is located according to the paper projection and a profile made, it may be found that, owing to inaccuracies in the platted topography, a considerable cut or fill comes on a hillside and may be eliminated by shifting a portion of the alinement. Occasionally other considerations than earthwork make such a shift desirable.

120. Problem 1. In Fig. 11, the line as originally located (shown dotted) involves considerable earthwork which could be eliminated by shifting the curve the distance p at or near its middle point.

The problem is to find the new degree of curve and the change in the position of the $T.C.$ and the $C.T.$ p is the difference in the external distances of the two curves, or

$$E_{\text{new curve}} = E_{\text{old curve}} + p$$

The new degree of curve may be computed from the new external by Eq. 11 or by means of Table 13, which is the quicker and more common method. The estimated value of p will usually give an odd value of D which is undesirable; hence the value of D to be used is taken only to the nearest ten minutes. The distance that

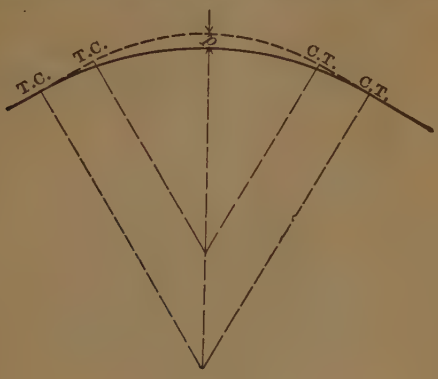

FIG. 11.

the $T.C.$ and the $C.T.$ are shifted is equal to the difference in T for the two curves.

121. Problem 2, Case 1.

In Fig. 12, the line as originally located (shown dotted) involved considerable earthwork between A and B, which could largely be eliminated by shifting the tangent the distance p parallel to its original position, which involves a change in the location of *TWO* curves.

The problem is to find the change in the $T.C.$'s and the $C.T.$'s of the two curves, the degrees of curve remaining unchanged. In triangle 1, the sides m and l can be readily computed since I and p are known. $m = n =$ the required change in the $T.C.$ and the $P.I.$ p will rarely exceed 100 ft.; hence the new tangent is best located

by offsets from the old tangent. The new *C.T.* is then located from the old *C.T.* by rectangular coordinates one of which is *p* and the other is the side *l* of triangle 1.

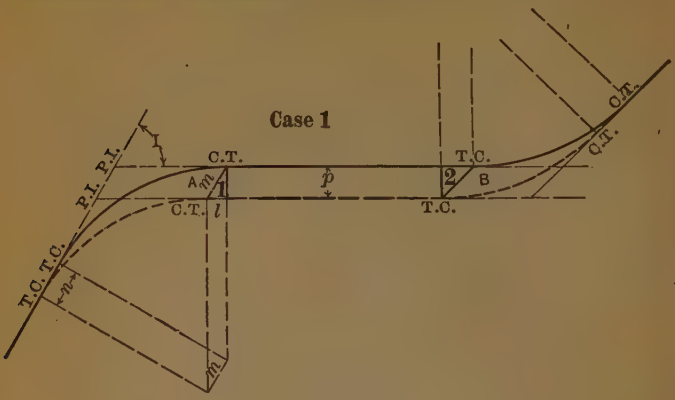

Fig. 12.

The second curve is located similarly using triangle 2.

122. Problem 2, Case 2.

Owing to some special cause, for example a stream-crossing as

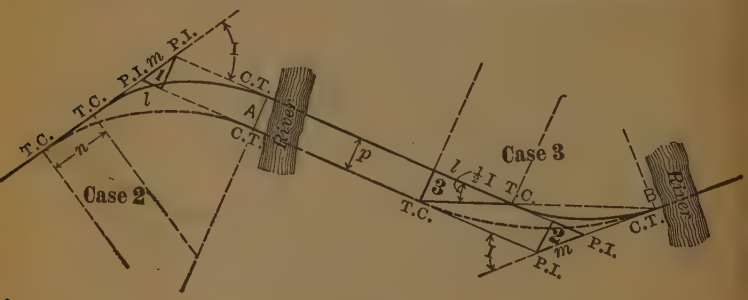

Fig. 13.

shown in Fig. 13, it may be undesirable to move the *C.T.* forward as in Case 1. This condition can usually be met by placing the new *C.T.* *approximately* on the same radial line as the old *C.T.*

This involves a change in the degree of curve and in the position of the $T.C.$

In triangle 1, I and p are known, and l and m are computed.

$$T_{\text{new curve}} = T_{\text{old curve}} - l$$

whence the new D is easily found from Table 13 and, as in Problem 11, is taken only to the nearest 10′. (The approximation in the position of the $C.T.$ is due to this not using the exact value of D.) A new value of T is then found which agrees with the D that is used. Then

$$n = T_{\text{old curve}} + m - T_{\text{new curve}}$$

123. Problem 2, Case 3. This case is similar to Case 2, except that the $C.T.$ is on the fixed tangent instead of the one being changed, hence the solution is somewhat different.

Solve triangle 2 for the side m, then

$$T_{\text{new curve}} = T_{\text{old curve}} - m$$

whence the new degree of curve is obtained from Table 13.

Solve triangle 3 for the side l. Then the new $T.C.$ can be located from the old $T.C.$ by the coordinates l and p.

As in the previous problem, D should be taken only to the nearest 10′. This necessitates a slight change in both the $T.C.$ and the $C.T.$, since T for this value of D is not the same as that used in the above computations. It must be remembered that the $T.C.$ becomes the $C.T.$, or vice versa, when the line is run in the opposite direction.

124. Problem 3. Sometimes curves are run in beginning at the $T.C.$ without previously locating the $P.I.$ and the $C.T.$, and checking the direction of the forward tangent. On reaching the $C.T.$ and projecting the forward tangent, it is found that the tangent does not pass through some controlling point. It is therefore necessary to change the direction of the tangent by shifting the $C.T.$ This is most readily accomplished by measuring the necessary change of direc-

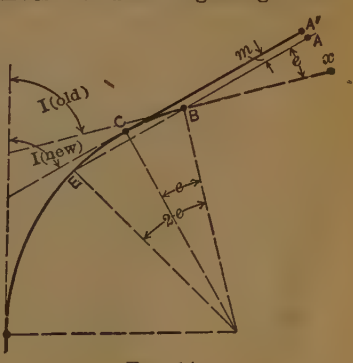

FIG. 14.

tion with the transit, and then changing the length of the curve to correspond. For example, in Fig. 14, the curve is run to B as a $C.T.$ and the tangent is found to pass through x, whereas it is necessary to pass through A. The angle e is measured with the transit. The length of curve corresponding to e is BC, whence the $C.T.$ is moved back to C. The forward tangent then takes the position CA', parallel to BA, and misses A by the distance m, which is the middle ordinate of the curve BE.

$$BE = 2BC = \frac{2e}{D}$$

$$m = \frac{7}{8} \times \frac{\overline{BE}^2}{4} \times D = 0.875 \frac{e^2}{D}$$

As m will rarely reach 5 ft., the line CA' meets the requirements.

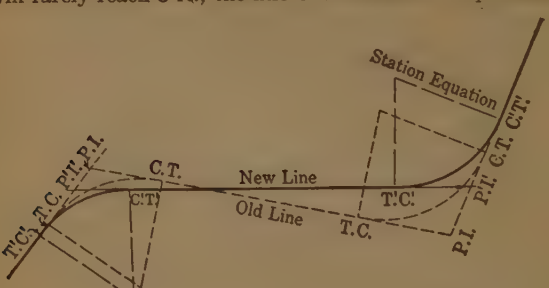

FIG. 15.

125. Problem 4. Occasionally it may be found desirable to change both the position of a tangent and its direction. In this case it is best to treat the problem as one in relocation. The new tangent is run out on the ground, the new $P.I.$'s located and the new values of I determined. The same degrees of curve may be used or others chosen to better fit the topography. The new alinement is then run in without reference to the old except that the distances along both are computed and a station equation made at the $C.T.$ of the second curve. This problem is illustrated in Fig. 15.

Compound Curves

126. Conditions for Use. A compound curve is a combination of two or more simple curves in the same direction with a common tangent at the point of junction. *A compound curve should never be used except under conditions where a simple curve will not meet the requirements.* In rough country it may happen that a large volume of earthwork can be avoided by making one part of a curve sharper than another, resulting in a compound curve.

For example, in Fig. 16, the line is located as a compound curve ABC, requiring little earthwork and keeping on the bank of the stream. If the simple curve AB had been produced to F to end

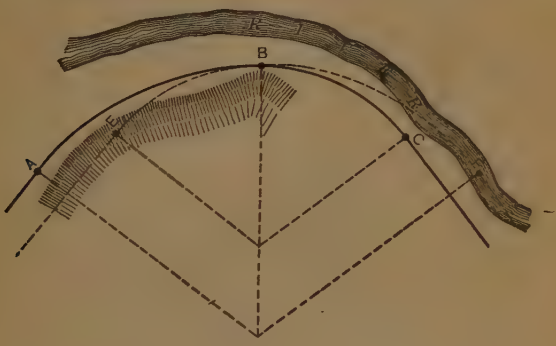

Fig. 16.

in a parallel tangent, it would have fallen in the river. Or if the simple curve CB had been prolonged to E, it would have pierced the cliff.

The degrees of curve and the point of compound curve, $C.C.$, are chosen to fit the contours and other governing conditions, and the central angles of the branches are scaled from the map.

This is one of many cases where a compound curve is applicable. From the standpoint of operation a compound curve is better than two simple curves separated by a short tangent, hence it should be used in such cases. *In flat country a compound curve is inexcusable on main line.* If the degrees of curve of two adjacent branches of a compound curve differ by more than 2° 00′, an easement curve should be inserted between the branches, and provision

for this should be made in locating the curve. (See Spirals, section 147.)

The nomenclature and positions of the functions are shown in Fig. 17.

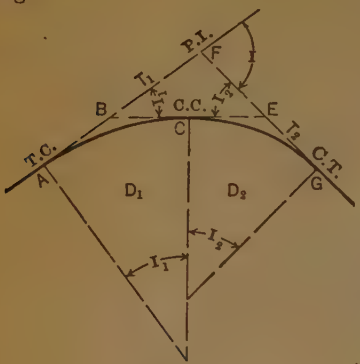

FIG. 17.

127. Problem 1.

In new work I will be measured and I_1, I_2, D_1, and D_2 will be determined from the preliminary maps. It is desired to know the values of T_1 and T_2 in order to set the $T.C.$ and the $C.T.$ and to run in the curve.

In Fig. 17, BE is the common tangent at the $C.C.$

$$BE = BC + CE$$

$BC = AB$ is the tangent distance for a D_1 curve for an intersection angle I_1; and $CE = EG$ is the tangent distance for a D_2 curve for an intersection angle I_2 and both are computed by Table 13.

Then in the triangle BEF, one side and the adjacent angles are known from which the sides BF and EF are computed.

$$T_1 = AB + BF, \quad \text{and} \quad T_2 = EF + EG$$

128. Problem 2. Occasionally the positions of the $T.C.$ and the $C.T.$ are fixed within narrow limits, giving unequal tangent distances, which necessitates a compound curve.

The degree of curve of one branch must be chosen before the problem can be solved. The problem then is to determine the degree of curve of the second branch and the central angles I_1 and I_2.

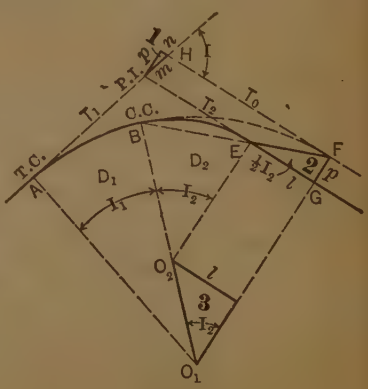

FIG. 18.

In Fig. 18, I, T_1, T_2, and D_1 are known. $AH = HF = T_0 =$ the tangent of D_1 curve for the intersection angle I.

In triangle 1,

$$m = T_0 - T_1$$

and hence the triangle can be solved for p and n.

In triangle 2,

$$l = T_0 + n - T_2$$

Since p is known, the triangle can be solved for the angle at E, which is $\frac{1}{2}I_2$.

$$I_1 = I - I_2$$

In triangle 3, since l and I_2 are known, the side $O_1O_2 = R_1 - R_2$ can be solved for, whence D_2 is determined from Table 10.

129. Field Work. A point on one branch of a compound curve can not be located with the transit set on a point on another branch, since in order to run in a circular curve with the transit and tape, the transit and the points to be located must be on the same circumference. Each branch of a compound curve may therefore be run in independently as a simple curve; but the following method is more convenient.

The total deflection angles for the first branch are computed from the $T.C.$ as for a simple curve to agree with D_1 and must check at the $C.C.$ by equaling $\frac{1}{2}I_1$. The total deflection angles for the second branch are computed to agree with D_2, but are added on continuously to those of the first branch; hence the check at the $C.T.$ is that the total deflection angle must equal $\frac{1}{2}I_1 + \frac{1}{2}I_2 = \frac{1}{2}I$. The transit is oriented at any point in the same manner as on a simple curve by setting off the total deflection of the station sighted at, remembering only that *a sight can not be taken past the* $C.C.$, since this would be sighting at a point not on the circle through the transit point and the deflection angle method would not apply. The transit may be set at the $C.C.$, however, since this point is common to both circles.

The errors of field work should be accumulated and corrected at some point on the curve other than at the $C.T.$ This may be done at the $C.C.$ if desired.

Change of Location Problems

130. Problem 1. This problem arises under conditions similar to those in Problem 2, Case 1, of simple curves, section **121.**

Given, as shown in Fig. 19, a located compound curve ABC. It is desired to move the forward tangent laterally a distance p. The simplest solution of this problem is to retain the same degrees of curve and to shift the $C.C.$ from B to E.

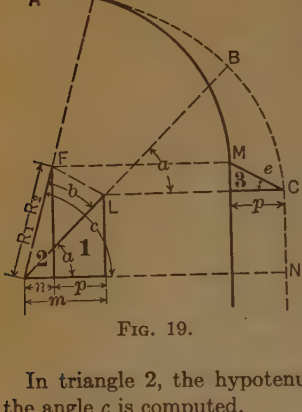

Fig. 19.

$$BE \text{ (in feet)} = \frac{b}{D_1} 100$$

Therefore the problem resolves itself into solving for the angle b.

In triangle 1, since a is known and the hypotenuse is the difference in the two radii, the base m can be computed. Then $n = m - p$.

In triangle 2, the hypotenuse and the base are known, whence the angle c is computed.

$$b = c - a$$

In triangle 3, $\qquad e = 90° - \tfrac{1}{2}b - a$

whence the coordinates of M, the new $C.T.$, from the old $C.T.$ can be determined.

Three other cases of this problem arise: (1) the same as above, except that the tangent is moved *outward*; (2) the curve of longer radius may fall on the tangent to be moved *inward*; and (3) the curve of longer radius may fall on the tangent to be moved *outward*.

The solution of each of these cases is the same as that given above except that the various points take different relative positions. If the same relative construction lines are drawn and the corresponding points are used, no confusion should arise.

131. Problem 2. This problem arises under conditions similar to those in Problem 2, Case 2, of simple curves, section **122**.

Given, as shown in Fig. 20, a located compound curve ABC. It is desired to move the forward tangent laterally a distance p, and to keep the $C.T.$ approximately on the same radial line, which involves a change in the degree of curve of the second branch and in the position of the $C.C.$

In triangle 1, a is known and the hypotenuse is the difference in the two given radii. Therefore the base m and the altitude n can be computed.

In triangle 2, the base n is now known and the altitude can be determined since R_1, R_2, p, and m are known. The angle at $F = \frac{1}{2}b$ (Why?) can therefore be determined. Then

$$c = b - a$$

and the $C.C.$ is shifted the distance BE (in feet) $= \frac{c}{D_1}100.$

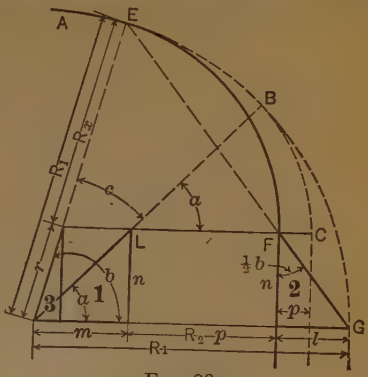

FIG. 20.

In triangle 3, the angle b and the altitude n being known, the hypotenuse r can be computed from which R_x is determined, and the corresponding degree of curve D_x is found from Table 10, and is taken to the nearest 10′. Then since E and b are already fixed, this will change the value of p slightly and will shift the $C.T.$ a short distance along the line EG from F, its theoretical location. The coordinates of this new position are given by the equations

$$x = h \sin b, \quad \text{and} \quad y = x \tan \tfrac{1}{2}b,$$

in which x is the movement along the tangent, y is the change in p, and h is the difference between the computed value of R_x and the value actually used.

Three other cases of this problem arise, depending on whether the longer or the shorter radius curve is on the tangent to be moved, and on whether the tangent moves inward or outward. In all of these cases the solution is relatively the same.

Reversed Curves

132. Conditions for Use. A reversed curve is a combination of two simple curves of opposite curvature with a common tangent at the point of junction.

A reversed curve should never be used on main lines of transport. The shock to trains due to the sudden reversal of curvature on railroads, the difficulty of properly steering motor vehicles on highways or vessels in canals, and the danger of erosion in drainage channels all combine to make reversed curves undesirable for major traffic movements. Furthermore, it is impossible to superelevate properly the outer rail, or outer edge of roadway, at or near the point of reversal, and this is essential to smooth and safe operation of trains or automobiles. If conditions require two curves of contrary curvature close together, they should be separated by sufficient tangent to run out the superelevation of each, or each should be provided with a spiral, in which case the "points of spiral" (*S.T.* and *T.S.*) may be made coincident.

In railroad yards and connections reversed curves may be permissible, since the speed is low. Since practically all such cases occur in connection with turnouts, further consideration of reversed curves on railroads will be given in Chapter 8.

The main alinement of roads and streets should also be free from reversed curves, but frequently such curves are required, or may be desirable, in curb lines and other minor locations.

Two general cases of reversed curves arise as follows:

133. Case 1. In Fig. 21, the two parallel tangents are to be connected by a reversed curve. R_1, R_2, and p are given.

From triangle 1,

$$l_1 = R_1 \sin a, \quad \text{and} \quad m_1 = R_1 \operatorname{vers} a$$

From triangle 2,

$$l_2 = R_2 \sin a, \quad \text{and} \quad m_2 = R_2 \operatorname{vers} a$$

Whence,

$$p = m_1 + m_2 = (R_1 + R_2) \operatorname{vers} a$$

$$\operatorname{vers} a = \frac{p}{R_1 + R_2} \tag{24}$$

And

$$L = l_1 + l_2 = (R_1 + R_2) \sin a \tag{25}$$

From these two equations and the ordinary functions of a simple curve, all ordinary cases of reversed curves between parallel tangents can be solved.

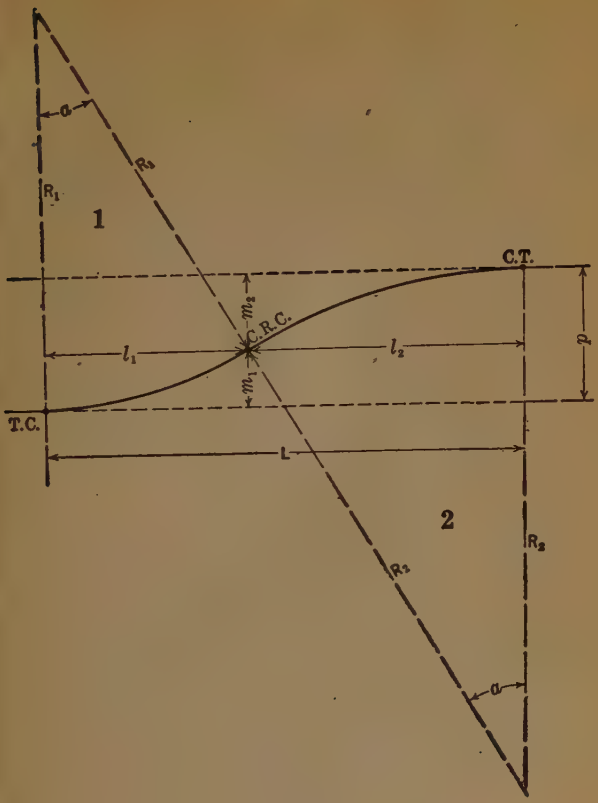

FIG. 21.

134. Case 2. In Fig. 22, the two tangents, intersecting with the angle I, are to be connected by the reversed curve in which T_1, R_1, and R_2 are known, and the tangent distance T_2 and the central angles of the two branches are required.

In triangle 1, the base T_1 and the angles are known, from which the sides l and m can be computed.

In triangle 2, the hypotenuse is $R_1 - m$, and the angles are known, whence the base p and the altitude n are determined.

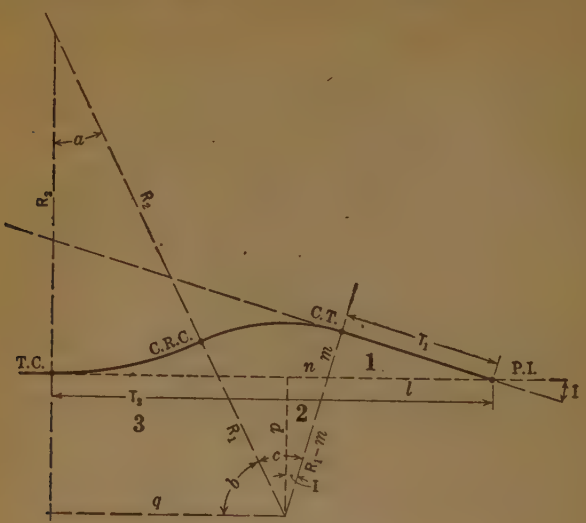

FIG. 22.

In triangle 3, the base is $R_2 + p$, and the hypotenuse is $R_1 + R_2$, whence the angles a and b and the distance q can be found. Then,

$$c = I + a$$

and

$$T_2 = l + n + q$$

Special Highway Problems

135. Widening Pavements on Curves. Observation has shown that the rear axle of a motor vehicle remains essentially radial on curves. Since the chassis frame is rigid, the front wheels must travel on a curve of longer radius than do the rear wheels. The pavement, therefore, must be wider on curves, if the clearance between passing vehicles is to remain constant.

Since the length of the vehicle is comparatively short, it is sufficient to consider that the widening per lane of traffic is equal to

the offset between the tangent and the curve in the length of the vehicle. Then from Fig. 23,

$$w = 2W = 2 \times \frac{L^2}{2R} = \frac{L^2}{R} \quad \text{(Approx.)} \tag{26}$$

where w is the widening of a two-lane road (or for each two lanes of a multiple-lane road), L is the length in feet of the vehicle measured

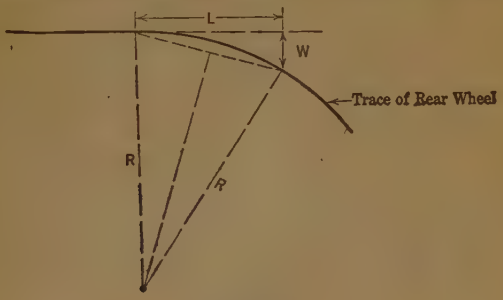

FIG. 23.

from the rear axle to the front bumper, and R is the radius of the curve in feet. It is sufficient to take R as the center-line radius. L should be about 20 ft. for ordinary conditions, and more if there are many large buses, trucks, etc.

Since it is more difficult to judge the clearance on curves than on tangents, it is desirable to have an additional width over that given by Eq. 26. This additional amount is hard to determine and is largely a matter of judgment. Mr. James T. Voshell of the U. S. Public Roads Administration suggests the following equation: [1]

$$w = 2(R - \sqrt{R^2 - L^2}) + \frac{35}{\sqrt{R}}$$

but the term $(R - \sqrt{R^2 - L^2})$ is the tangent offset, hence this equation reduces to the simpler form

$$w = \frac{L^2}{R} + \frac{35}{\sqrt{R}} \tag{27}$$

[1] *Public Roads*, April 1927, page 35.

The second term, therefore, is an additional width to allow for the difficulty of judging the clearance.

Example. A pavement 20 ft. wide on a curve with a radius of 400 ft. is to be widened for vehicles 20 ft. long.

From Eq. 26, $w = \frac{20^2}{400} = 1.0$ ft., making a total required width of 21.0 ft. From Eq. 27, $w = \frac{20^2}{400} + \frac{35}{\sqrt{400}} = 2.75$ ft., making a required width of 22.75 ft., of which 1.75 ft. is "psychological" widening.

The need for widening is another argument for flatter curves, so as to reduce the amount of widening, or entirely eliminate it, and thus reduce the cost of construction.

The full amount of widening is required as soon as the entire vehicle is on the curve and hence should be required within a vehicle length of the *T.C.* and the *C.T.* of a simple curve. Obviously, it would be unsatisfactory to change the width abruptly at these points, and consequently the change of width must be made gradually in some convenient distance suitable to the character and speed of traffic.

Formerly the widening was frequently placed on the outside of the normal pavement, and the change of width was made as shown in Fig. 24*a* in a distance of 150 to 200 ft. This was objectionable, because it made reversed alinement on the outer edge, which was

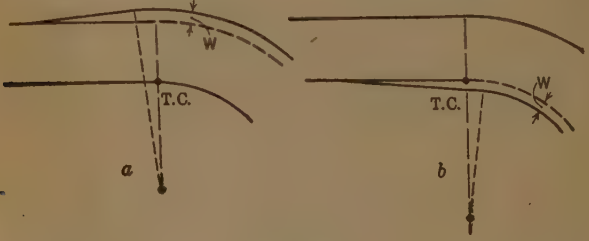

FIG. 24.

disliked by traffic and was poor in appearance. Later and better practice was to put the widening on the inner edge and make the change of width as shown in Fig. 24*b*, but this method was also objectionable on account of appearance and the difficulty of pro-

viding proper runoff for the superelevation. The best method is to place the widening on the inner edge, or symmetrically about the center line, and to make the change of width by means of spirals as explained in Chapter 5.

136. Sight Distance. On highways it is essential for safety that the drivers of vehicles be able to see approaching vehicles in time to avoid collision.

The *sight distance* is the maximum distance at which two vehicles traveling in the same traffic lane are mutually visible. Experience dictates that a sight distance of at least 700 ft. is necessary on curves, but the increase in motor-vehicle speeds indicates that not less than 1000 ft. should be provided.

The sight distance can be taken as the chord of the given curve, corresponding to a middle ordinate equal to the distance from the center line of the inner traffic lane to the object which obstructs the view. The full sight distance is not necessary at the level of the roadway, but is required at the height of the normal line of sight, which should be not more than 4.0 ft. above the roadway.

Thus in Fig. 25, M is the clearance from the center line to the

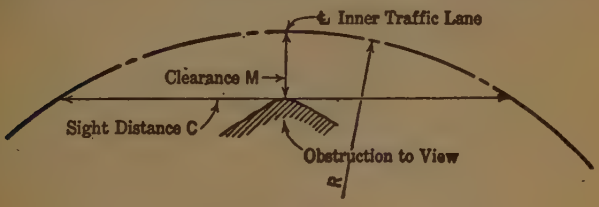

FIG. 25.

obstruction and C is the sight distance. Since the middle ordinate in the distance C is equal to the tangent offset in the distance $\frac{1}{2}C$, we have from Eq. 20

$$M = 0.0000218C^2D = \frac{0.125C^2}{R}$$

in which M is the clearance in feet, C the sight distance in feet, D is the degree of curve, and R is the radius in feet.

In designing the alinement, especially on 2-lane roads, it is highly important to provide ample sight distances at suitable intervals for passing, since some vehicles travel faster than others. If possible, such passing facilities should be provided on the tangents, but, if the topography demands a winding road, it may be necessary to

consider safe passing on curves. The flatter the curves, the more easily this can be done. The distance required to pass is not accurately known, since it depends on both the difference in speed of the two vehicles and the actual speed of the slower vehicle. The indications are, however, that from 500 to 1000 ft., in addition to the sight distances as above indicated, may be required. Thus, at as frequent intervals as possible, sight distances of one-quarter mile or more should be sought. These passing spaces should then be compared with the grade line to insure that vertical curves do not restrict them. Where the topography does not permit such provisions, immediate consideration should be given to providing at least three lanes to avoid congestion and to promote safety.

137. Safe Speeds. Since 1930, motor vehicles capable of much higher speeds than formerly have become universal, with the result that there is an upward trend in general traffic speeds. This has led to serious consideration of safe highway speeds, especially on curves.

The U. S. Bureau of Public Roads suggested that a portion of the coefficient of friction between the tires and the roadway, amounting to 0.16, might be utilized in overcoming unbalanced centrifugal force, and hence speeds in excess of the theoretical speed used in computing the superelevation could be safely permitted. The Illinois Division of Highways, after studying this proposal, reached the conclusion that an allowance of 0.16 was too great and adopted the value of 0.10. Professor Moyer* from his experiments on skidding found that, as the excess speed increased, a greater coefficient of friction was required to avoid skidding. This is another way of saying that the permissible amount of coefficient of friction to resist unbalanced centrifugal force decreases as the speed increases. His data indicate that the value varies from about 0.04 to about 0.12. Combining this idea with the usual equation for superelevation, the following equation may be derived:

$$V^2 = 15R(e + f) \qquad (28)$$

where e is the superelevation as actually provided and f is the allowance in coefficient of friction to overcome unbalanced centrifugal force and may be taken as any of the values indicated above.

This equation has been erroneously considered as giving the "safe speed" on which to design new alinements. In reality it merely gives the mechanical relation between speed and a safe

*Bulletin 120, Iowa Engineering Experiment Station. 1934.

factor against lateral skidding, and fails to take into account the physio-psychological limitations of the driver and the mechanical limitations of the vehicle. This is obvious, since the equation is that of a parabola and, consequently, the speed increases indefinitely with the radius and would reach infinity on a tangent, which is an impossibility for both the car and the driver.

Several other equations have been proposed for safe speed. Some of these have been based on existing curves and, although superior to the above equation, are not reliable in speed ranges above about 50 to 60 miles per hour. Present information does not permit the derivation of a really rational equation because the safe speed on tangent is unknown. If we assume that an average car and driver can safely travel 100 miles per hour on tangent, and that a radius in excess of say 8000 feet may be considered as tangent, we may set up the elliptical equation,

$$V^2 = 2.5R - 0.000156R^2 \qquad (29)$$

This equation agrees closely with the best equations for speeds, based on observed curves, up to about 70 miles per hour, and gives conservative values beyond. This equation may be used to determine a reasonably safe speed, and this value can then be used in Eq. 59 to determine the required superelevation. It will generally be found that the maximum permissible superelevation will be required, hence the curves should always be spiraled. In view of the lack of more definite information, Eq. 29, combined with maximum superelevation, is recommended for determining the minimum radius for new highways.

Problems

1. If the *P.I.* is in a stream or is otherwise inaccessible, how can *I* and the positions of the *T.C.* and the *C.T.* be determined?

2. If the *T.C.* is inaccessible, how can the curve be run in and checked?

3. If the line of a curve passed through a building or similar obstruction, how can the curve be run in and checked?

4. Given $I = 63° 43'$. Find *T*, *L*, *R*, and *E* for each of the following values of *D*—0° 30′, 2° 00′, 5° 00′, and 7° 30′. Solve by equation and check by Table 13.

5. Given the *P.I.* at Sta. 118 + 60.0, $I = 57° 48'$, and $D = 5° 00'$. Find the station numbers of the *T.C.* and the *C.T.*

6. Write transit notes for the preceding problem.

7. The *P.I.* is at Sta. 205 + 75.0, $I = 68° 28'$. It is desired to locate a curve which will pass through a point 63.0 ft. to the right of Sta. 203 + 35.0. Find R and D.

Answer. $R = 814.2$, $D = 7° 02'$ (use $7° 00'$.)

8. The *C.T.* of an $8° 00'$ curve is at Sta. 46 + 27.5, and the *T.C.* of a $6° 00'$ curve is at Sta. 50 + 27.5. It is desired to eliminate the broken back by using a $3° 00'$ curve compounded with the given curves. Determine the Sta. of the $C.C._1$, the Station Equation at the $C.C._2$, and the amount the alinement is moved laterally.

Answer. Sta. $C.C. = 45 + 00.1$; Sta. Eq. $52 + 19.9 = 52 + 17.5$; $z = 18.8$ ft.

9. Given, on original location, a $6° 00'$ curve with $I = 78° 21'$ and the *T.C.* at Sta. 1041 + 72.6. Topography requires the middle point of the curve to be moved toward the center *about* 45 ft. What is the new degree of curve and the station number of its *T.C.*?

10. Given an established curve with $I = 69° 38'$, $D = 3° 00'$, and the *T.C.* at Sta. 982 + 41.1. It is desired to move the forward tangent *outward* a distance of 50 ft. Find the new station numbers of the *T.C.* and the *C.T.*, and the coordinates from the old *C.T.* to the new *C.T.*

Answer. *T.C.* = 982 + 94.4; *C.T.* = 1006 + 15.5; $l = 18.56$, and $p = 50.0$.

11. Data same as in Problem 10, except that the forward tangent is moved *inward*.

Answer. *T.C.* = 981 + 87.8; *C.T.* = 1005 + 08.9; $l = 18.56$, and $p = 50.0$.

12. Given an established curve with $I = 47° 23'$, $D = 2° 30'$, and the *T.C.* at Sta. 1841 + 83.7. It is desired to move the forward tangent *outward* a distance of 60 ft., keeping the *C.T.* on the same radial line *approximately*. Find the new degree of curve, the station number of the *T.C.*, and the distance that the *C.T.* moves forward or backward due to using D to the nearest ten minutes only.

Answer. D, computed = $2° 38.7'$; D, used = $2° 40'$; *T.C.* = 1843 + 28.1; *C.T.* moves 7.6 ft. backward.

13. Data same as in Problem 12, except that the forward tangent is moved *inward*.

Answer. D, computed = $2° 22.2'$; D, used = $2° 20'$; *T.C.* = 1840 + 30.3; *C.T.* moves 16.7 ft. forward.

14. Given an established curve with $I = 50° 35'$, $D = 4° 30'$, and the *T.C.* at Sta. 155 + 24.5. It is desired to move the initial tangent 35 ft. *inward* without changing the position of the

C.T. (except the small amount due to using approx. value of *D*). Find the new degree of curve, the station numbers of the new *T.C.* and *C.T.*, the coordinates of the new *T.C.* from the old *T.C.*, and the small amount that the *T.C.* and the *C.T.* must shift due to the approx. value of *D* used.

Answer. *D*, computed = 4° 51.8′; *D*, used = 4° 50′; *T.C.*, computed = 155 + 98.6; *T.C.*, used = 155 + 94.8; *C.T.*, computed =166 + 38.7; *C.T.*, used = 166 + 41.3; $l = 70.3$; $p = 35.0$; *T.C.* and *C.T.* shifted 3.8 ft.

15. Data same as in Problem 14 except that the tangent is moved *outward.*

Answer. *D*, computed = 4° 11.1′; *D*, used = 4° 10′; *T.C.*, computed = 154 + 50.4; *T.C.*, used = 154 + 47.6; *C.T.*, computed =166 + 59.1; *C.T.*, used = 166 + 61.6; $l = 76.9$; $p = 35.0$; *T.C.* and *C.T.* shifted 2.8 ft.

16. After running in a 3° 00′ curve to the right, using paper location notes, it is found that the forward tangent passes to the right of the governing point by 2° 20′ as measured at the *C.T.* (a) What distance must the *C.T.* be shifted? (b) How much does the new tangent miss the controlling point?

Answer. (a) = 77.8 ft.; (b) = 1.6 ft.

17. Given the *P.I.* at Sta. 837 + 00, $I = 64° 44'$, $I_1 = 29° 00'$, $I_2 = 35° 44'$, $D_1 = 4° 00'$, and $D_2 = 5° 30'$. Find the station numbers of the *T.C.*, *C.C.*, and *C.T.*

Answer. *T.C.* =828+73.4; *C.C.* = 835 + 98.4; *C.T.* =842+48.1.

18. Given the *P.I.* at Sta. 1846 + 50.0, $I = 57° 18'$, $T_1 = 835.0$, $T_2 = 687.0$, and $D_1 = 3°.00'$. Find I_1, I_2, D_2, and the station numbers of the *T.C.*, *C.C.*, and *C.T.*

Answer. $I_1 = 16° 17.2'$; $I_2 = 41° 00.8'$; $D_2 = 4° 47.7'$; *T.C.* = 1838 + 15.0; *C.C.* = 1843 + 57.9; *C.T.* = 1852 + 13.2.

19. A 2° 30′ curve compounds with a 4° 00′ curve at Sta. 8792 +27.6. The central angle of the 4° 00′ curve = 26° 45′. It is desired to move the forward tangent *inward* 30 ft., but to retain the same degree of curve. Find the station number of the new *C.C.* and the coordinates of the new *C.T.* referred to the old *C.T.*

Answer. *C.C.* = 8790 + 61.7; $l = 54.5$ ft.; $p = 30.0$ ft.

20. Data the same as in Problem 19, except that the tangent is to be moved *outward.*

Answer. *C.C.* = 8794 + 21.9; $l = 66.4$ ft.; $p = 30.0$ ft.

21. A 5° 00′ curve compounds with a 3° 00′ curve at Sta. 147 +63.3. The central angle of the 3° 00′ curve is 19° 00′. The for-

ward tangent is to be moved 25 ft. *inward*, but the same degree of curve is to be retained. Find the station number of the new *C.C.* and the coordinates of the new *C.T.* referred to the old *C.T.*

Answer. *C.C.* = 149 + 03.8; l = 90.2 ft.; p = 25.0 ft.

22. Data same as in Problem 21, except that the tangent is to be moved *outward*.

Answer. *C.C.* = 146 + 61.2; l = 63.3 ft.; p = 25.0 ft.

23. A 4° 30′ curve compounds with a 7° 30′ curve at Sta. 999 +67.0. The central angle of the 7° 30′ curve is 39° 24′. The forward tangent is to be moved *inward* 50 ft. and the *C.T.* is to be kept approximately on the same radial line. Find the new degree of curve for the second branch, the station number of the new *C.C.*, and the coordinates of the new *C.T.* (actual) referred to the old *C.T.*

Answer. D, computed = 6° 32.8′; D, used = 6° 30′; *C.C.* = 996 +36.1; x = 5.16 ft. forward; p' = 47.35 ft.

24. Data same as in Problem 23, except that the tangent is to be moved *outward*.

Answer. D, computed = 12° 51.3′; D, used = 12° 50′; *C.C.* = 1003 + 31.6; x = 0.3 ft. forward; p' = 50.06.

25. A 5° 30′ curve compounds with a 3° 40′ curve at Sta. 1888 +36.2. The central angle of the 3° 40′ curve is 27° 50′. The forward tangent is to be moved *inward* 30 ft. and the *C.T.* is to be kept approximately on the same radial line. Find the new degree of curve for the second branch, the station number of the new *C.C.*, and the coordinates of the new *C.T.* (actual) referred to the old *C.T.*

Answer. D, computed = 2° 49.0′; D, used = 2° 50′; *C.C.* = 1890 + 84.4; x = 2.96 ft. backward; p' = 30.37 ft.

26. Data the same as in Problem 25, except that the tangent is to be moved *outward*.

Answer. D, computed = 4° 03.0′; D, used = 4° 00′; *C.C.* = 1886 + 01.9; x = 11.66 ft. forward; p' = 34.32 ft.

27. What is the amount of widening required on a 7° 00′ curve for busses 30.0 ft. long, neglecting any allowance for judging clearance?

Answer. 1.10 ft.

28. What would be the widening for the curve in Problem 27 by Voshell's formula?

Answer. 2.33 ft.

29. What is the clearance required from the center line of a 20-ft. pavement on an 8° 00′ curve for a sight distance of 500 ft.?

Answer. 43.6 ft.

CHAPTER 5

SPIRALS

General Characteristics

When circular curves are used there is an abrupt change from rectilinear to curvilinear motion, and vice versa, at the *T.C.* and the *C.T.* which results in shock to both the track and the rolling stock on railroads and adds to the difficulty of steering and to the danger of skidding on highways. A similar condition exists at the *C.C.* of compound curves.

On tangents the track or roadway should be level transversely, whereas on curves the outer rail or edge of roadway should be superelevated above the inner to counteract the effect of centrifugal force. If the theoretical need for superelevation were strictly followed there would be a vertical jog in the outer rail or across the roadway at the *T.C.* and the *C.T.* which obviously is impossible from the standpoint of operation. It is necessary, therefore, to attain the superelevation in some convenient distance termed the *runoff*.

If the runoff is placed entirely on the tangent, an objectionable tilting of the vehicle occurs before the curve is reached, and, since this tilt is not balanced by centrifugal force, the vehicle drifts inward. This causes a lateral pressure on the inner rail on railroads and adds to the steering difficulties on highways. If the runoff is placed entirely on the curve, the tilting on the tangent is eliminated, but the curve is entered with no superelevation to balance the centrifugal force. The result, again, is a lateral drift, this time outward with pressure on the outer rail or with increased steering troubles for the motor vehicle. It has been customary on railroads, therefore, to place part of the runoff on the tangent and part on the curve, thereby effecting a compromise but satisfying neither condition. On highways it has been customary to place the runoff entirely on the tangents, largely for the sake of appearance, thus placing the entire burden of proper steering on the driver.

In order to overcome these objections an easement curve, or transition curve, of varying radius and of the same length as the runoff, is placed at the ends of simple curves and between the branches of compound curves in such a manner that the change of curvature is smooth and gradual and the superelevation is at all times in agreement with the degree of curve. The form of such curve commonly used is a spiral based on the cubic parabola.[1]

138. Characteristics. The spiral is best defined by stating its principal characteristics, which are:

1. The curvature of the spiral increases uniformly from its beginning to its end. At the beginning, the *T.S.*, the curvature is zero; and at the end, the *S.C.*, it is the same as that of the circular curve for which the spiral serves as a transition. If the rate of increase in curvature is 1° per 100 ft., the degree of curve of the spiral will be 1° at Sta. 1, 2° at Sta. 2, 3° at Sta. 3, and so on. On a circular curve all points have the same degree of curve, whereas on a spiral every point has a different degree of curve.

2. Offsets from the tangent at the *T.S.* to points on the spiral vary approximately as the *cubes* of the distances along the curve, instead of approximately as the *squares* of the distances as in the case of circular curves. This characteristic is illustrated in Fig. 26.

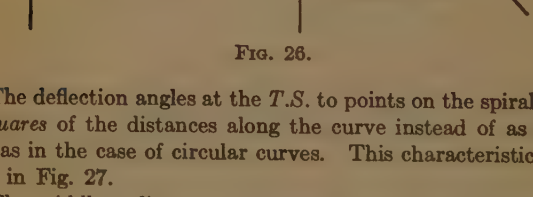

FIG. 26.

3. The deflection angles at the *T.S.* to points on the spiral vary as the *squares* of the distances along the curve instead of as the *first power* as in the case of circular curves. This characteristic is illustrated in Fig. 27.

4. The middle ordinates of successive equal chords increase uniformly as the distance along the spiral increases, instead of being

[1] Practically all of the spirals and easements curves in common use are based on this curve. They differ principally in the methods of computation and field location. The spiral here given can be substituted for any of them with no appreciable variation. For example, the so-called "10-chord spiral" is obtained by using ten chords of equal length in the formulas and methods here given.

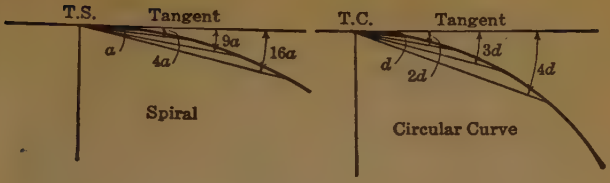

FIG. 27.

equal as in the case of circular curves. This characteristic is illustrated in Fig. 28.

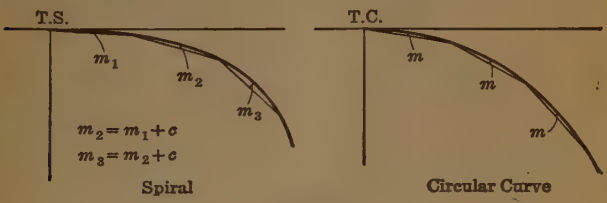

FIG. 28.

5. For a spiral, the triangle formed by the long chord as a base and the *P.I.* as a vertex is oblique, and the larger base angle is twice the smaller; whereas, for the circular curve the corresponding triangle is isosceles and the base angles are equal. This characteristic is illustrated in Fig. 29.

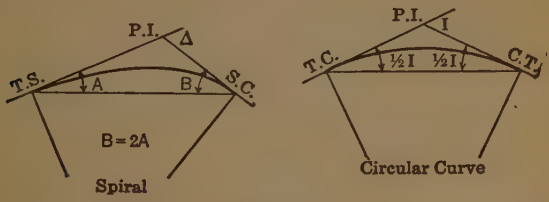

FIG. 29.

Classification of Spirals. Circular curves are classified or designated by their degrees of curve; for example, a 1° curve, a 2° curve, a 3° curve, and so on. Spirals are classified or designated by their rates of change of curvature per 100 ft. of distance. The symbol k is used to represent this rate of change. Spirals, then, are designated as $k = 1°$ spiral, $k = 2°$ spiral, $k = 3°$ spiral, etc.

139. Spiral Functions. The functions of the spiral are shown in Fig. 30, and are defined as follows:

T.S.—the point of change from tangent to spiral. The degree of curve at this point is zero.

S.C.—the point of change from spiral to circular curve. The degree of curve at this point is the same as that of the circular curve being spiraled.

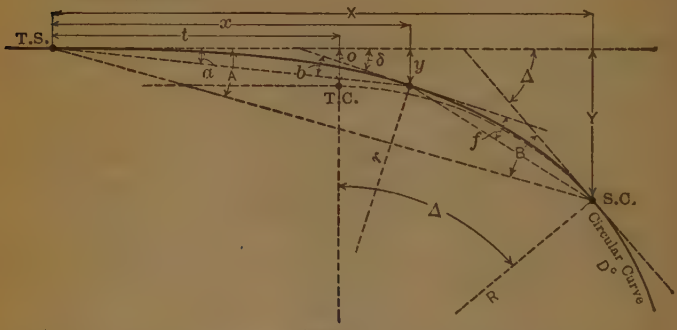

FIG. 30.

C.S.—the point of change from circular curve to spiral. It is geometrically the same as the *S.C.*

S.T.—the point of change from spiral to tangent. It is geometrically the same as the *T.S.*

a—the angle at the *T.S.* between the tangent at that point and a chord to any other point on the spiral—*the spiral deflection angle.*

A—*a* for the *S.C.*

b—the angle at any point on the spiral between a tangent at that point and a chord to the *T.S.*

B—*b* at the *S.C.*

d—degree of curve of the spiral at any point.

D—degree of curve of the circular curve to which the spiral connects.

f—the angle at any point on the spiral between a tangent at that point and a chord to any other point (*b* is a special case of *f*).

δ—the central angle of the spiral from the *T.S.* to any point.

Δ—the central angle of the spiral from the *T. S.* to the *S.C.*

k—the rate of change in the degree of curve of the spiral in degrees per station.

l—the length of the spiral in feet from the $T.S.$ to any point.

L—the length of the spiral in feet from the $T.S.$ to the $S.C.$

s—the length of the spiral in stations from the $T.S.$ to any point $= \frac{l}{100}$.

S—the length of the spiral in stations from the $T.S.$ to the $S.C.$ $= \frac{L}{100}$.

r—the radius of curvature of the spiral at any point (radius corresponding to d).

R—the radius of curvature of the circular curve to which the spiral connects.

x—the abscissa, in feet, of any point referred to the $T.S.$

X—the abscissa, in feet, of the $S.C.$ referred to the $T.S.$

y—the ordinate, or tangent offset, in feet, of any point.

Y—the ordinate, or tangent offset, in feet, of the $S.C.$

$T.C.$—the point of curve of central curve, produced back from the $S.C.$ to a tangent parallel to the initial tangent through the $T.S.$ This becomes the $C.T.$ when the stationing is in the opposite direction.

o—the ordinate, in feet, from the tangent through the $T.S.$ to the $T.C.$

t—the abscissa, in feet, of the $T.C.$ referred to the $T.S.$

$P.I.$—the point of intersection of the tangents of the spiraled curve.

$I.$—the total central angle of the spiraled curve.

T_s—the tangent distance of a symmetrically spiraled curve. ($T.S.$ to $P.I.$ and $P.I.$ to $S.T.$)

T—the tangent distance of an unspiraled curve of the same D and I.

E_s—the external distance of the spiraled curve.

E—the external distance of an unspiraled curve of the same D and I.

140. Formulas.

From definition,

$$\left.\begin{aligned} d = ks &= \frac{kl}{100} \\ D = kS &= \frac{kL}{100} \end{aligned}\right\} \qquad (30)$$

The derivations of the following formulas require the use of the calculus and are given in the Appendix.

$$\delta \text{ (in degrees)} = \tfrac{1}{2}ks^2,\ \Delta = \tfrac{1}{2}kS^2 \tag{31}$$

$$\left.\begin{aligned} a \text{ (in degrees)} &= \tfrac{1}{3}\delta = \tfrac{1}{6}ks^2 \\ a \text{ (in minutes)} &= 10ks^2, \\ A \text{ (in degrees)} &= \tfrac{1}{3}\Delta = \tfrac{1}{6}kS^2 \end{aligned}\right\} \tag{32}$$

$$\left.\begin{aligned} b \text{ (in degrees)} &= \tfrac{2}{3}\delta = 2a, \\ B \text{ (in degrees)} &= \tfrac{2}{3}\Delta = 2A \end{aligned}\right. \tag{33}$$

$$\left.\begin{aligned} y &= 0.291ks^3 - 0.00000158k^3s^7 \\ Y &= 0.291kS^3 - 0.00000158k^3S^7 \end{aligned}\right\} \tag{34}$$

$$\left.\begin{aligned} x &= l - 0.000762k^2s^5 \\ X &= L - 0.000762k^2S^5 \end{aligned}\right\} \tag{35}$$

$$o = 0.0727kS^2 \tag{36}$$

$$t = \tfrac{1}{2}L - 0.000127k^2S^5 \tag{37}$$

$$T_s = T + o\tan\tfrac{1}{2}I + t = (R + o)\tan\tfrac{1}{2}I + t \tag{38}$$

$$E_s = E + \frac{o}{\cos\frac{1}{2}I} = (R + o)\operatorname{exsec}\tfrac{1}{2}I + o \tag{39}$$

The spiral departs from any osculating circle at the same rate as from the tangent through the $T.S.$ (40)

The spiral and o mutually bisect. (41)

Table 12 gives the values of L, Δ, o, t, X, and Y for values of k and D commonly used.

Application to Simple Curves

141. Field Work. Although it is possible to have spirals with different values of k at the two ends of a simple curve, there is rarely, if ever, any sound reason for so doing; hence the curves are always spiraled symmetrically.

The first step in the field work is to determine the station numbers of the $T.S.$, $S.C.$, $C.S.$, and $S.T.$ The $T.S.$ and the $S.T.$ are located on the ground by measuring the distance T_s from the $P.I.$

From Fig 31,

$$T_s = T + o\tan\tfrac{1}{2}I + t \tag{42}$$

T is found from Table 13, o is found from Table 12 or Eq. 36, and t is found from Table 12 or Eq. 37.

From Fig. 31 it is evident that the portion of the circular curve to be retained has a central angle of $I - 2\Delta$, and hence its length is

$$L_c = \frac{I - 2\Delta}{D} = \frac{I}{D} - S \qquad (43)$$

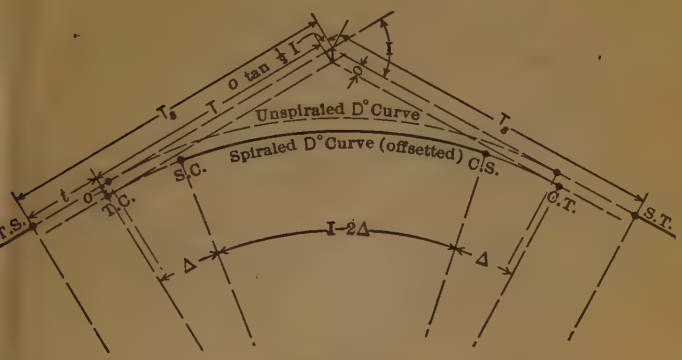

FIG. 31.

It should therefore be remembered that, in checking the deflection angles for the circular portion of the layout, the final deflection angle, or the *C.S.*, should be $\frac{1}{2}(I - 2\Delta)$.

Example. Given:—*P.I.* = 21 + 21.1, $D = 3° 40'$, $I = 51° 20'$, and $k = 1°$.

Sta. *P.I.* =		21 + 21.1
From Table 13,	$T = 750.9$	
From Table 12, $o = 3.58$,	$o \tan \frac{1}{2}I = 1.7$	
From Table 12,	$t = 183.3$	
	$T_s =$	9 35.9
	Sta. *T.S.* =	11 + 85.2
	$S =$	3 66.7
	Sta. *S.C.* =	15 + 51.9
	$L_c =$	10 33.3
	Sta. *C.S.* =	25 + 85.2
	$S =$	3 66.7
	Sta. *S.T.* =	29 + 51.9

Spirals are staked out by deflection angles, by offsets, or by middle ordinates.

142. Deflection Angles. The process of staking out spirals by deflection angles is the same as that for simple curves. Spirals rarely exceed 500 ft. in length, and the total deflection angle to the *S.C.* rarely exceeds 5° 00′, therefore *the entire spiral can be run in from the T.S.*[3]

Since a spiral is more difficult to aline than a simple curve, stakes should be placed closer together; and since the spiral becomes sharper as it increases in length, stakes should be placed at shorter intervals at the end than at the beginning.

RULE:—*Place stakes 50 ft. apart on all spirals up to the point where the degree of curve (d) becomes about 3°; beyond this point place them 25 ft. apart.*

For ease in computation, stakes are placed the above distances apart *beginning at the T.S.;* hence in general *all* the stakes will fall at plusses.

The spiral deflection angles are from Eq. 32

$$a \text{ (in minutes)} = 10ks^2$$

Note that the spiral deflection angles vary with the *square* of the distance instead of the first power as in simple curves.

In Fig. 32 point 1 is located by a 50-ft. chord from the *T.S.* and the angle a_1; point 2 by a 50-ft. chord from 1 and the angle a_2; and so on, noting the change in chord length when d becomes 3°. From Eq. 32,

$$a_1 = 10'k(0.5)^2$$

$$a_2 = 10'k(1.0)^2 = 4a_1$$

$$a_4 = 10'k(1.75)^2 = \frac{49}{4}a_1$$

$$a_5 = 10'k(2.0)^2 = 16a_1$$

$$A = a_6 = 10'k(2.25)^2 = \frac{81}{4}a_1 = \frac{1}{3}\Delta$$

It is thus seen that the deflection angle for the first station only needs to be determined from Eq. 32. The other values are found by multiplying this value by the square of the ratio of the other distances to the first distance.

[3] For running in spiral from an intermediate point on spiral, see section 146.

Example. Assume that in Fig. 32, *T.S.* is at Sta. 711 + 44.0, = 2, and $S = 2.25$. Then $D = 4° 30'$.

Deflection angle of *T.S.*, Sta. 711 + 44.0 = 0° 00′.

" " " 1 " 711 + 94.0 = 0° 05′
" " " 2 " 712 + 44.0 = 0° 20′
" " " 3 " 712 + 94.0 = 0° 45′
" " " 4 " 713 + 19.0 = 1° 01′ (1° 01.25′ exact)
" " " 5 " 713 + 44.0 = 1° 20′
" " *S.C.*(*A*) " 713 + 69.0 = 1° 41′ (1° 41.25′ exact)

$A = \frac{1}{2}\Delta$, $\Delta = 5° 04'$

$B = \frac{2}{3}\Delta = 2A = 3° 22.5'$

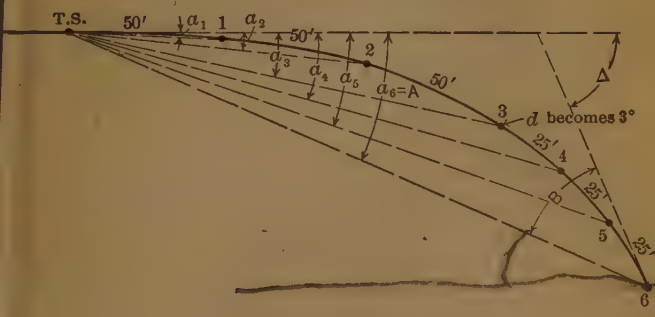

Fig. 32.

To orient the transit at the *S.C.*, back-sight on the *T.S.* with the plates set at 3° 22.5′ (angle *B*); then turn the plates to zero and the telescope is on tangent and the circular curve is run in as from the *T.C.* of a simple curve, *stakes being placed at the regular stations* (in the above problem at 714 + 00, 714 + 50, etc.). After running in about half of the circular curve, move the transit to the *S.T.* and run in the second spiral with the same deflection angles. Then move to the *C.S.* and *back-in* the remainder of the circular curve, thus placing the adjustment of the *errors of surveying* at the center of the curve instead of at the end of the spiral as is often but *unwisely* done. The form of notes is shown in Fig. 33.

143. Offsets. Evidently the entire spiral can be located by means of the coordinates x and y (tangent offsets), and this is a satisfactory method for short flat spirals where y is less than about 10 ft. When y becomes greater than this, the spiral can not be

TRANSIT NOTES FOR LINE L

STATION	ALINEMENT	TOTAL DEFL. ANGLE	CALC. BEARING	MAG. BEARING	REMARKS
1027	Tan.		S 64°-14′ W	S 64°-10′ W	
26+23.1	S.T. ⊙	0°-00′			
+73.1	Spiral $k = 1°$ Backed in from S.T.	0°-02.5			
25+23.1		0-10			
+73.1		0-22.5			
24+23.1		0-40			
+73.1		1-02.5			
+23.1	C.S. ⊙	1-30			
+23.1	C.S. ⊙	8°-45′	$= \frac{1}{2} I_c$	(check)	
23		8-24			
22		6-54			P.I. 1020+40
21	3°-00′ C.R.	5-24			I = 26°-30′
1020		3-54			D = 3°-00′
19		2-24			T_s = 600.2
18		0-54			
17+39.8	S.C. ⊙	0°-09′			
17+39.8	S.C. ⊙	1-30			$k = 1°$
+89.8		1-02.5			L = 300.0
16+39.8	Spiral $k = 1°$	0-40			Δ = 4°-30′
+89.8		0-22.5			A = 1°-30′
15+39.8		0-10			B = 3°-00′
+89.8		0-02.5			
+39.8	T.S. ⊙	0°-00′			
1014	Tan.		S 37°-44′ W	S 37°-50′ W	

FIG. 33.

Mike McCarthy - Inst.
Heine Heinrichson - H.C.
Oley Olsen - R.C.
S. Garibaldi - R.F.

28

June 17, 1930
Clear - Hot

Set up at S.T. plates at 0°00′; B.S. along Tangent;
Back in Spiral to C.S.

$I_c = I - 2\Delta = 17° - 30'$

B.S. at T.S., Plates at 3°-00′. Plunge and turn to
0°-00′ for Tangent.

FIG. 33.

located with sufficient accuracy unless the offsets are turned off with an instrument.

On location, since it is usually desirable to advance the line as rapidly as possible, the best method is to run-in the circular curve from the *T.C.* to the *C.T.* (offsetted curve, see Fig. 31), and to insert the spiral later by offsets from both the tangent and the circular curve, or by deflection angles.

Since the spiral departs from an osculating circle at any point at the same rate that it departs from the tangent at the T.S. (see Eq. 40 and Sec. 264 in Appendix), it follows that the offset from the circular curve to the spiral at any distance from the *S.C.* is the same as the offset from the tangent at the *T.S.* for the same distance. Since the offset o and the spiral bisect each other (see Eq. 41 and Sec. 263 in Appendix), it is evident that the maximum offset is $\frac{1}{2}o$ and it will therefore be small; also, that it will be necessary to compute offsets for half the spiral only. Half of the spiral is then located by offsets y from the tangent at the *T.S.*, and the other half by the same offsets measured normal to the circular curve.

From Eq. 34 it is seen that the offsets vary approximately as the cube of the distances, and for this method of location, and for highway spirals, they can be so considered with inappreciable error. Since the maximum ordinate is $\frac{1}{2}o$, and o is determined to offset the *T.C.*, it is seen that the offsets can be determined directly from o instead of from Eq. 34. Furthermore, it is not necessary to compute the offsets for even chord lengths, but the half-spiral can be divided into any number of equal parts and the ordinates to these points will be in the ratio of 1^3, 2^3, 3^3, 4^3, etc.

144. Middle Ordinate Method. The spiral can be staked out by the middle ordinate method. This method is especially useful in realining existing curves, where the movement of trains interferes seriously with the deflection angle method, since that method requires that the transit be set up in the center of the track.

The value of the middle ordinate of a chord of length n (in 100-ft. units), at any portion of the spiral, is approximately

$$m = 0.218\, n^2 \frac{(D_1 + D_2)}{2} \qquad (44)$$

in which D_1 and D_2 are the degrees of curve of the spiral at the ends of the chord. For values of k less than 1.5°, this equation will give values of m which are correct to the nearest 0.01 ft.

Table 5 gives the correct values of successive middle ordinates, r 100-ft. chords, for spirals whose k's are 0.25°, 0.50°, 1.00°, 1.25°, .50°, and 2.00°. For spirals with other values of k, the middle rdinates can be obtained by interpolation. It is to be noted that ue values in the table for Sta. 0 + 50 are tangent offsets, rather than iddle ordinates, as it is more convenient to locate that station by its ngent offset.

TABLE 5

Middle Ordinates for Spirals

Station on spiral	Middle Ordinates					
	$k = 0.25°$	$k = 0.50°$	$k = 1°$	$k = 1.25°$	$k = 1.50°$	$k = 2°$
	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.
T.S. = 0+00	0.00	0.00	0.00	0.00	0.00	0.00
+50	0.01*	0.02*	0.04*	0.05*	0.06*	0.07*
+75	*0.01*	*0.02*	*0.04*	*0.05*	*0.06*	*0.08*
1+00	0.03	0.05	0.11	0.14	0.16	0.22
+25	*0.02*	*0.03*	*0.07*	*0.09*	*0.10*	*0.14*
+50	0.06	0.11	0.22	0.27	0.33	0.44
+75	*0.02*	*0.05*	*0.10*	*0.12*	*0.14*	*0.19*
2+00	0.08	0.16	0.33	0.41	0.49	0.65
+25	*0.03*	*0.06*	*0.12*	*0.16*	*0.18*	*0.24*
+50	0.11	0.22	0.44	0.55	0.65	0.87
+75	*0.04*	*0.07*	*0.15*	*0.19*	*0.22*	*0.29*
3+00	0.14	0.27	0.55	0.68	0.82	1.08
+25	*0.04*	*0.09*	*0.18*	*0.22*	*0.26*	*0.34*
+50	0.16	0.33	0.65	0.81	0.97	1.28
+75	*0.05*	*0.10*	*0.20*	*0.25*	*0.30*	*0.38*
4+00	0.19	0.38	0.76	0.94	1.12	1.46
+25	*0.06*	*0.11*	*0.23*			
+50	0.22	0.44	0.86			
+75	*0.06*	*0.13*	*0.25*			
5+00	0.25	0.49	0.95			

* This is a tangent offset instead of a middle ordinate.

On new work, the spiral is staked out as follows, assuming that the T.S. is at Sta. 0 and using a $k = 1°$ spiral for illustration. Sta. + 50 is located by extending the tangent 50 ft. beyond the T.S. nd driving a stake at this point. To extend the tangent accurately, strong, light, 100-ft. string (a silk fishline is generally used) is

stretched tightly with its rear end at a point 50 ft. back of the *T.S.* and its middle over the point at the *T.S.* A pencil mark is then made on the stake. The tangent offset, which in this problem is 0.04 ft., is then measured toward the center of the curve, and a second mark is made on the stake to give the position of Sta. 0 + 50 on the spiral. After this pencil mark has been used in the location of subsequent points on the spiral, a tack should be driven, but *not before.*

Sta. 1 is located by holding the rear end of a 100-ft. steel tape at the *T.S.* and swinging the tape in an arc until the middle point of the tape is a distance from Sta. 0 + 50 equal to the middle ordinate for that station, which in this example is 0.11 ft. It is to be noted that this value is given in Table 5 opposite Sta. 1 and is the value used in locating Sta. 1, but it is the middle ordinate measured at Sta. 0 + 50. After a stake has been driven at Sta. 1, and a 100-ft. distance from the *T. S.* marked on it, the point marking the exact location of Sta. 1 is obtained by the use of the 100-ft. string.

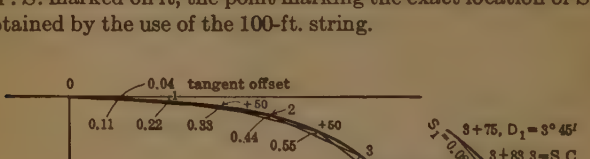

FIG. 34.

Sta. 1 + 50 is similarly located by measuring a middle ordinate of 0.22 ft. from the 100-ft. chord from Sta. 0 + 50 to Sta. 1 + 50, and by a similar procedure the remaining 50-ft. stations on the spiral are located as shown in Fig. 34. After this has been done, if it is desired to place stakes at 25-ft. intervals, the middle ordinates given in italics in Table 5 opposite the +25's and +75's are used. The procedure here, however, is somewhat different. For example, if a stake is desired at Sta. 3 + 25, a 50-ft. string is stretched between Sta. 3 + 00 and Sta. 3 + 50, and a middle ordinate of 0.18 ft. is used to locate Sta. 3 + 25.

Suppose the circular curve being spiraled is a 3° 50′ curve. With $k = 1$, the length of the spiral will be 383.3 ft. To place a stake at Sta. 3 + 83.3, the *S.C.*, a temporary stake is placed at Sta. 4, and a permanent stake at Sta. 3 + 75, as above described. The string is then stretched between Sta. 3 + 75 and Sta. 4, and Sta. 3 + 83.3 is

located by the ordinate m' at a distance of 8.3 ft. from Sta. 3 + 75. This ordinate is computed from the equation,

$$m' = 0.873\, S_1 S_2 \frac{D_1 + D_2}{2} \text{ (Approx.)} \qquad (44a)$$

in which S_1 and S_2 are the two segments, in stations, into which the 25-ft. chord is divided (in this case, 0.083 and 0.167), and D_1 and D_2 are the degrees of the spiral at Stas. 3 + 75 and 4 + 00, respectively, which in this problem are 3° 45′ and 4° 00′. The substitution of these values in Eq. 44a gives m' a value of 0.05 ft.

If the ground is fairly level and free from vegetation, and care is taken in stretching the string and in measuring the middle ordinates, a high degree of accuracy can be obtained by this method. The principal objection to the method, as compared with the deflection angle method, is that an error made in locating any station affects all subsequent stations and becomes larger as the distance from the point of error increases. For this reason, the deflection angle method is preferable for new work. For the realinement of existing curves, however, the middle ordinate method is the better of the two, especially if the ends of the spiral were marked by permanent monuments originally. This method is considered further under the subject of String-lining Curves in Chapter 6.

Application to Compound Curves

At the *C.C.* of a compound curve—as at the *T.C.* of a simple curve—there is a change in the rate of curvature and in the amount of superelevation, and if this is great enough to be objectionable, a spiral should be inserted between the two branches.

Evidently only that part of the spiral of curvature intermediate between the degrees of the two curves is required, and—as at the *T.C.* of a simple curve—the two curves must be offsetted at the *C.C.*

145. Problem 1. In Fig. 35, *HBC* is a D_1 curve and *EFJ* is a D_2 curve having parallel tangents at *C* and *E* (the position of the *C.C.* if the curve were unspiraled). It is desired to connect the two curves by the spiral *BF*. Consider the spiral run backward to its *T.S.* at *A*. Since the degree of curve of the spiral at *B* must be D_1 and at *F* must be D_2, then the spiral from *B* to *F* is that part of a regular spiral from where $d = D_1$ to where $d = D_2$. The value of l_s depends upon the maximum speed permissible on the *sharper* curve and upon the difference in the two degrees of curve.

Since the spiral departs from every osculating circle at the same rate as from the tangent at the *T.S.*, $BC = EF$, the spiral bisects CE, and CE is equal to o for a spiral whose $D = D_2 - D_1$ for the chosen value of k.

To insert a spiral between the curves, find o for a $D_2 - D_1$ curve

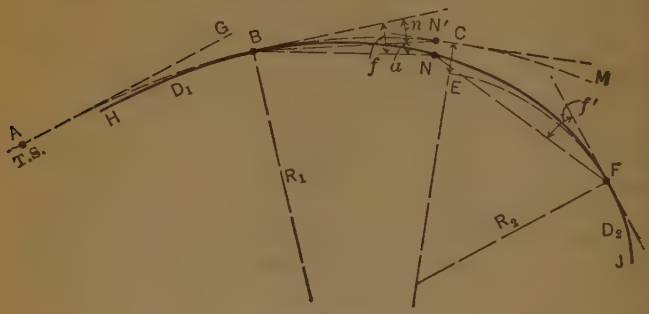

FIG. 35.

and make the offset from C to E. The length of the spiral in stations is

$$S_1 = \frac{D_2 - D_1}{k} \qquad (45)$$

Locate B and F by measuring $\frac{1}{2}S_1$ from C and E.

The curves may be staked out by continuing the first branch to C, offsetting to E, and running-in the second branch; and then inserting the spiral by offsets from the circular curves in exactly the same way as explained in section 143.

146. Deflection Angles may also be used, but, since the transit is at a point on the spiral, B or F, and not at the *T.S.*, the deflection angles to be used are values of f instead of a.

From a point on the tangent at B in Fig. 35, the deflection angle n to any point N' on the circular curve BCM is $\frac{1}{2}D_1 \times BN'$.

Since the spiral departs from the osculating circle at the same rate as from the tangent at the *T.S.*, the angle in degrees between the circular curve and a point N on the spiral is $a = \frac{1}{6}k(BN)^2$.

But BN and BN' are equal, whence

$$f = n + a = \tfrac{1}{2}D_1(BN) + \tfrac{1}{6}k(BN)^2 \qquad (46)$$

the transit were set at F, the deflection angle to N would be

$$f' = n - a = \tfrac{1}{2}D_2(FN) - \tfrac{1}{6}k(FN)^2 \qquad (46a)$$

Example. It is desired to place a spiral with $k = 1°$ between the branches of a compound curve, where D_1 is 2° and D_2 is 6° and the *C.C.* is at Sta. 50.

From Eq. 45, the length of the spiral will be 400 ft.; therefore, the point B, the *C.S.*, in Fig. 35 comes at Sta. 48, and the point F, the *C.C.*, at Sta. 52. With the transit on tangent at Sta. 48, and the vernier set at zero degrees, the deflection angles for locating Stas. 49, 50, 51, and 52 on the spiral are found from Eq. 46, as follows:

Station	*Deflection Angles*	
C.S. 48	On tangent	$= 0°\,00'$
49	$f = n+a = \frac{1}{2}\times2\times1+\frac{1}{6}\times1\times(1)^2 = 1°\,00' + 0°\,10' = 1°\,10'$	
50	$f = n+a = \frac{1}{2}\times2\times2+\frac{1}{6}\times1\times(2)^2 = 2°\,00' + 0°\,40' = 2°\,40'$	
51	$f = n+a = \frac{1}{2}\times2\times3+\frac{1}{6}\times1\times(3)^2 = 3°\,00' + 1°\,30' = 4°\,30'$	
C.C. 52	$f = n+a = \frac{1}{2}\times2\times4+\frac{1}{6}\times1\times(4)^2 = 4°\,00' + 2°\,40' = 6°\,40'$	

If the transit had been set-up over Sta. 52, the deflection angles, computed from Eq. 46a, would be as follows:

Station	*Deflection Angles*	
C.C. 52	On tangent	$= 0°\,00'$
51	$f' = n - a = \frac{1}{2}\times6\times1 - \frac{1}{6}\times1\times(1)^2 = 3°\,00' - 0°\,10' = 2°\,50'$	
50	$f' = n - a = \frac{1}{2}\times6\times2 - \frac{1}{6}\times1\times(2)^2 = 6°\,00' - 0°\,40' = 5°\,20'$	
49	$f' = n - a = \frac{1}{2}\times6\times3 - \frac{1}{6}\times1\times(3)^2 = 9°\,00' - 1°\,30' = 7°\,30'$	
C.S. 48	$f' = n - a = \frac{1}{2}\times6\times4 - \frac{1}{6}\times1\times(4)^2 = 12°00' - 2°40' = 9°\,20'$	

147. Problem 2. Fig. 36 shows a compound curve in which the *P.I.*, I_1, I_2, D_1, and D_2 are given. This curve is to be spiraled at the ends and between the two branches with spirals k_1, k_2, and k_3, in order. The tangent distances T_{s1} and T_{s2} are required. The solution is as follows:

From a point on the first given tangent opposite the *T.C.* of the D_1 curve draw an auxiliary circle (shown dotted) with a radius of $R_1 + o_1$ through the angle I_1 to the *C.C.* From this point draw a second circle with a radius of $R_2 + o_2 + o_1$ through the angle I_2 and ending in an auxiliary tangent parallel to the second given tangent at a point opposite the *C.T.* of the D_2 curve. This auxiliary tangent

will lie outside or inside the given tangent, depending on whether $o_1 + o_2$ is greater or less than o_3, and will intersect the first given tangent at a point other than the given $P.I.$ These two auxiliary circles form an ordinary compound curve concentric with the given circles as offsetted to provide room for the spirals.

The tangent distances of this auxiliary compound curve may now be computed in the usual way, as given in section 127. In triangle 1, the base is equal to the sum of the two tangent distances of the auxiliary circles, which are $(R_1 + o_1) \tan \frac{1}{2} I_1$ and $(R_2 + o_2 + o_1) \tan \frac{1}{2} I_2$, respectively, and the angles are I_1, I_2, and $180° - I$. The other two

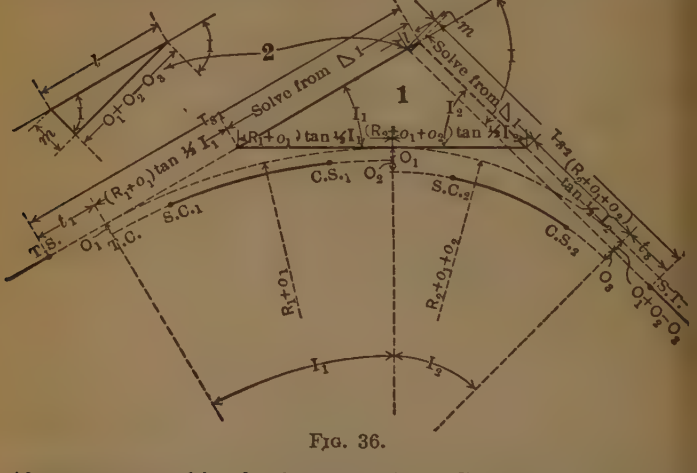

FIG. 36.

sides are computed by the sine proportion. Combining these values with the corresponding auxiliary tangent distances and the values of t_1 and t_3, there is obtained the distances from the auxiliary $P.I.$, $P'.I'$, to the $T.S.$ and the $S.T.$ It is now necessary to correct these distances to the given $P.I.$ To do this, form triangle 2 by dropping a perpendicular from the auxiliary $P.I.$ to the second given tangent. In this triangle the base is $o_1 + o_2 - o_3$ and the given angle is I. Compute the sides l and m.

If the auxiliary tangent lies outside the given tangent, $o_1 + o_2 - o_3$ is positive, as shown in Fig. 36, and the side l is subtracted from the computed distance to the $T.S.$ to obtain the desired T_{s1}. At the same time, the side m is added to the computed distance to the $S.T.$ to obtain T_{s2}.

If the auxiliary tangent lies inside the given tangent, and this occurs when o_3 is greater than $o_1 + o_2$, then $o_1 + o_2 - o_3$ is negative, and l and m are applied with signs opposite to those above.

If I is greater than 90°, triangle 2 changes position with respect to the *P.I.*, and the signs of l and m are opposite to those indicated above in each case. To avoid mistakes, however, it is always desirable to draw a sketch of this triangle in proper position with respect to the *P.I.* and determine by inspection the proper signs for l and m.

Multiple-centered compound curves can be computed by an extension of this method.

Railroad Spirals

148. Choosing S and k. Since the function of the spiral is to ease the entrance to a circular curve, D will always be known. Then in Eq. 30, it is necessary only to choose a value of k or S, and the spiral is fixed. Obviously, any value of k could be chosen and the spiral would fit, but such a spiral may be so short as to require an excessive rate of attaining the superelevation, or it may be needlessly long. Similarly any value of S might be chosen and the spiral would fit, but in this case an odd value of k is likely to result, and this complicates the calculation and the field work. The logical method, therefore, is first to determine a satisfactory length of spiral to give a suitable rate of superelevation, and from this to compute an approximate value of k from which a simple value, convenient for computations and field work, may be chosen.

The length of the runoff, and hence the length of the spiral, is a direct function of the total superelevation and of a rate of attaining this superelevation that will be comfortable to passengers. The total superelevation for standard-gage track depends upon the speed and the degree of curve. It is given by the equation,

$$e^* = 0.00069\, DV^2 \qquad (47)$$

in which e is the superelevation in inches, D is the degree of curve, and V is the speed in miles per hour. The value of e should not exceed about 8 in., on account of slowly moving trains, but the track should be superelevated for the fastest trains up to this limit. The American Railway Engineering Association states that curves requiring a superelevation of less than 2 in. need not be spiraled, but many railroads spiral for superelevations of 1 in. With the coming of the

* For derivation, see Appendix, section 265.

high-speed "streamliners" there has been a distinct tendency to superelevate for lower values of e, down to $\frac{1}{2}$ in.

The maximum rate at which superelevation may be attained without discomfort to passengers may be taken as $1\frac{1}{8}$ in. per second. Therefore the minimum desirable length of runoff is

$$L = \tfrac{8}{7}\, ev$$

where v is the velocity in feet per second. Reducing v to V, the speed in miles per hour,

$$L = 1.26\, eV$$

Substituting the value of e from Eq. 47, and changing D to R,

$$L = 0.00087\, DV^3 = \frac{5V^3}{R} \text{ (approximate)} \qquad (48)$$

Substituting the value of D from Eq. 30 and solving for k,

$$k = \frac{115{,}000}{V^3} \text{ (approximate)} \qquad (49)$$

It is evident, therefore, that a suitable value of k can be determined directly when the speed of operation is known. Consequently the proper procedure in any given case is to determine first the normal maximum speed to be expected. Then from Eq. 49 compute a trial value of k from which a simple value, convenient for computation and field use, is selected.

The diagram, Fig. 37, is plotted from values computed by the preceding equations. It gives k, L, and e for curves from 0° 30′ to 10° 00′ and speeds from 30 to 100 miles per hour. From this diagram, with any two of the five values, D, V, k, e, or L given, the other three may be determined within the accuracy of scaled values. It is especially useful in determining k and e when D and V are known. The relation between k and L should always be determined from Eq. 30 for the sake of accuracy.

Examples. (*a*) A 4° curve is to be operated at 50 miles per hour. Entering the diagram with $V = 50$ and following the horizontal line to its intersection with the curved line marked $D = 4°\ 00'$, we can read directly that the superelevation should be about 7 in., that the theoretical length should be a little more than 425 ft., and that k should be a little less than 1°. The nearest convenient value of k is 1°, and, when this value is substituted in Eq. 30, L is found to be

RAILROAD SPIRALS
$k = \frac{115,000}{V^3}$
$e = 0.00069 DV^2$
Speed-miles per hour
0
100
200
300
400
500
100
90
80
70
60
50
40
30
k=⅛
k=¼
k=⅜
k=½
k=⅝
k=¾
k=1
k=1¼
k=1½
k=1¾
k=2
k=2½
k=3
k=4
e=1"
e=2"
e=3"
e=4"
e=5"
e=6"
e=7"
e=8"
e=9"
e=10"
D=0°30'
D=0°46'
D=1°00'
D=1°30'
D=2°00'
D=2°30'
D=3°00'
D=4°00'
D=5°00'
D=6°00'
D=7°00'
D=8°00'
D=9°00'
D=10°00'

FIG. 37.

400 ft., which is verified by the diagram. Thus the curve should be spiraled with a $k = 1°$ spiral 400 ft. long, and the superelevation should be 7 in. to agree with the speed of 50 miles per hour.

(*b*) A given 2° 30′ curve is superelevated 6 in. Following the curved line marked 2° 30′ to its intersection with the inclined line, marked $e = 6$ in., we find that V is about 59 miles per hour, and L is 450 ft. From Eq. 30 the value of k corresponding to $L = 450$ is 0.5555, which is an inconvenient value to use. The diagram shows that the nearest convenient value is $k = \frac{1}{2}$, which would be used, giving a spiral length of 500 ft.

(*c*) It is proposed to use a superelevation of 4 in. with a runoff of 300 ft. From the intersection of the $e = 4$-in. line with the $L = 300$-ft. line, we find that the speed is about 60 miles per hour, the degree of curve is 1° 30′, and the value of k is $\frac{1}{2}°$.

149. Variation in Speeds. Since the centrifugal force due to curvature varies with the square of the speed, while the amount of superelevation in the track remains fixed, it follows that a curve can be theoretically superelevated for only one speed, although the trains will actually operate at different speeds.

If the speed is below that for which the curve is theoretically superelevated, there will be an unbalanced, inward component which may be increased somewhat by a component of the drawbar pull. This force must be resisted by flange pressure against the inner rail. This force will tend to overturn the rail; hence rail braces are frequently used on the inside rail of heavily superelevated tracks. At the same time this pressure tends to move the track inward and often results in putting the track out of alinement. Also, in case of slowly moving passenger trains, the unbalanced tilt may be objectionable to passengers. These are the reasons for limiting the maximum superelevation to about 8 in.

If the speed is greater than that for which the track is superelevated, the centrifugal force exceeds the component due to slope, and a pressure on the outer rail results. Thus, rail braces are often found also on the outside rails to resist the tendency for the rail to overturn. This pressure also tends to throw the track out of line. When this is combined with the tendency for slow trains to move the track inward, it often happens that the track gets out of alinement very quickly.

The theoretical speed for which the track is superelevated is known as the "equilibrium speed." Trains may, however, operate at a maximum speed in excess of the equilibrium speed. According to

the American Railway Engineering Association, the maximum speed may exceed the equilibrium speed by an amount equal to an increase of 3 in. in the superelevation. For example, a track superelevated 6 in. may be safely operated at a speed requiring a theoretical superelevation of 9 in.

Application to Existing Curves

To insert a spiral in an existing track, it is necessary to shift the ends of the curve inward to provide room for the spiral. If the same degree of circular curve were retained, this would shift the *entire curve* inward. Since the amount of such shifting may be con-

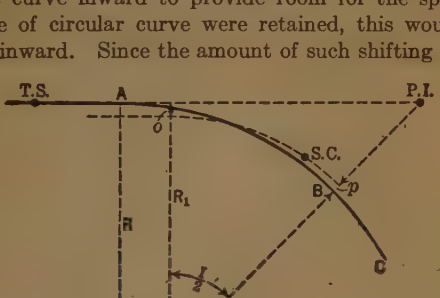

FIG. 38.

siderable (o sec $\frac{1}{2}I$ at its center) the new alinement may not be on the old roadbed and considerable cost of earthwork would be entailed in making the change. To obviate this, the degree of curve may be changed in such a way that the new alinement will permit the insertion of the spiral and at the same time require little or no additional earthwork.

150. SIMPLE CURVES. Existing simple curves may be spiraled in two ways.

(1) By shifting the center of the curve *outward*[4] a small amount, and by sharpening the curve sufficiently to give the desired offset. This method is particularly applicable for curves whose lengths are less than about four times the length of the spiral to be used.

[4] The shifting must be *outward* to make the change in alinement a minimum.

(2) By sharpening the curve slightly at the ends to give the necessary offset, leaving the middle portion of the curve unchanged. This method is preferable for long curves.

151. First Method.

In Fig. 38, ABC is the existing unspiraled curve with B as its middle point. It is desired to shift the track *outward* at B the *assumed* distance p, and to sharpen the curve to give the necessary offset o.

From the figure, p is the difference between the external distances of the existing curve and the spiraled curve. Therefore from Eqs. 11 and 39, we have

$$p = R \operatorname{exsec} \tfrac{1}{2}I - [(R_1 + o) \operatorname{exsec} \tfrac{1}{2}I + o]$$

$$= (R - R_1 - o) \operatorname{exsec} \tfrac{1}{2}I - o \qquad (50)$$

In Eq. 50, both R_1 and o are unknown, but $o = 0.0727 \dfrac{(5730)^3}{R_1{}^3 k^2}$. This value of o could be substituted in Eq. 50 and an expression for R_1 found, but obviously the expression would be a difficult one to use. It is faster and simpler to solve the equation by successive approximations as follows: Choose a value of R_1 less than R (preferably to agree with an even 10 minutes of degree of curve) and solve Eq. 50 for p. If this value agrees sufficiently closely with the desired shifting of the track at B, the solution is complete. If not, re-estimate R_1 (using the previous value as a guide), and repeat until a satisfactory value of p is obtained.

The distance from the $T.C.$ of the original curve to the $T.S.$ is the difference between T_s for the new curve and T for the old curve. The $T.S.$ and the $S.T.$ are then located from the $T.C.$ and the $C.T.$, respectively, of the old curve, and the new curve is staked out as in new work.

Example. $I = 40° 00'$, $D = 4° 00'$, $k = 1°$, and p = about 1.5 ft.

Choose a trial value of $R_1 = 1322.3$ ft. for $D_1 = 4° 20'$. Then from Eq. 50, $p = 0.8$ ft. which is less than that desired. Choosing $R_1 = 1312.2$ ft. for $D_1 = 4° 22'$, p becomes 1.3 ft., showing that a change of $2'$ in D_1 increases p about 0.5 ft. Then to make $p = 1.5$ ft., D_1 by interpolation would have to be about $4° 23'$. Since an odd value of D_1 is undesirable, and since it is not necessary to make p exactly 1.5 ft., it is sufficient to take $D_1 = 4° 22'$.

In the foregoing discussion it was assumed that the degree of the existing curve was known and that the $T.C.$ and $C.T.$ were monumented. It will generally happen, however, that the degree

of curve is unknown, that the track is in poor alinement, and that the *T.C.* and *C.T.* are not monumented. In this case, run out the tangents to an intersection and measure the intersection angle. Then measure the external distance of the existing curve and from it determine the degree of the curve that will connect the tangents and pass through the middle point of the existing curve. Use the value of R corresponding to this value of D in solving Eq. 50.

152. Second Method. Case 1. In Fig. 39, ABC is the existing unspiraled curve, whose degree of curve, D, is known, whose alinement is good, and whose *T.C.* and *C.T.* are monumented. At some point on this curve, such as B, it is desired to compound with an assumed curve of slightly shorter radius, R_1, which, when run to a tangent parallel to the initial tangent, will be a distance from it equal to o for a R_1 curve, thus providing room for the spiral.

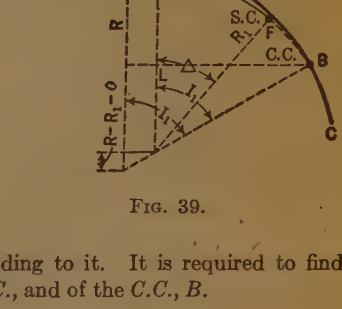

FIG. 39.

In the figure, EFB is the R_1 curve, and EG is o corresponding to it. It is required to find the location of the *T.S.*, of the *S.C.*, and of the *C.C.*, B.

From the figure,

$$o = EG = GL - EL = (R - R_1) \text{ vers } I_1$$

and

$$\text{vers } I_1 = \frac{o}{R - R_1} \tag{51}$$

The *T.S.* is located by measuring back from the old *T.C.*

$$T.C. \text{ to } T.S. = t - (R - R_1 - o) \tan I_1 \tag{52}$$

The *S.C.* is located by measuring the spiral length from the *T.S.* The *C.C.*, B, is located by measuring the distance FB from the *S.C.*

$$FB \text{ (in feet)} = \frac{I_1 - \Delta}{D_1} 100 \tag{53}$$

The location of the point B may be checked by measuring the distance AB along the old curve.

$$AB \text{ (in feet)} = \frac{I_1}{D} 100$$

The distance from the *T.S.* to the *C.C.* along the new alinement is shorter than along the old alinement, and the track will have to be shortened. This shortening is equal to the difference in the station numbers of the *C.C.* as computed along the respective alinements.

The limits of R_1 are such as will make the point B come at the middle of the original curve, or will make B and F coincide. In general, R_1 should be chosen between $0.8R$ and $0.9R$.

Example. $D = 3° 30'$, D_1 (assumed) $= 4° 00'$, $k = 1°$, and $o = 4.65$. From Eq. 51, $I_1 = 12° 14.3'$. From Eq. 52 the distance

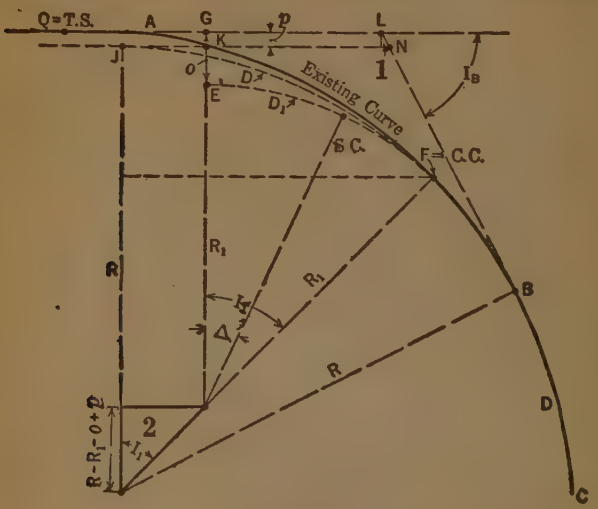

Fig. 40.

from the old *T.C.* to the *T.S.* is 156.5 ft. The spiral is 400 ft. long. From Eq. 53, the distance from the *S.C.* to the *C.C.* is 106.0 ft. The new alinement will be about 0.3 ft. shorter than the old alinement.

153. Second Method. Case 2. In Fig. 40, *ABC* is the existing unspiraled curve, whose degree of curve is unknown, whose alinement may be poor, and whose *T.C.* and *C.T.* are not monumented. First set-up in the center of the track at some point *C* near the middle of the curve and by trial deflection angles find the degree of the curve, *D*, that will most nearly conform to the existing track. Then run-in this *D* curve to a point *B* which is about 500 ft. from the end of the curve. If this curve were continued, its *T.C.* would fall at *J*

on a tangent parallel to the initial tangent QL and at a distance p from it. At B run-out the tangent to the D curve to an intersection with the initial tangent at L and measure the intersection angle I_B and the distance BL. BN is the tangent distance of the curve corresponding to I_B. Then, in triangle 1,

$$p = NL \sin I_B$$

The D curve is to be compounded with a chosen D_1 curve at some point F to give the required offset o.

$EK = o - p = (R - R_1)$ vers I_1, whence

$$\text{vers } I_1 = \frac{o - p}{R - R_1} \tag{54}$$

The $T.S.$ is located at Q by measurement from L.

$$QL = QG + GL = t + JN - JK - NL \cos I_B$$

The $S.C.$ and the $C.C.$ are located as in Case 1, and the spirals are staked out as in new work.

J may fall outside of QL, in which case p and o are numerically added; again, p may be greater than o, and in this case R_1 must be greater than R.

154. COMPOUND CURVES.

An existing compound curve can be spiraled in the following manner.

In Fig. 41, ABC is the existing compound curve with its $C.C.$ at B. To make room for the spiral it is necessary to compound the sharper branch, D_2, at some point C with a still sharper curve of chosen degree, D_3, and to continue the first branch, D_1, to the point E where its tangent is parallel to the tangent of the D_3 curve at G. EG is the offset o for a spiral whose $D = D_3 - D_1$. The problem is to find $BE = n_1$ and $CG = n_2$.

For small central angles the ordinates between two circular curves are equal to the difference in their tangent offsets. Therefore EF can be considered as the difference in the tangent offsets of the D_1 and D_2 curves in the distance n_1; similarly FG is the difference in the tangent offsets of the D_2 and D_3 curves for the distance n_2. It must be remembered that the figure is very much exaggerated.

Then from Eq. 20,

$$o = EF + FG = 0.873(D_2 - D_1)n_1^2 + 0.873(D_3 - D_2)n_2^2.$$

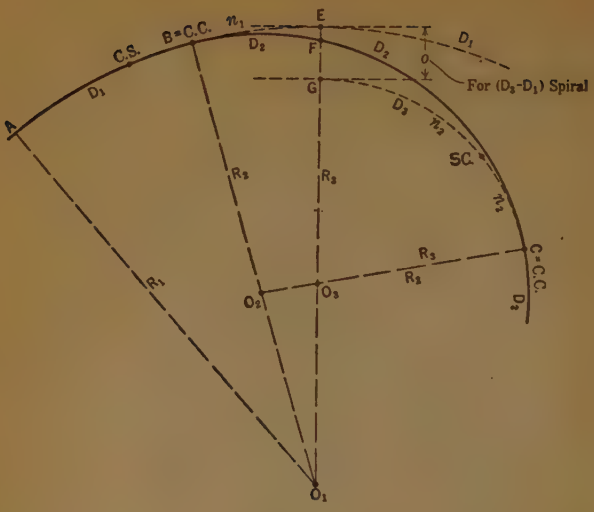

FIG. 41.

Since the central angle of the arc BC equals the sum of the central angles of the arcs BE and CG,

$$D_2(n_1 + n_2) = D_1 n_1 + D_3 n_2$$

or

$$(D_2 - D_1)n_1 = (D_3 - D_2)n_2.$$

Solving these simultaneous equations for n_1 and n_2 we have

$$n_1 = 1.07\sqrt{\frac{(D_3 - D_2)o}{(D_2 - D_1)(D_3 - D_1)}} \tag{55}$$

$$n_2 = 1.07\sqrt{\frac{(D_2 - D_1)o}{(D_3 - D_2)(D_3 - D_1)}} \tag{56}$$

C is located by measuring the distance $n_1 + n_2$ along the D_2 curve from B. The $C.S.$ is located by running the D_1 curve from B the distance $n_1 - \frac{1}{2}S_1$, and the $S.C.$ is located by running the D_3 curve from C the distance $n_2 - \frac{1}{2}S_1$. The spiral is staked out as usual for compound curves.

Highway Spirals

Spirals seem to have been first used on highways merely to run out the changes in width when pavements were widened on curves. But as motor vehicle speeds increased until they equaled or exceeded

rain speeds, it began to be realized that highway spirals were distinctly desirable. Indeed a careful analysis reveals the fact that spirals are even more necessary on highways than on railroads because the motor vehicle is steered by the driver instead of being guided by the rails.

No motor vehicle can change instantly from rectilinear to curvilinear movement. A finite interval of time is required to turn the steering wheel, during which the car moves forward; and the natural action of the driver is to turn the wheel at a rate that is convenient and that makes smooth riding. If it is assumed that the wheel is turned at a uniform rate, it follows that the angular change of direction is increasing at a constant rate, which may be designated k. If the car moves forward at a constant speed, the total change of direction will vary with this change in angle and the distance traveled. Consequently,

$$\delta = \int ks\, ds = \tfrac{1}{2}ks^2$$

in which δ is the total change of direction in the distance s, and k is the unit rate of change in the steering angle.

This equation is identical with Eq. 31, which gives the total change of direction of the spiral, and shows that the natural path of the vehicle under the natural process of steering is a spiral.

Another important factor in determining the path of the car and the need for spiraling is the effect of superelevation and the manner in which it is introduced. To analyze this factor, consider a car approaching an unspiraled curve at the speed for which the curve is superelevated and with all the runoff on the tangent, as is common practice.

On normal tangent the pavement is level transversely; hence the only lateral forces acting on the car are those incident to normal driving and these are met by the ordinary processes of steering. But, as the car traverses the runoff, it is subjected to an increasing lateral force due to the increasing tilt of the pavement. Since the alinement is straight, there is no balancing centrifugal force; hence this lateral force tends to cause a lateral inward "drift" of the car. This force can by no means be neglected, since it amounts to 20 lb. per ton for each percent of cross slope. Thus for a superelevation of 0.10 ft. per ft. of width, it amounts to 200 lb. per ton, or about four times the tractive effort required to drive the car at 50 m.p.h. on an average level road.

If the driver wishes to stay in his proper lane, he must overcome this lateral force by steering against it. This steering movement is outward, or opposite in direction to that required by the approaching curve, and hence is not a natural steering movement. Consequently, few drivers do it, but permit the car to take the natural drift inward. This lateral force due to the tilt is resisted only by a component of the "rolling resistance" of the road. Since the total drift occurs in the total length of the runoff, it is made at a very small angle with line of travel, and consequently the total value of the component is small and may be neglected.

Since the lateral force is a direct component of gravity, and the tilt is increasing at a uniform rate, we may write $df = n\,dx$, where n is a constant depending on the total superelevation and length of runoff. From the principles of mechanics, we know that $df = m\,da$ and, also, that the movement caused by an acceleration is $dy = \frac{1}{2}t^2 da$, where t is the time. Since the car is moving at a constant speed, it follows that $t = \frac{x}{v}$. Combining these equations we find that

$$y = \int \frac{nx^2}{2\,mv^2}\,dx = \frac{nx^3}{6\,mv^2} \tag{57}$$

where y is the lateral drift in feet in the distance x in feet, n is the unit increase in the tilt (equal to the total superelevation in ft. per ft. of width divided by the length of runoff in feet), m is the unit of mass (equal to $\frac{1}{32}$), and v is the speed of the car in feet per second.

Obviously this is the equation of a *cubic parabola*, and it is well known that the common transition spiral is based on the cubic parabola. It is thus shown that the natural effect of placing the runoff on the tangent is to cause the car to tend to move in a curved path that is essentially a spiral.

It has already been shown that the natural steering process also tends to cause the car to move on a spiral. That this is true is abundantly shown by the frequency with which cars run off the pavement on the inside at the entrance to curves to the right and encroach on the adverse traffic lane at the entrance to curves to the left, with many consequent accidents and fatalities.

If a spiral of the same length as the runoff had been used, the increasing lateral force due to the increasing tilt would have been balanced by an equal centrifugal force due to the increasing curvature. The effect of the unbalanced tilt would thus have been eliminated, and the driver would have been relieved of any necessity

If trying to compensate for it, while the spiral alinement would have conformed to his natural steering practice. Thus, the introduction of the spiral would have simplified the steering process and at the same time promoted comfort and safety by eliminating the tendency to lateral drift. *The use of the spiral simply means that the road is built on the path which the car tends to follow under the forces acting on it and under the natural process of driving.*

As an example, assume a 2° curve superelevated 0.10 ft. per ft. of width, with a runoff distance of 200 ft. and a car speed of 60 m.p.h. Then $n = 0.0005$, $x = 200$, $m = \frac{1}{32}$, and $v = 88$. Substituting these values in Eq. 57, $y = 2.75$ ft. This is sufficient, if uncompensated in the steering, to cause the car to encroach dangerously on the adverse lane or to run off the pavement. On the other hand, if a spiral with $k = 1°$, giving the same length of runoff, had been used, Y at its end would have been 2.33 ft. (Table 12). This is only 0.42 ft. different from the natural path of the car, instead of 2.75 ft. as in the case of the unspiraled curve, and this small variation is permissible in the ordinary width of traffic lane. In traversing the spiral the driver would have 200 ft. (equivalent to about $2\frac{1}{4}$ seconds at the given speed) in which to turn his wheel at a gradually increasing rate; hence the steering would be a simple, natural, and convenient process.

If a spiraled curve is approached at a speed less than that for which it is superelevated, part of the force due to tilt will not be balanced by centrifugal force, which will result in some tendency of the car to drift inward. To overcome this drift the driver must steer against it by turning the steering wheel a less amount than required by the curvature. The net movement, however, will be in the direction of curvature and, with the entire length of spiral in which to accomplish it at a gradual rate, the entire steering process will be done in a simple and natural manner. On the other hand, if the spiraled curve is approached at a speed greater than that for which it is superelevated, part of the centrifugal force will not be balanced by tilt; hence there will be some tendency for the car to drift outward. To overcome this the driver must turn his wheel more than required by the curvature. But again the steering movement is all in the direction of curvature with the entire length of spiral available in which to make it gradually; hence the steering process is again simple, natural, and convenient. Thus, whether the speed be above, below, or at the theoretical speed, the steering is simplified, and consequently the curve is made safer and more convenient.

155. Choosing D or R. Horse-drawn traffic permitted the use of very sharp curves, whence arose the practice of designating highway curves by the radius instead of the degree of curve. Although modern traffic requires flatter curves, the custom of designating curves by the radius still persists, even though the degree of curve is more desirable for exactly the same reasons as on railroads. For all radii greater than about 300 ft., a simple value of the degree of curve should be chosen, rather than an even value of the radius.

The use of the radius instead of the degree of curve does not, however, greatly complicate the use of the spiral. From Eq. 30, $d = ks$, and from Eq. 15, $d = \frac{5730}{r}$. Substituting the latter in the former, we have

$$r = \frac{5730}{ks} \quad \text{and} \quad R = \frac{5730}{kS} \tag{30a}$$

Equation 30 is used when D is chosen, and Eq. 30a is used when R is chosen.

156. Choosing S or k. It has been the common practice in highway work to choose the length of runoff, and thus the length of spiral, and then, with D or R known, to determine the other factors. This generally gives odd values of k, which are distinctly inconvenient in both computations and field work. It is better to determine a simple value of k and permit S to be odd, if necessary, since this method simplifies both computations and field work.

Owing to the manner of steering, motor vehicles do not change abruptly from rectilinear to curvilinear motion, but each provides its own spiral to a limited extent, as previously pointed out. Furthermore, recent investigations show that motorists will readily accept a higher rate of tilt and more unbalanced centrifugal force than will passengers on a train. These factors tend to reduce the required length of spiral, or, what is the same thing, permit a higher value of k.

Professor Moyer * from some extended investigations of skidding of automobiles arrived at the conclusion that the minimum length of highway spiral is given by the equation

$$L_s(\text{in feet}) = \frac{1.58\ V^3}{R}$$

* Bulletin 120, Iowa Engineering Experiment Station, Ames, Iowa, 1934.

hich is equivalent to a value of k of

$$k = \frac{363000}{V^3}$$

Professor Moyer distinctly states that this is the sharpest spiral nat should be employed. Consequently, a spiral of greater length, r lower value of k, should be used wherever possible.

From a study of Moyer's work, railroad practice, and existing ighway spirals, the authors recommend that the following equation e used to determine the approximate *desirable* value of k.

$$k = \frac{230000}{V^3} \tag{58}$$

This equation gives values of k twice as great as for railroads, or pirals one-half as long. It provides spirals about 58% longer than he minimum suggested by Moyer. Fig. 42 is a highway spiral dia- ram based on Eq. 58 and is used in the same manner as Fig. 37.

Example. Assume that a curve with a radius of 800 ft. is to be perated at 40 m.p.h. From Moyer's equation the minimum length f spiral would be

$$L_s = \frac{1.58 \times 40^3}{800} = 126.4 \text{ ft.}$$

rom Eq. 58, the approximate desirable value of k would be

$$k = \frac{230000}{40^3} = 3.6$$

om which the more convenient value of $k = 3.5$ would be chosen. he length of the spiral from Eq. 30a would be

$$L_s = 100S = \frac{573000}{3.5 \times 800} = 204.6 \text{ ft.}$$

Assume, however, that the radius of 800 ft. with a spiral length of 00 ft. was chosen with stakes to be located at 50-ft. intervals; then om Eq. 30a,

$$k = \frac{5730}{800 \times 2} = 3.58125$$

rom Eq. 31,

$$\Delta = \tfrac{1}{2} \times 3.58125 \times 2^2 = 7.1625^\circ = 7^\circ 09.75'$$

HIGHWAY SPIRALS
$k = \frac{220{,}000}{V^3}$
$e = \frac{0.067V^2}{R} = 0.0000117DV^2$
Speed-miles per hour
Length-feet
100
90
80
70
60
50
40
0
100
200
300
400
500
600
700
k=¼
k=⅜
k=½
k=⅝
k=¾
k=1
k=1¼
k=1½
k=1¾
k=2
k=2½
k=3
k=4
k=5
k=6
k=7
k=8
e=0.02
e=0.04
e=0.06
e=0.08
e=0.10
e=0.12
e=0.14
e=0.16
e=0.18
e=0.20
D=0°30'
D=0°45'
D=1°00'
D=1°30'
D=2°00'
D=2°30'
D=3°00'
D=4°00'
D=5°00'
D=6°00'
D=7°00'
D=8°00'
D=9°00'
D=10°00'

FIG. 42.

From Eq. 32,

$$a_1 = 10 \times 3.58125 \times 0.5^2 = 0°\,08.953'$$
$$a_2 = 2^2 \times 8.953 = 0°\,35.81'$$
$$a_3 = 3^2 \times 8.953 = 1°\,20.58'$$
$$a_4 = 4^2 \times 8.953 = 2°\,23.26' = A = \tfrac{1}{3}\Delta$$

From Eq. 15, $D = \dfrac{5730}{800} = 7°\,09.75'$ on which the deflection angles for the circular curve must be based.

If k had been chosen as 3.5, then

$$S = \frac{5730}{3.5 \times 800} = 2.0464 \quad \text{and} \quad \Delta = 7°\,19.6'$$

$$a_1 = 10 \times 3.5 \times 0.5^2 = 0°\,08.75'$$
$$a_2 = 2^2 \times 8.75 = 0°\,35.00'$$
$$a_3 = 3^2 \times 8.75 = 1°\,18.75'$$
$$a_4 = 4^2 \times 8.75 = 2°\,20.00'$$
$$a_5 = A = \tfrac{1}{3}\Delta = 2°\,26.53'$$

The circular deflection angles would be the same as before, but the spiral deflection angles have been simplified.

Assume that a 7° 00′ curve, which has approximately the same radius, had been used with $k = 3.5°$. Then $S = 2.0$ and $\Delta = \frac{1}{2} \times 3.5 \times 2^2 = 7°\,00'$. a_1, a_2, and a_3 would be the same as in the last example while $a_4 = A = \frac{1}{3}\Delta = 2°\,20'$, and the circular deflections would be for a 7° curve. This shows that even values of D and k are more desirable than even values of the radius and length of spiral. Table 11 gives the necessary data for even radius curves.

157. Superelevation. The theoretical superelevation for highways is given by the equation

$$e = \frac{0.067V^2}{R} = 0.0000117DV^2 \qquad (59)$$

where e is the superelevation in feet per foot of width, V is the speed in miles per hour, and R and D are the radius and the degree of curve, respectively. Table 6 gives the theoretical superelevation for several different radii and speeds.

TABLE 6.—THEORETICAL SUPERELEVATION. Transverse slope per foot of width $e = 0.067\ V^2/R$.

Speed, miles per hour	Radius, feet (above); degree of curve (below)																	
	50	60	75	100	150	200	300 19°06′	400 14°20′	500 11°28′	600 9°33′	700 8°12′	800 7°10′	900 6°22′	1,000 5°44′	1,500 3°49′	2,000 2°52′	3,000 1°55′	5,000 1°09′	10,000 0°34.4′
5	7/16 **0.034**	5/16 **0.028**	1/4 **0.022**	3/16 **0.017**	1/8 **0.011**	1/8 **0.008**	1/16 **0.006**												
10	1 5/8 **0.135**	1 5/16 **0.112**	1 1/8 **0.090**	1 3/16 **0.067**	9/16 **0.045**	7/16 **0.034**	1/4 **0.022**	3/16 **0.017**	3/16 **0.013**	1/8 **0.011**	1/8 **0.010**	1/8 **0.008**	1/16 **0.007**	1/16 **0.006**					
15	3 3/8 **0.303**	3 **0.253**	2 7/16 **0.202**	1 13/16 **0.151**	1 3/16 **0.101**	15/16 **0.076**	5/8 **0.050**	7/16 **0.038**	3/8 **0.030**	5/16 **0.025**	1/4 **0.022**	1/4 **0.019**	3/16 **0.017**	3/16 **0.015**	1/8 **0.010**	1/16 **0.007**	1/16 **0.005**		
20	6 7/16 **0.538**	5 15/16 **0.495**	4 3/16 **0.359**	3 1/4 **0.269**	2 3/16 **0.180**	1 5/8 **0.135**	7/8 **0.090**	11/16 **0.067**	5/8 **0.054**	9/16 **0.049**	7/16 **0.038**	7/16 **0.034**	3/8 **0.030**	5/16 **0.027**	1/4 **0.018**	3/16 **0.013**	1/8 **0.009**	1/16 **0.005**	
25	10 1/8 **0.842**	8 7/16 **0.701**	6 3/4 **0.562**	5 1/16 **0.421**	3 1/4 **0.271**	2 1/2 **0.211**	1 11/16 **0.140**	1 1/4 **0.105**	1 **0.084**	1 1/16 **0.070**	3/4 **0.060**	5/8 **0.053**	9/16 **0.047**	1/2 **0.042**	5/16 **0.027**	1/4 **0.021**	3/16 **0.014**	1/8 **0.008**	1/16 **0.004**
30			7 1/4 **0.606**	4 7/8 **0.405**	3 3/8 **0.303**	2 3/16 **0.202**	1 13/16 **0.151**	1 7/16 **0.121**	1 1/4 **0.101**	1 1/16 **0.087**	15/16 **0.076**	13/16 **0.067**	3/4 **0.061**	1/2 **0.040**	3/8 **0.030**	1/4 **0.020**	1/8 **0.012**	1/16 **0.062**	
35				6 5/8 **0.550**	4 15/16 **0.413**	3 5/16 **0.275**	2 1/2 **0.206**	2 **0.165**	1 5/8 **0.137**	1 7/16 **0.118**	1 1/4 **0.103**	1 1/8 **0.092**	1 **0.082**	1 1/16 **0.055**	1/2 **0.041**	5/16 **0.027**	3/16 **0.016**	1/16 **0.082**	
40					6 5/16 **0.538**	4 5/16 **0.359**	3 1/4 **0.270**	2 9/16 **0.216**	2 3/16 **0.180**	1 7/8 **0.154**	1 5/8 **0.135**	1 7/16 **0.120**	1 5/16 **0.108**	7/8 **0.072**	5/8 **0.054**	7/16 **0.036**	1/4 **0.022**	1/8 **0.011**	
45					8 3/16 **0.682**	5 7/16 **0.454**	4 1/16 **0.340**	3 1/4 **0.273**	2 3/4 **0.227**	2 5/16 **0.195**	2 1/16 **0.170**	1 13/16 **0.151**	1 5/8 **0.136**	1 1/4 **0.091**	1 3/16 **0.068**	9/16 **0.045**	5/16 **0.027**	1/8 **0.0135**	
50						6 3/4 **0.562**	5 1/16 **0.421**	4 1/16 **0.337**	3 3/8 **0.281**	2 7/8 **0.241**	2 1/2 **0.211**	2 1/4 **0.187**	2 **0.168**	1 5/16 **0.112**	1 **0.084**	11/16 **0.056**	7/16 **0.034**	3/16 **0.017**	
60							7 1/4 **0.602**	5 3/4 **0.482**	4 13/16 **0.402**	4 1/8 **0.345**	3 5/8 **0.302**	3 3/16 **0.268**	2 7/8 **0.241**	1 15/16 **0.161**	1 7/16 **0.121**	15/16 **0.080**	9/16 **0.048**	1/4 **0.028**	
70								7 7/8 **0.656**	6 9/16 **0.547**	5 5/8 **0.468**	4 15/16 **0.411**	4 3/8 **0.365**	3 15/16 **0.328**	2 5/8 **0.219**	2 **0.164**	1 5/16 **0.109**	13/16 **0.066**	3/8 **0.033**	
80									8 3/16 **0.715**	7 3/8 **0.614**	6 7/16 **0.537**	5 3/4 **0.477**	5 1/8 **0.429**	3 7/16 **0.286**	2 9/16 **0.215**	1 3/4 **0.143**	1 **0.086**	1/2 **0.043**	
90										9 5/16 **0.777**	8 1/8 **0.679**	7 1/4 **0.603**	6 1/2 **0.543**	4 5/16 **0.362**	3 1/4 **0.271**	2 3/16 **0.181**	1 5/16 **0.109**	11/16 **0.055**	
100											10 1/8 **0.838**	8 15/16 **0.744**	8 **0.670**	5 3/8 **0.447**	4 **0.335**	2 11/16 **0.223**	1 5/8 **0.134**	13/16 **0.067**	

Bold-face figures give decimals of a foot, light-face figures in inches.

Because of the effect on slow-moving vehicles, especially when the pavement is slippery, the maximum amount of superelevation must be limited. The consensus of opinion of leading highway engineers is that a value of $e = 0.10$ may be safely used, although values as high as 0.16 may be permitted in special cases. This corresponds closely to the 6 to 10 in. used on high-speed, standard-gage railroads. With the general increase in highway speeds, there is a distinct tendency towards using higher values of e more freely.

It has been common practice to omit the superelevation entirely if the computed value of e is less than about 0.04. From a consideration of safe speeds as given in section 137, and of Moyer's data previously referred to, it appears that, in order to provide a suitable factor of safety against lateral skidding at high speeds on flat curves, a minimum value of e of about 0.02 should be used.

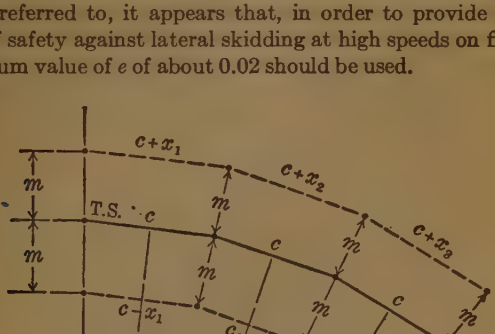

FIG. 43.

158. Spiraling Unwidened Curves. When the pavement width is the same on both tangents and curves, the spirals are best inserted on the center line as outlined in the preceding pages, and the edges of the pavement located from the center-line stakes. This means that offset stakes at a fixed distance from the center line must be located. This is best done by measuring the required offset distance from each center-line stake to the corresponding offset stake. It is essential that the measurements be made *radial* to the curves,

else the alinement of the edges and the width of pavement will be irregular.

Alining by eye. When the offset distance is short, say less than 15 to 20 ft., the offset stakes can usually be lined in with all necessary accuracy by eye.

Using chord and offset. Where the offsets are larger, or greater accuracy is desired, the following method shown in Fig. 43 is convenient: Offset stakes for the inner and the outer edges are set opposite the *T.S.*, using the transit or other accurate method to place them at right angles to the center line of the tangent. The other stakes are then located successively by measuring the offset distance from the corresponding center-line stake and the chord from the preceding offset stake. Since the radius of curvature is different on the offset line from that on the center line, the chord lengths must be changed accordingly. This change in chord length may be computed from the easily derived equation

$$x = \frac{cmd}{5730} = \frac{cm}{r} \qquad (60)$$

where x is the change in chord length, to be added for the outside edge and subtracted for the inside edge, c is the chord length on the center line, m is the distance from the center line to the line of offset stakes, and r and d are the radius and degree of curve, respectively, of the center line at the middle of the chord c. Values of x can be computed with all necessary accuracy on the slide rule.

On spirals d and r vary from chord to chord, hence x also varies. On the circular curve d and r become D and R, respectively, and x is constant for given values of m and c.

Using transit and tape. Occasionally it may be desired to locate or rerun the offset line with the transit and tape. In this case the same values of a as those used on the center line may be used in conjunction with the chord lengths as given in the preceding paragraph. Although this is not mathematically correct on the spiral, since the chord lengths vary slightly, the discrepancies are exceedingly small and can be neglected. This method, however, is exact on the circular curve. This procedure gives concentric curves with stakes opposite each other, enabling the width of pavement to be checked before the forms are set.

It is, of course, possible to compute a spiral to fit the given values of R, D, and S for each offset line, but this method is cumbersome and adds nothing in the way of accuracy; and, although the curves

re concentric, the stakes are not opposite, making it difficult to heck the pavement width before the forms are set.

Some highway departments compute separate spirals for each edge f the pavement, generally using the same length of spiral, or some- imes the same value of k. The alinement so obtained is entirely atisfactory, but the pavement varies somewhat in width along the piral.

159. Spiraling Widened Curves. As pointed out in section 135, curve is usually widened by placing the additional width along the

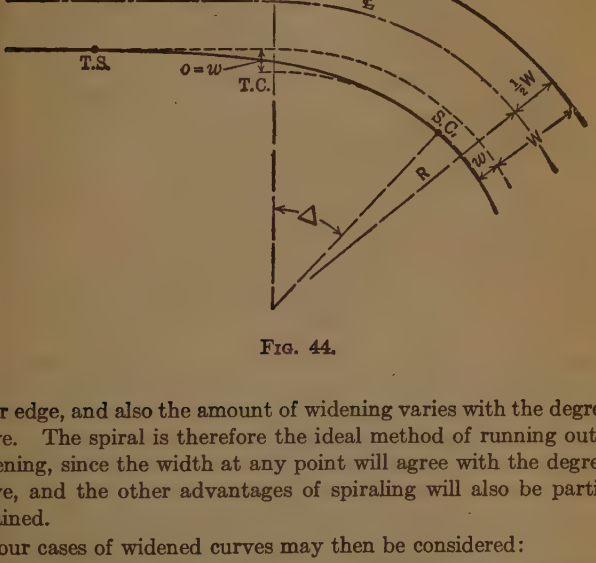

FIG. 44.

nner edge, and also the amount of widening varies with the degree of curve. The spiral is therefore the ideal method of running out the videning, since the width at any point will agree with the degree of curve, and the other advantages of spiraling will also be partially btained.

Four cases of widened curves may then be considered:

1. Curves widened on the inner edge with both the center-line nd the outer edge unspiraled.

2. Curves widened on the inner edge with the center-line and the uter edge concentrically spiraled.

3. Curves widened on the inner edge with separate spirals on the dges without regard to a center-line spiral.

4. Curves widened symmetrically about the center-line with separate spirals on the edges.

160. Case 1. In Fig. 44 the inner edge is widened the amount w and the center-line is unspiraled. The outer edge is then a circle concentric with the center-line. The inner edge must therefore be spiraled with a curve which must conform to the given radius and width of pavement, and whose value of o must equal the widening w.

From Fig. 44

$$R_1 = R - \tfrac{1}{2}W - w - p$$

where p is the distance from the edge of the pavement to the form stakes.

From Eqs. 30 and 30a,

$$R_1 = \frac{5730}{kS} \quad \text{and} \quad D_1 = kS$$

From Eq. 36, and the given conditions,

$$o = w = 0.0727kS^2$$

Solving for S and reducing,

$$S = \sqrt{\frac{R_1 w}{416.6}} = \sqrt{\frac{w}{0.0727 D_1}} \tag{61}$$

With S now known, k is determined from Eq. 30 or 30a.

It is evident that with a given widening and a given radius both k and S will probably have odd values. It is possible, however, to overcome this difficulty by computing S and k as above and then choosing a convenient value of k and computing back for the corresponding value of S and for a new value of $o = w$. The difference between this new value of w and the original value will usually be quite small and may be neglected, or the difference may be adjusted in setting the pavement forms.

Since the alinement will normally be staked out on the center-line as a tangent and circular curve, and the stakes for the outer edge located by offsets, it is often convenient to locate the stakes for the inner edge also by offsets. To do this, locate points on the center-line at convenient, equal intervals between the *T.S.* and the *T.C.*,

nd also at the same intervals between the $T.C.$ and the $S.C.$ Since he spiral bisects the ordinate o at the $T.C.$, and $o = w$, the offset rom the center-line at the $T.C.$ to the line stake at the middle of the piral will be $\frac{1}{2}W + \frac{1}{2}w + p$. The offset from the center-line to the $T.S.$ will be $\frac{1}{2}W + p$. The ordinates for the intermediate points can hen be found by dividing $\frac{1}{2}w$ in proportion to the cubes of the dis- ances from the $T.S.$ The quantity $\frac{1}{2}W + p$ is added to each of these rdinates to obtain the corresponding offsets from the center-line oints. For the other half of the spiral, the offsets are measured rom the points on the center-line circle. The offset at the $S.C.$ is $\frac{1}{2}W + w + p$. The intermediate offsets are computed from the same proportional parts of $\frac{1}{2}w$ as for the first part. They are used in the reversed order, working back from the $S.C.$ to the $T.C.$, by subtract- ing them from $\frac{1}{2}W + w + p$. This will give the same value for the offset at the $T.C.$ as previously computed.

Example. Assume a spiral 200 ft. long and a widening of 2.00 ft. on the inner edge of a pavement which is 20.00 ft. wide on tangents. Form stakes are to be set 2.00 ft. from the edge of the pavement. Divide each half of the spiral into four parts by points 25 ft. apart on the center-line, measuring each way from the $T.C.$ The spiral ordinate at the $T.C.$ is $\frac{1}{2}o = \frac{1}{2}w = 1.00$ ft., and at the $T.S.$ it is 0.00. At the first point, 25 ft. from the $T.S.$, the ordinate is $\frac{1.00}{4^3} = 0.0156$, at the second point it is $0.0156 \times 2^3 = 0.12$, and at the third point it s $0.0156 \times 3^3 = 0.42$ ft. Adding each of these to $\frac{1}{2}W + p = 12.00$ t., the offsets from the $T.S.$ to the $T.C.$ are 12.00, 12.02, 12.12, 12.42, nd 13.00 ft., respectively. Now using these same ordinates, and eginning at the $S.C.$ and working back, each is subtracted from $\frac{1}{2}W + w + p = 14.00$ ft., whence the offsets are 14.00, 13.98, 13.88, nd 13.58 ft., respectively, with 13.00 ft. at the $T.C.$ as before.

161. Case 2. In this case a suitable spiral is selected for the enter-line and located in the usual way. The outer edge is then ocated as a concentric spiral by measuring offsets of $\frac{1}{2}W + p$ from he center-line stakes to the form stakes.

For the inner edge a spiral must be computed for the corresponding adius and for a value of o equal to o for the center-line spiral plus he widening w. The problem therefore becomes identical in form vith that in Case 1. Using the subscript 1 for the inner spiral, we ave

$$o_1 = o + w \quad \text{and} \quad R_1 = R - \tfrac{1}{2}W - w - p$$

By analogy with Eq. 61,

$$S_1 = \sqrt{\frac{R_1(o + w)}{416.6}} = \sqrt{\frac{o + w}{0.0727 D_1}} \qquad (61a$$

S_1 and k_1 are computed as in Case 1, and may be corrected bac for an even value of k_1 if desired.

The spiral may be located by offsets in a manner similar to that i Case 1. The center-line tangent is produced through the *T.S.* to th *T.C.* and divided into equal parts as before. The spiral ordinate are computed from $\frac{1}{2}o_1$ and added to $\frac{1}{2}W + p$ to obtain the offset For the second half of the spiral it is necessary to extend the cente line circle back from the *S.C.* to the *T.C.* and to locate the require points on it. The offsets from these points are $\frac{1}{2}W + w + p$ minu the spiral ordinates, as before.

162. Case 3. This case differs from Case 2 in that no cente line spiral is considered. A spiral is chosen directly for the outc edge and is located independently. Using the subscript 1 for th inner spiral, to correspond with the preceding cases, and the subscrip 2 for the outer spiral, as shown in Fig. 45, we have by analogy,

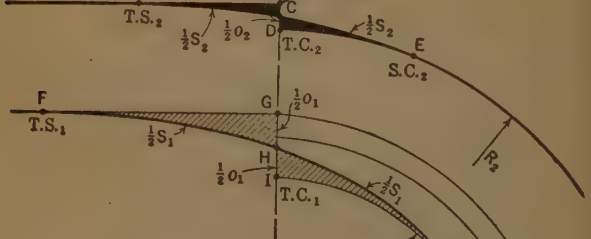

FIG. 45.

$$o_1 = o_2 + w \quad \text{and} \quad R_1 = R - \tfrac{1}{2}W - w - p$$

S_1 and k_1 are determined as before and may be corrected back fo an even value of k if desired.

Both spirals may be located by offsets by applying the same prin ciples as used in the previous cases.

163. Symmetrical Widening. Case 4. Satisfactory results can often be obtained by widening the curve symmetrically about the center-line with the center-line spiraled. There are two methods of doing this.

First Method. A spiral for the center-line is chosen, computed, and laid out in the usual way. Spirals are then computed for the outer and inner edges to agree in a manner analogous to the preceding cases.

For the outer edge,

$$o_2 = o - \tfrac{1}{2}w \quad \text{and} \quad R_2 = R + \tfrac{1}{2}W + \tfrac{1}{2}w + p$$

For the inner edge,

$$o_1 = o + \tfrac{1}{2}w \quad \text{and} \quad R_1 = R - \tfrac{1}{2}W - \tfrac{1}{2}w - p$$

The equations for S_2 and S_1 are analogous to Eq. 61a, from which values of S_2, k_2, S_1, and k_1 can be computed. These may be corrected back for even values of k_1 and k_2.

Second Method. The center-line spiral is provided as before. The widening is then made in successive increments by dividing $\frac{1}{2}w$ into ordinates for equally spaced points on the spiral in proportion to the cubes of the distances from the *T.S.* These ordinates added to $\frac{1}{2}W + p$ give the offsets to be measured from the proper points on the center-line to both the inner and outer lines of form stakes. This method is simple and easily applied and gives results approximating those obtained by the first method.

It is to be especially noted that if $o = \frac{1}{2}w$ there will be no spiral on the outer edge, and the problem reduces itself to Case 1. If o is less than $\frac{1}{2}w$, the outside widening, if brought around to the *T.C.*, would lie outside the tangent. This is undesirable and therefore symmetrical widening should not be used.

164. Example. Given: center radius $R = 500$ ft., pavement width 20 ft., stakes to be placed 2.0 ft. outside of edge of pavement.

a. Pavement to be widened 2.0 ft. on inside edge with outer edge unspiraled. Then $o_1 = 2.0$ and R_1 (radius of line of stakes) $= 486.0$ ft.

From Eq. 61,

$$S = \sqrt{\frac{486 \times 2}{416.6}} = 1.527$$

From Eq. 30a,

$$k = \frac{5730}{486 \times 1.526} = 7.72$$

Taking $k = 7.7$ and solving back for S and w, we find $S = 1.532$ and $o = w = 2.01$. The difference in width is negligible, and hence the simple value of k would be used.

b. The pavement is to be widened 2.0 ft. on the inside, with the outside edge spiraled with a spiral about 100 ft. long. Then $R_2 = 512$ and $S_2 = 1.0$, whence from Eqs. 30a and 36, $k_2 = 11.2$ and $o_2 = 0.81$, which values are satisfactory.

Then,

$$o_1 = 2.00 + 0.81 = 2.81 \quad \text{and} \quad R_1 = 486$$

From Eq. 61a,

$$S_1 = \sqrt{\frac{486 \times 2.81}{416.6}} = 1.811$$

From Eq. 30a,

$$k = \frac{5730}{486 \times 1.811} = 6.51 \quad \text{(Use 6.5)}$$

c. The pavement is to be widened 1.0 ft. on each side of the center line, and the center-line is to be spiraled with $k = 7.0$. Solving the equations, we find $o = 2.23$ and $S = 163.7$, whence $o_1 = 3.23$ and $o_2 = 1.23$.

Also $R_1 = 487$ and $R_2 = 513$, whence

$$S_1 = \sqrt{\frac{487 \times 3.23}{416.6}} = 1.943$$

$$S_2 = \sqrt{\frac{513 \times 1.23}{416.6}} = 1.230$$

$$k_1 = \frac{5730}{487 \times 1.943} = 6.06$$

$$k_2 = \frac{5730}{513 \times 1.230} = 9.08$$

165. Computations and Field Work. The computations and field work for highway spirals are identical with those for railroad spirals as previously outlined. It should be remembered, however, that, if spirals at the edges of a pavement are run separately by transit and tape, they should be computed for the *radius at the stake line* and not for the radius at the edge of the pavement or that of the center-lines.

166. Compound Curves. As pointed out in Chapter 4, compound curves are very undesirable on highways from the standpoint of

raffic, since the unexpected change in curvature introduces hazards. f, however, the topography demands a compound curve and the difference between the two degrees of curve is more than about 1°, ne road will be made safer by the introduction of a spiral of generous imensions between the two circles. Such spirals are computed and aid out in the same manner as previously explained.

167. Area of Pavement due to Spiraling. Since the spiral departs rom any osculating circle at the same rate as from the initial tan- ent, and since o and S mutually bisect, it follows that the area etween the initial tangent, the first half of the spiral, and the upper alf of the ordinate o is equal to the area between the second half f the spiral, the lower half of the ordinate o and the circular curve rom $T.C.$ to $S.C.$ Thus in Fig. 45 the area ABC equals the area DE, and likewise the area FGH equals the area HIJ.

It is therefore evident that the insertion of a spiral does not of self affect the area of the pavement. The area of pavement on a idened curve, when spirals are used, is therefore equal to the area ound by multiplying the total width of the widened portion by the ength of the circular arc whose radius is equal to $\frac{1}{2}(R_1 + R_2)$ and hose central angle is the intersection angle I of the given curve. n other words, compute the area of pavement as if it were to be idened abruptly at the $T.C.$ and $C.T.$ without any kind of tran- ition.

Problems

1. What is the maximum speed for 10°, 7°, 3°, and 2° curves?

2. Given. $P.I.$ at Sta. $741 + 60.0$; $I = 35° 42'$; $k = 2°$; and $= 5° 00'$. Determine the station numbers of the $T.S.$, $S.C.$, $.S.$, and $S.T.$, and the external distance of the curve.

Answer. $T.S. = 736 + 65.3$, $S.C. = 739 + 15.3$, $C.S. = 743 + 9.3$, $S.T. = 746 + 29.3$, $E_s = 60.3$.

3. Write transit notes for Problem 2.

4. Compute notes for locating the spiral in Problem 2 by offsets.

5. On new location a 5° 00′ curve is to be offsetted from a 2° 00′ urve at Sta. $333 + 00.0$ for a $k = 1°$ spiral. Find the station umbers of the $C.S.$ and the $S.C.$ and the required offset. Write otes for spiral.

6. Write notes to locate spiral in Problem 5 by offsets.

7. Given. $P.I.$ at Sta. $267 + 0.0$; $I = 69° 20'$; $I_1 = 21° 40'$; $_2 = 47° 40'$; $D_1 = 1° 30'$; $D_2 = 4° 00'$; $k_1 = k_2 = 1°$; $k_3 = 2°$.

Find T_{s1}, T_{s2}, and the station numbers of the $T.S.$, $S.C._1$, $C.S._1$, $S.C._2$, $C.S._2$, and $S.T.$

Answer. $T_{s1} = 1883.8$, $T_{s2} = 1271.9$, $T.S. = 248 + 16.2$, $S.C._1 = 249 + 66.2$, $C.S._1 = 262 + 10.6$, $S.C._2 = 264 + 60.6$, $C.S._2 = 274 + 27.3$, $S.T. = 276 + 27.3$.

8. Write transit notes for Problem 7.

9. An existing curve is to be spiraled. $I = 39° 18'$, $D = 3° 20'$, $k = 1°$. The track is to be shifted outward at the middle about 1.0 ft. Find D_1 and the actual value of p.

Answer. $D_1 = 3° 28'$; $p = 0.86$ ft.

10. Given an existing $2° 40'$ curve to be spiraled by compounding near the ends with a $3° 00'$ curve. The $T.C.$ is at Sta. $66 + 66.6$ and $k = 1°$. Find the station numbers of the $T.S.$, $S.C.$, and $C.C.$, and the amount that the track will be shortened.

Answer. $T.S. = 65 + 47.2$, $S.C. = 68 + 47.2$, $C.C. = 69 + 42.2$. The track is shortened 0.06 ft.

11. An existing compound curve has its $C.C.$ at Sta. $488 + 50.0$. $D_1 = 4° 00'$, $D_2 = 7° 00'$, $D_3 = 7° 40'$, and $k = 1°$. Find the station numbers of the $C.S.$, $S.C.$, and $C.C.$

Answer. $C.S. = 487 + 16.5$, $S.C. = 490 + 83.2$, $C.C. = 491 + 24.2$, $n_1 = 49.87$, $n_2 = 224.40$.

12. A highway curve has a normal center radius of 400 ft. and a normal pavement width of 18.0 ft. The curve is to be widened 3.0 ft. with form stakes placed 2.0 ft. from the edge of the slab. With the outer edge unspiraled, find the value of S and k for a spiral on the stake line of the inner edge.

Answer. $S = 1.667$; $k = 8.9$.

13. The curve in Problem 12 is to have a spiral about 100 ft. long on the outer edge. Find the computed value of k_2, choose an even value, find the corresponding value of S_2, and o_2, and then compute the values of S_1 and k_1.

Answer. k_2 computed $= 13.94$; k_2 used $= 14.0$; corrected $S_2 = 0.996$; $S_1 = 1.927$; $k_1 = 7.67$; $o_1 = 4.01$; $o_2 = 1.01$.

14. The curve in Problem 12 is to be widened symmetrically about the center-line, using $k = 10.0$ for the center-line spiral. Find o, o_1, o_2, S_1, S_2, k_1, and k_2.

Answer. $o = 2.14$; $o_1 = 3.64$; $o_2 = 0.64$; $S_1 = 1.840$; $S_2 = 0.796$; $k_1 = 8.04$; and $k_2 = 17.45$.

CHAPTER 6

STRING-LINING METHOD OF REALINING RAILROAD CURVES

If a curved track is in correct alinement and the outer rail is correctly superelevated at all points for a chosen speed, and if all trains move at this speed, there would be no lateral forces acting on the track, and it would remain in correct alinement. These conditions, however, are never completely satisfied. All trains do not move at the chosen speed, nor is the correct superelevation at all points maintained. Expansion and contraction of the rails also tend to shift the track. The result is that lateral forces do exist, and in some places the track will move outward under traffic, making that portion of the curve sharper; and in other places the track will move inward, making that portion of the curve flatter. Periodically, then, the track has to be realined. This can be done by the deflection angle method, but, if traffic is heavy, it is generally more satisfactory to use the string-lining method, which is another name for the middle ordinate method. Before proceeding further, sections 7 and 144, which treat of the middle ordinate method of staking out simple curves and spirals, respectively, should be reviewed.

168. Length of String. The length of string used is the same as the desired length of chord. Any length of chord can be used, but the American Railway Engineering Association recommends a length of 62 ft. As this is a commonly used length, it will be the measurement referred to in this discussion. The principal reason for choosing this odd length is that the middle ordinate (in inches) for a 62-ft. chord is the same as the degree of curve, as shown in Fig. 23. The use of a 62-ft. chord means that the stakes on the curve will be spaced at one-half this distance, or 31 ft. The 31-ft. points on the track are called stations and are numbered consecutively.

169. Locating Station 0. The first step in the field work is to determine the location of Station 0, the beginning of the curve. It may be that an old center stake can be found at this point, or possibly a superelevation marker, placed when the curve was staked

out originally. In this case Sta. 0 can be located quite definitely. Otherwise, Sta. 0 will have to be located by eye or by use of the string. By stretching the string along the gage-line of the rail, the point at which the track begins to curve can be located within a few feet. This is a very important step, since upon it depends the amount that the track will have to be shifted to place it in correct alinement.

170. Locating Other Stations. After Sta. 0 has been located, the other stations are obtained by measuring along the *outside* rail of the curve with a steel tape. The 31-ft. stations are marked on the

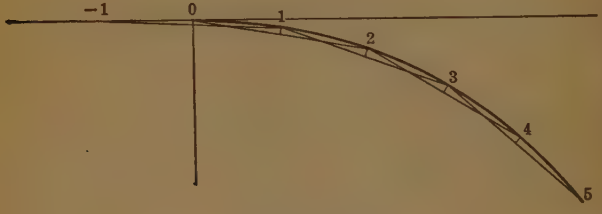

FIG. 46.

head of the rail with a soft pencil and on the flange on the inside of the rail with keel. The number of each station, or that of every fifth station, is marked on the flange. Two or three stations are chained on the tangent back of Sta. 0. These are designated Sta. −1, Sta. −2, etc. One such minus station is always necessary, and more may be, if it is found desirable to start the curve back of Sta. 0. The stationing is continued for several stations beyond the end of the curve for the same reason.

171. Measuring the Existing Middle Ordinates. Three men are required to measure the old, or existing, middle ordinates. Two men stretch the 62-ft. string between Sta. −1 and Sta. 1, and the third man measures the middle ordinate at Sta. 0 and records it in the notes. The party then moves forward 31 ft.; the string is stretched between Sta. 0 and Sta. 2, and the middle ordinate at Sta. 1 is measured and recorded. This procedure is continued throughout the curve, as illustrated in Fig. 46.

For greater accuracy and convenience in measuring the middle ordinates, two paddles similar to that shown in Fig. 47*a* may be used. The ends of the string pass through the holes in the paddles

1 the paddles are held against the head of the rail. The holes are
: inch from the gage line of the rail, and this amount has to be
)tracted from the middle ordinate as measured with a scale which
rts with zero. It is convenient also to construct a special scale
)ut 12 in. long with the zero of the scale one inch from the end to
)w for the 1-in. offset in the string. The divisions on such a scale
1 be any desired length and should be numbered consecutively

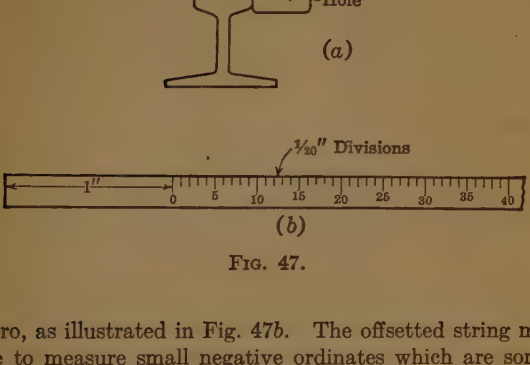

FIG. 47.

)m zero, as illustrated in Fig. 47*b*. The offsetted string makes it
ssible to measure small negative ordinates which are sometimes
ınd at the ends of curves.

172. Measuring Unit. There is a difference of opinion as to how
)sely the middle ordinates should be measured. The American
ilway Engineering Association recommends a unit of 0.10 inch.
ıe English practice is to use 1 mm. (0.04 in.). The authors
ommend a value of 0.05 in., because this value not only gives
eater accuracy than the 0.10-in. value, but also is much easier to
e in the computations necessary to adjust the curve. It is believed
ıt the last statement will be concurred in by all those who have
d practical experience in string-lining curves. As far as the theory
string-lining is concerned, it makes no difference whether $\frac{1}{8}$, $\frac{1}{10}$,
or $\frac{1}{20}$ inch is used. In the problems which follow, all middle
linates are given in units of 0.05 in.

173. The Problem. The middle ordinates, obtained as above
scribed, will vary, because the alinement of the existing track does
t conform to a circle, or to a circle with true spirals at its ends.

To be realined the track will have to be moved, inward here and outward there, until all the middle ordinates on the circular curve have the same value, and the ordinates on the spirals vary by a constant increment. The problem is to determine how much the track will have to be moved, or thrown, at each station in order to bring about this condition. The distance the track is moved at each station is called the *throw* at that station. If the throw is outward, it is a plus throw; and if inward, it is a minus throw.

174. Effect of Throwing Track at One Station. The effect on the middle ordinates of a throw at a given station is illustrated in Fig. 48. A, B, C, D, and E are stations on an existing curve. The middle ordinates at B, C, and D are BG, CH, and DI, respectively. Suppose it is found desirable to move the track at C outward to C'. What effect will this have upon the three middle ordinates? The

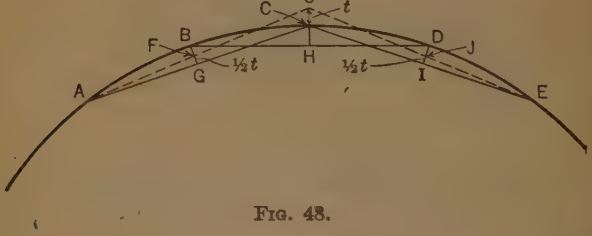

FIG. 48.

ordinate at C has been increased by the amount of the throw, t. At the same time the ordinates at B and D have been decreased by an amount equal to $\frac{1}{2}t$, since $FG = JI = \frac{1}{2}CC'$. This does not show clearly in the figure, because it is not drawn to scale, but FG, CC', and DJ are practically parallel and may be so considered. From this, the following rules may be stated:

Rule 1. Throwing the track *outward* at any station *increases* the middle ordinate at that station by an amount equal to the throw, and *decreases* the middle ordinates at the two adjacent stations by an amount equal to *one-half* the throw.

Rule 2. Throwing the track *inward* at any station *decreases* the middle ordinate at that station by an amount equal to the throw, and *increases* the middle ordinates at the two adjacent stations by an amount equal to *one-half* the throw.

175. Sum of Middle Ordinates. A fundamental principle of the

ing-lining method is that, for a constant length of chord, the sum all the middle ordinates is a constant, because this sum is a function of the central angle of the curve. It makes no difference what gree of curve is used to join the tangents, or how irregular the rve may be, the sum of the middle ordinates must be a constant any given length of chord. This being true, the sum of the new ddle ordinates for the realined curve must be the same as that of e old ordinates.

176. The New Ordinates. The next step is to determine the new dinates. This can be done only by trial. A new series of ordinates ust be found which will satisfy the following conditions:

1. The sum of the new ordinates for the entire curve, including irals, must be the same as that of the old ordinates.
2. The ordinates at all points on the circular curve must be equal e section 117).
3. The ordinates at successive points on the two end spirals must crease or decrease at a constant rate (see section 138).
4. The use of the new ordinates must not require the existing ack to be thrown at any point more than a specified distance, nerally 8 in., which is equivalent to 160 units when 0.05 in. is used the unit of measurement.

It was shown in Eq. 23 that the middle ordinate (in inches) of a rcular curve for a 62-ft. chord is, approximately, the same as e degree of curve. Therefore m, in units of 0.05 in., is equal $20D$.

It was shown in section 144 that the middle ordinate for spirals ay be computed from Eq. 44, which is repeated below.

$$m \text{ (in ft.)} = 0.218\, n^2 \frac{D_1 + D_2}{2}$$

For $n = 0.62$,

$$m \text{ (in ft.)} = 0.0419\,(D_1 + D_2) = 0.0419\,(s_1 + s_2)k$$

$$m \text{ (in 0.05-in. units)} = 10.056\,(s_1 + s_2)k$$

For the chord between Sta. 0 and Sta. 2, this becomes

$$m = 10.056\,(0.00 + 0.62)k = 6.235k \qquad (62)$$

For the chord between Sta. 1 and Sta. 3,

$$m = 10.056\,(0.31 + 0.93)k = 12.47k$$

TABLE 7

Middle Ordinates for Spirals, Based on 62-ft. Chords

Sta.	Middle Ordinates in 0.05-in. Units									
	Increment = 3 units, $k = 0.48$	Increment = 4 units, $k = 0.64$	Increment = 5 units, $k = 0.80$	Increment = 6 units, $k = 0.96$	Increment = 7 units, $k = 1.12$	Increment = 8 units, $k = 1.28$	Increment = 9 units, $k = 1.44$	Increment = 10 units, $k = 1.60$	Increment = 11 units, $k = 1.76$	Increment = 12 units, $k = 1.92$
0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	$1\frac{1}{6}$	$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{5}{6}$	2
1	3	4	5	6	7	8	9	10	11	12
2	6	8	10	12	14	16	18	20	22	24
3	9	12	15	18	21	24	27	30	33	36
4	12	16	20	24	28	32	36	40	44	48
5	15	20	25	30	35	40	45	50	55	60
6	18	24	30	36	42	48	54	60	66	72
7	21	28	35	42	49	56	63	70	77	84
8	24	32	40	48	56	64	72	80	88	96
9	27	36	45	54	63	72	81	90	99	108
10	30	40	50	60	70	80	90	100	110	120
11	33	44	55	66	77	88	99	110	121	132
12	36	48	60	72	84	96	108	120	132	144
13	39	52	65	78	91	104	117	130	143	156
14	42	56	70	84	98	112	126	140	154	168

rom these equations it is evident that the value $6.24k$ is not ly the middle ordinate at Sta. 1 on the spiral, but is also the crement used to obtain the middle ordinate at Sta. 2. This me increment is used to obtain the middle ordinates at succes- ve stations.

Table 7 gives the middle ordinates for spirals of different values of Fractional values of k have been used in this table in order that e middle ordinates may be in whole numbers. For other values k, the ordinate may be computed from Eq. 62, or obtained from e table by interpolation.

The determination of the new ordinates is unquestionably the ost important step in the realinement of a curve by the string- ing method, and should be done by experienced men. The prin- ble is quite simple, but the application of that principle, so as to tain the best and most economical results, is a more difficult atter. To illustrate the principles involved and the procedure, e following examples are given.

177. Example 1. The computations are recorded in tabular form shown in Example 1. The stations are recorded in column 1, d the old ordinates in column 2. The new ordinates, which are termined by trial, are recorded in column 3. The difference tween the old ordinate and the new ordinate at any station is lled the *error* and is recorded in column 4. If the old ordinate is eater than the new ordinate, the error is positive and is preceded a plus sign. If the old ordinate is less than the new ordinate, e error is negative and is preceded by a minus sign. In column 5 e recorded the algebraic sums of all the errors up to and including at at the station opposite which it is recorded. To obtain the m of the errors at any station it is necessary only to add the error that station to the sum of the errors at the preceding station, as dicated by the arrows. Since the sum of the new ordinates must ual the sum of the old ordinates, the algebraic sum of all the errors ust be zero. Column 6 gives the half-throws necessary at each ation. The successive values in this column are also obtained by ccessive algebraic additions. The half-throw at any station is the gebraic sum of the half-throw at the preceding station plus the m of the errors at that station. The addition is made as indicated the arrows. Column 7 gives the throws at each station.

That the throws in column 7 will result in the new ordinates ven in column 3 is proved as follows: As shown in Fig. 48, the l ordinate at any station is changed by the full amount of the throw

at that station and by one-half the throws at the adjacent stations.

Old ordinate at Sta. 11 =	70
Inward throw at Sta. 11 =	−56
	14
½ Inward throw at Sta. 10 =	+30
	44
½ Inward throw at Sta. 12 =	+18
New ordinate at Sta. 11 =	62

The first example is a very simple one and is given to illustrate the general method of determining the throws. This curve was originally staked out by the deflection angle method and the location plans show that it is a 3° 06′ curve with end spirals whose $k = 1°$. The circular curve is 434 ft. in length, or 14 chord lengths of 31 ft. With these data, we also know the original middle ordinates which must be reestablished by realining the track. Also, superelevation stakes are found at the *T.S.* and the *S.T.*, so there is no question as to the location of these points.

For this curve we know from Eq. 23 that the middle ordinates of the circular curve are,

$$m \text{ (in inches)} = D^\circ = 3.1$$

$$m \text{ (in 0.05-in. units)} = 62$$

Since $k = 1°$, the values of the spiral ordinates in Table 7, in the column headed $k = 0.96$, have all been increased by one-twenty-fourth to obtain the ordinates to be used.

In this problem the amount by which the successive middle ordinates increase is 6.2 units. It will be observed that the ordinate at Sta. 0 in Table 7 is only one unit, or one-sixth this value. This is because the chord is between Sta. −1 and Sta. 1, and half the chord is on the tangent and half on the curve. The increment, therefore, is only one-sixth full value as shown in Fig. 49. A similar condition exists at the other end of the spiral where the chord is between Sta. 9 on the spiral and Sta. 11 on the circular curve, so that the difference between the middle ordinate at Sta. 9 and that

EXAMPLE 1

Final Solution

Sta.	Old Ordinates	New Ordinates	Errors	Sum of Errors	Half-throws	Throws
(1)	(2)	(3)	(4)	(5)	(6)	(7)
−1	0	0	0	0	0	0
L.S. 0	5	1	+ 4	+ 4	0	0
1	9	6	+ 3	+ 7	+ 4	+ 8
2	14	12	+ 2	+ 9	+11	+22
3	16	19	− 3	+ 6	+20	+40
4	20	25	− 5	+ 1	+26	+52
5	24	31	− 7	− 6	+27	+54
6	30	37	− 7	−13	+21	+42
7	40	44	− 4	−17	+ 8	+16
8	54	50	+ 4	−13	− 9	−18
9	61	56	+ 5	− 8	−22	−44
L.C 10	71	61	+10	+ 2	−30	−60
11	70	62	+ 8	+10	−28	−56
12	68	62	+ 6	+16	−18	−36
13	64	62	+ 2	+18	− 2	− 4
14	55	62	− 7	+11	+16	+32
15	53	62	− 9	+ 2	+27	+54
16	58	62	− 4	− 2	+29	+58
17	56	62	− 6	− 8	+27	+54
18	59	62	− 3	−11	+19	+38
19	68	62	+ 6	− 5	+ 8	+16
20	64	62	+ 2	− 3	+ 3	+ 6
21	60	62	− 2	− 5	0	0
22	58	62	− 4	− 9	− 5	−10
23	69	62	+ 7	− 2	−14	−28
T.S. 24	65	59	+ 6	+ 4	−16	−32
25	60	54	+ 6	+10	−12	−24
26	52	48	+ 4	+14	− 2	− 4
27	38	42	− 4	+10	+12	+24
28	32	36	− 4	+ 6	+22	+44
29	23	30	− 7	− 1	+28	+56
30	18	24	− 6	− 7	+27	+54
31	15	18	− 3	−10	+20	+40
32	15	12	+ 3	− 7	+10	+20
33	10	6	+ 4	− 3	+ 3	+ 6
S.T. 34	4	1	+ 3	0	0	0
35	0	0	0	0	0	0

at Sta. 10 is five-sixths the full value, or five units. The proof of this is shown in Fig. 49.

Another point to be noted is that the last throw must be zero, or else the forward tangent would have to be moved in order to be tangent to the new curve. Because of the conditions in this example, no difficulty was experienced in replacing the old curve with a theoretically correct new curve.

178. Difference in Results Obtained by Two Methods. It might be well at this point to consider the difference between a curve staked out by the deflection angle method and one realined by the string-lining method. In the deflection angle method the two tangents are

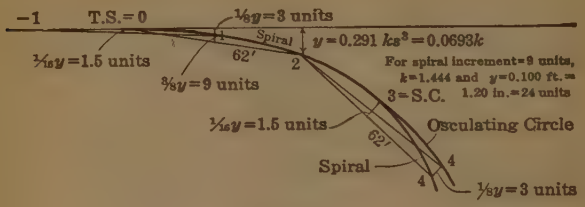

Fig. 49.

equal, the curve is symmetrical, the circular portion of the curve is generally an odd number of feet in length, and the end spirals are the same in length. In the string-lining method the two tangent distances are not quite equal, the curve is not symmetrical, the length of the circular portion of the curve is a multiple of one-half the chord length, and the end spirals are generally not of the same length. This follows from the procedure used in the string-lining method. From a practical standpoint, however, the results of the string-lining method are entirely satisfactory.

179. Example 2. The problem in Example 1 is not a typical one, and was chosen to explain the general procedure in computing the throws. The problem which follows is a typical one and introduces certain modifications which are generally necessary. Its solution required a number of trials, only the final one being shown.

In column 2 of Example 2 are given the existing ordinates of an old curve which requires realinement. Many solutions are possible, but some will require greater throws than others. The problem is to

ld that solution which will require the least shifting of the track ld at the same time result in a satisfactory curve. The successive ps in the solution of this problem are as follows:

1. The first step is to obtain the sum of the ordinates in column 2. his sum is found to be 2504 units. The sum of the new ordinates column 3 must be the same.

2. The second step is to examine the fluctuating ordinates of the rcular portion of the curve and to determine a constant value which ll replace them satisfactorily. From inspection it seems that the *C.* comes at Sta. 10 and the *C.S.* at Sta. 31. Several trials proved is to be correct. The sum of the 20 ordinates between these ations is 1664 units, or an average of 83.2. A trial value of 83 was osen and found to be satisfactory. The nearest whole unit, how- er, will not always be the most satisfactory, since the end spirals ve an effect on the circular curve. Twenty ordinates of 83 units ch make a total of 1660 units for the circular portion of the curve. his sum of 1660 units subtracted from 2504 units leaves 844 units r the two end spirals.

3. The third step is to determine the ordinate at Sta. 10, the *S.C.*, hen the string is stretched between Sta. 9 on the spiral and Sta. 11 n the circular curve. For all spirals whose value of k is between 48° and 1.44°, or whose spiral increment is between 3 and 9, this dinate will be $\frac{1}{2}$ to $1\frac{1}{2}$ units less than the ordinate for the circular rve. For values of k greater than 1.44°, the ordinate will be $\frac{1}{2}$ to 2 units less. In this problem, a brief inspection of the old dinates in column 2 and the spiral ordinates in Table 7 will show at a suitable spiral will have a spiral increment of about 9 units; nce, the ordinate at Sta. 10 will be 83 minus 1 or 2 units. After veral trials, a value of 82 was chosen.

4. The fourth step is to choose a suitable spiral. Spirals with crements of 8, 9, and 10 were tried in turn, and 9 was found to give e best results. For the spiral whose increment is 9 units, the fference between the ordinates at Sta. 9 and at Sta. 10 will be either or 2 units less than the increment 9, as shown in Fig. 49. There- re, the ordinate at Sta. 9 can be either 75 or 76 units. By trial, 5 was found more satisfactory than 76. With the ordinate at a. 9 fixed at 75, the ordinate at Sta. 8 will be 9 less, or 66, and the dinates at Stas. 7, 6, 5, 4, 3, and 2 will be 57, 48, 39, 30, 21, and 12, spectively. At Sta. 1 the ordinate is 3, which is 9 less than the dinate at Sta. 2, but this is a coincidence.

The *T.S.* is at Sta. 0 + 20.7, and a stake should be driven at this

point to mark the beginning of the curve. The method of determining the plus of the *T.S.* is shown in Fig. 50.

The ordinate at Sta. 2 is 12 units. This is measured from the 62-ft. chord *BB'* between Sta. 1 and Sta. 3. If the chord *BB'* is shifted backward along the spiral toward the *T.S.*, the value of the middle ordinate will decrease at a uniform rate. This rate of decrease for this spiral is 9 units in 31 ft., or 1 unit in 3.44 ft. When the chord *BB'* reaches the position *AA'*, *A* will be at the *T.S.* and the middle ordinate will be 9 units, or 3 less than the ordinate to *BB'* at Sta. 2. To change the ordinate by 3 units, the chord will

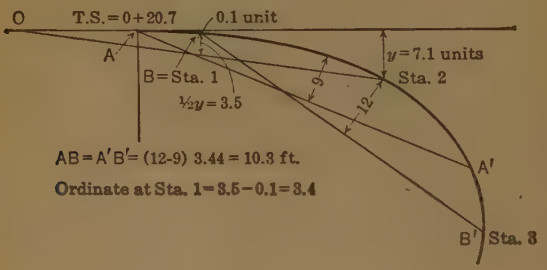

FIG. 50.

have to be shifted 3×3.44 ft. $= 10.33$ ft., and $A'B' = AB = 10.33$ ft. But point *B* is at Sta. 1; therefore the *T.S.* will be 10.33 ft. to the left of Sta. 1, or 20.7 ft. to the right of Sta. 0. When the string is stretched between Sta. 0 and Sta. 2, the ordinate at Sta. 1 will be 3.

5. Now that the new ordinates for the spiral have been tentatively determined, the next step is to compute the error, the sum of the errors, the half-throw, and the full throw at each station, and to decide whether these throws are excessive. For the first six stations the throws are outward and reach a maximum of 1.4 in. From Sta. 7 to Sta. 18 the throws are inward and reach a maximum value of 4.2 in. at Sta. 11. The throws are then outward again to the end of the circular curve. These throws are about as small as it is possible to get, and represent the result of ten trial solutions.

6. Since the result thus far is satisfactory, the next step is to choose a spiral for the forward end of the curve. The sum of all the ordinates must be 2504 units. The first spiral required 433 units and the circular curve 1660 units, leaving 411 units for the second spiral.

Starting with an ordinate of 82 at the *S.C.* and using a spiral

EXAMPLE 2

Final Solution

Station	Old Ordinates	New Ordinates	Errors	Sum of Errors	Half-throws	Throws
(1)	(2)	(3)	(4)	(5)	(6)	(7)
−1	0	0	0	0	0	0
0	3	0	+ 3	+ 3	0	0
T.S.+20.7						
1	7	3	+ 4	+ 7	+ 3	+ 6
2	9	12	− 3	+ 4	+10	+20
3	16	21	− 5	− 1	+14	+28
4	28	30	− 2	− 3	+13	+26
5	33	39	− 6	− 9	+10	+20
6	41	48	− 7	−16	+ 1	+ 2
7	68	57	+11	− 5	−15	−30
8	66	66	0	− 5	−20	−40
9	70	75	− 5	−10	−25	−50
S.C. 10	85	82	+ 3	− 7	−35	−70
11	97	83	+14	+ 7	−42	−84
12	82	83	− 1	+ 6	−35	−70
13	75	83	− 8	− 2	−29	−58
14	89	83	+ 6	+ 4	−31	−62
15	81	83	− 2	+ 2	−27	−54
16	87	83	+ 4	+ 6	−25	−50
17	92	83	+ 9	+15	−19	−38
18	75	83	− 8	+ 7	− 4	− 8
19	80	83	− 3	+ 4	+ 3	+ 6
20	88	83	+ 5	+ 9	+ 7	+14
21	76	83	− 7	+ 2	+16	+32
22	85	83	+ 2	+ 4	+18	+36
23	82	83	− 1	+ 3	+22	+44
24	74	83	− 9	− 6	+25	+50
25	87	83	+ 4	− 2	+19	+38
26	86	83	+ 3	+ 1	+17	+34
27	77	83	− 6	− 5	+18	+36
28	81	83	− 2	− 7	+13	+26
29	88	83	+ 5	− 2	+ 6	+12
30	82	83	− 1	− 3	+ 4	+ 8
C.S. 31	77	82	− 5	− 8	+ 1	+ 2
32	80	74	+ 6	− 2	− 7	−14
33	67	65	+ 2	0	− 9	−18
34	61	55	+ 6	+ 6	− 9	−18
35	45	46	− 1	+ 5	− 3	− 6
36	34	36	− 2	+ 3	+ 2	+ 4
37	22	26	− 4	− 1	+ 5	+10
38	15	18	− 3	− 4	+ 4	+ 8
S.T. 39	13	9	+ 4	0	0	0
40	0	0	0	0	0	0
Sum....	2504	2504				

increment of 10 units, the successive ordinates would be 82, 74, 64, 54, 44, 34, 24, 14, and 5, a total of 395 units, which is too small. Using an increment of 9 units, the successive ordinates would be 82, 75, 66, 57, 48, 39, 30, 21, 12, and 3, a total of 433 units, which is too large. By trial it was found that an increment of 9.4 units is about right. This increment gives successive ordinates of 82, 74, 65, 55, 46, 36, 27, 18, and 9, a total of 412 units, which is still one unit too large. One of the last series of ordinates, or one on the circular curve, will have to be reduced one unit. Before deciding which ordinate to change, the half-throws and throws are computed from the *C.S.* onward as follows:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	30	82	83	−1	−3	+4	+ 8
C.S.	31	77	82	−5	−8	+1	+ 2
	32	80	74	+6	−2	−7	−14
	33	67	65	+2	0	−9	−18
	34	61	55	+6	+6	−9	−18
	35	45	46	−1	+5	−3	− 6
	36	34	36	−2	+3	+2	+ 4
	37	22	27	−5	−2	+5	+10
	38	15	18	−3	−5	+3	+ 6
	39	13	9	+4	−1	−2	− 4
	40	0	0	0	−1	−3	− 6

At Sta. 39 the sum of the errors is −1, and the half-throw is −2. Both of these must be zero. One of the ordinates will have to be reduced one unit, and the last half-throw of −2 units indicates that the reduction must be made two stations back, or at Sta. 37. By changing the ordinate at Sta. 37 from 27 to 26, both the sum of the errors and the last half-throw at Sta. 39 will be zero.

Following the procedure described above, it is quite easy to choose the ordinates so that the sum of the errors will be zero, but to make the last half-throw zero is a more difficult matter. In the above example the fact that the last half-throw is small—only 2 units—is a coincidence. It might easily have been much larger. Suppose it had been −20 instead of −2. In this case, the ordinate 20 stations back, at Sta. 19, would have to be reduced one unit, instead of the ordinate at Sta. 37.

It may happen that the sum of the errors is zero, but the last half-throw is not zero. In this case, one of the ordinates will have to be reduced one or more units and another ordinate increased by

the same amount, in order to keep the sum of the errors equal to zero. The distance apart of the stations at which the ordinates are changed multiplied by the change in the value of the ordinate will equal the resulting change in the last half-throw. From this it follows, that, if in the preceding example, the new ordinate at Sta. 1 had been chosen as 2 units instead of 3 units, the half-throw at Sta. 39 would have been 38 units. This explains why so many trials are necessary before the best solution can be obtained. It also bears out the statement previously made to the effect that a more satisfactory solution can be obtained if the unit of measurement is 0.05 in. rather than 0.10 in.

180. Compound Curves. If the curve to be realined is a compound curve, it will be necessary to place a spiral between the two branches of the curve. The procedure in determining the new ordinates for this spiral can be explained best by an example.

Suppose a study of the old ordinates indicates that a uniform ordinate of 51 can be used for the first branch of the curve up to Sta. 29, and a value of 82 can be used for the second branch beyond Sta. 34. The middle spiral would then be five stations in length. Since the purpose of the spiral is to bring about a gradual change of the ordinate from 51 for the first branch to 82 for the second branch, and since this change must be accomplished in five stations, the increment per stations must be one-fifth of 31, or approximately 6. Hence the spiral whose increment is 6 will best meet the requirements. The ordinate at Sta. 29, when the string is stretched between Sta. 28 on the circular curve and Sta. 30 on the spiral, will be 51 plus one-sixth the spiral increment, or 52. The ordinates at Stas. 30, 31, 32, and 33 will be 58, 64, 70, and 76, respectively. At Sta. 34, the *S.C.*, the ordinate will be 76 plus five-sixth the spiral increment, or 81. The difference between the ordinates at Sta. 34 and Sta. 35 is one unit, or one-sixth the spiral increment. The numerical values in this example were specially chosen to illustrate the procedure. Generally, a fractional value of the spiral increment would have to be used.

181. Staking the Curve. After satisfactory throws have been computed, stakes are set opposite each station by measurement from the gage of the outside rail. The stakes are so set that they will be in the center of the track after it has been thrown the computed amount at each station. If a station falls on a tie, the stake will have to be moved so as to miss the tie. The ballast around the stakes should be removed so that the stakes will not be pushed out of line when the track is shifted.

CHAPTER 7

PARABOLIC CURVES

PARABOLIC curves are used for vertical curves on grade lines, for pavement crowns, as substitutes for circular curves under certain conditions, and in other cases where it is more convenient to locate the curve by coordinates than by angles and distances.

182. Properties. The equation of the parabola is $y = px^2$. In general the axes are oblique, the X axis coinciding with a tangent and the Y axis paralleling a diameter. From the equation, the following properties can be derived:

1. The offsets from a tangent vary directly with the square of the distance from the point of tangency.

2. A line from the middle of a chord to the intersection of the tangents from the ends of the chord is bisected by the curve. In other words, the external distance E is equal to the middle ordinate M.

3. The second differences between the values of y for equal increments of x are equal.

These fundamental properties make the parabola particularly adaptable to certain work.

183. Compound Parabolas. Occasionally a single symmetrical parabola does not seem to fit the conditions. In the case of horizontal curves this can usually be rectified by using an unsymmetrical curve with unequal tangents. This can not always be done on account of objects interfering with the field work. In this case two or more parabolas can be compounded. The point of compounding is located and the common tangent laid out, and then each branch of the curve is located by the methods given.

On vertical curves, since the coordinates are measured horizontally and vertically, unsymmetrical curves theoretically can not be used. Practically, however, they can be used provided the difference in tangent lengths is not too great, since the effect is merely to distort the curve somewhat and this is of no moment. Occasionally, the best solution lies in using compound curves. The

various tangents are laid out of such lengths that, when the lengths of the vertical curves are fixed, their ends become coincident. Each curve is computed separately.

Vertical Curves

The intersections of the several straight portions of a grade line must be rounded off to avoid undue stresses in drawbars, insure smooth riding, improve the appearance, and provide safe sight distance for automobile drivers. Since the intersection angles are always small, and gravity tends to hold the vehicles in contact with the roadway, the effect of centrifugal force is imperceptible, and the installation of vertical curves is less complicated than that of horizontal curves.

184. Length of Vertical Curve. The length of a vertical curve is dependent on (1) the total change of grade between the two tangents, (2) the safe rate of change of grade per station, (3) the necessary sight distance, and (4) convenience in computing and staking out the curve.

The minimum change of grade which requires a vertical curve is about 0.1 per cent. On roads and streets a greater change than this without a vertical curve will show a distinct break of grade which looks bad, although the natural processes of construction will round the change enough to be acceptable to traffic. On railroads the natural stiffness of the rails will automatically supply a curve of sufficient length.

The tangents of a vertical curve are made equal, and the *P.I.* should be made to come at a regular grade stake. It is convenient, therefore, to make each tangent some multiple of the grade-stake spacing, and hence the total length of vertical curve should be some multiple of twice the grade-stake spacing. On pavements where grade stakes are commonly placed 25 ft. apart, the minimum length of vertical curve becomes 50 ft., and the greater lengths become some multiple of 50 ft. On railroads, grade stakes are normally 100 ft. apart, hence the length of vertical curves should be some multiple of 200 ft., although multiples of 100 ft. are sometimes used.

The maximum length of vertical curve is governed by the topography and the judgment of the engineer. Ordinarily the curves should be made as long as conditions will permit. A grade line with long, easy vertical curves looks better and rides better than one

with short curves on either a highway or a railway. There is a mistaken belief that long vertical curves add to the amount of earthwork. If the grade line is properly designed, both tangents and curves being considered together, the long vertical curves need not add to the earthwork and frequently will actually reduce it.

185. Rate of Change. By definition

$$r = \frac{G_1 - G_2}{L} \tag{63}$$

where r is the rate of change in per cent per station (100 ft.), G_1 and G_2 are the two given grades in per cent, and L is the length of the vertical curve in stations. The grades G_1 and G_2 are considered positive when ascending and negative when descending, and are used with their proper algebraic signs in this equation.

The American Railway Engineering Association recommends $r = 0.1$ for summits and $r = 0.05$ for sags on first-class steam roads. Twice these amounts are recommended on less important lines and on electric roads. Many first-class systems are using the higher rates with apparent satisfaction.

The rate of change is little used in highway work, and then usually in an incidental or auxiliary manner.

186. Sight Distance. Sight distance becomes a controlling factor at summits on highways when the total change of grade is large. On single-lane roads the sight distance should always be something

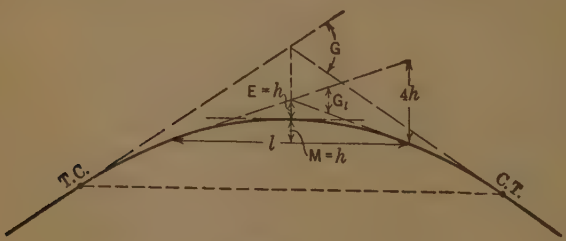

FIG. 51.

more than the sum of the distances in which two vehicles approaching each other can stop. On double-lane roads it is presumed that traffic will stay in the proper lane, and hence the sight distance

should be such that a driver can recognize an approaching vehicle without being startled when traveling at normal road speeds and with the corresponding degree of attention given the road. Experience seems to fix the minimum sight distance on vertical curves at the same value as for horizontal curves, or 700 ft., but with a tendency to use at least 1000 ft. on account of the increasing road speeds.

In Fig. 51.

Let L = total length of vertical curve in stations.

l = the sight distance in stations.

h = height of line of sight above the roadway at the point of observation, assumed to be the same for both vehicles, and usually taken as 5.0 ft.

G = total change of grade in per cent.

G_l = change of grade in the sight distance l.

Then from the figure and the properties of the parabola,

$$E = M = h \quad \text{and}$$

$$G_l = \frac{4h}{\frac{1}{2}l} = \frac{8h}{l} \tag{64}$$

By proportion,

$$L = \frac{lG}{G_l}$$

whence

$$L = \frac{l^2G}{8h} \tag{65}$$

Eq. 65 is exact when G is greater than G_l and approximate, but on the safe side, when G is less than G_l.

187. Computing Vertical Curves. There are two common methods of computing vertical curves, each of which has certain advantages under particular conditions. The first method may be called the *tangent offset method* and is generally used on road and street work where the stakes are placed less than a full station of 100 ft. apart. The second method may be called the *rate of change method* and is frequently used on railroads when the stakes are placed at 100-ft. intervals.

188. First Method. In Fig. 52 the elevation of the *P.I.* at *A* is known, the grades G_1 and G_2 are known, and the length *L* of the vertical curve has been chosen. Since all distances are theoretically measured on the horizontal, the tangent distances are each equal to $\frac{1}{2}L$, whence the stations of the *T.C.* and *C.T.* at *B* and *C*, respectively, can be determined. The elevations of these points can then be found since the grades are known. *D*, *E*, *F*, *G*, etc., are points where grade stakes are to be placed at equal distances apart, there being *n* such distances on each side of the *P.I.* Figure 52 is very much exaggerated, as angles *ABC* and *ACB* are generally less than 2°.

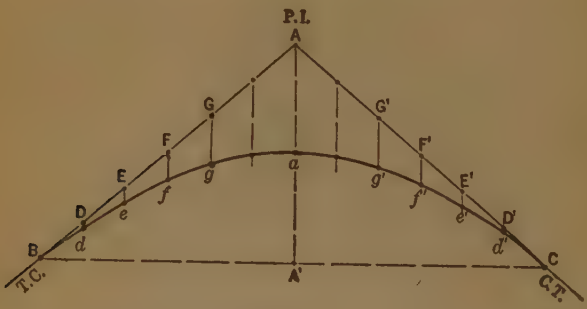

FIG. 52.

The elevations of the points on the tangent at *D*, *E*, *F*, *G*, etc., are computed, since the distances and grades are known.

The elevation of *A'*, the midpoint of the long chord *BC*, is readily found, as it is the average of the elevations of *B* and *C*.

From the properties of the parabola, $Aa = aA'$, hence the elevation of *a* and the ordinate *Aa* can be determined. Also,

$$Dd : Aa :: (BD)^2 : (BA)^2$$

But *BD* is one unit of *n* equal units between *B* and *A*, hence

$$Dd : Aa :: 1^2 : n^2, \quad \text{or} \quad Dd = \frac{Aa}{n^2} \quad \text{and} \quad Aa = n^2 Dd$$

and

$$Ee = 2^2 \times Dd = 4Dd$$

$$Ff = 3^2 \times Dd = 9Dd$$

$$Gg = 4^2 \times Dd = 16Dd$$

and so on for the number of spaces *n*.

Having now computed the offsets from the tangent, the elevations of the points on the curve are found by correcting each elevation on the tangent by the corresponding offset. If the curve is at a summit, as shown in Fig. 52, the ordinates are subtracted from the tangent elevations. If the curve is in a sag, the ordinates are found in the same way, but are added to the tangent elevations.

Since the curve is symmetrical about the *P.I.*, it is necessary to compute the offsets for only one-half the curve, since the offsets at the corresponding points on the second half, as at D', E', F', G', etc., are the same as on the first half.

189. Second Method. This method is based on the third property of the parabola as given in section 182. To use this method it is necessary to know, or to be able to find, the station number and elevation of the *P.I.*, the two grades G_1 and G_2, and an approximate value of r.

The first step is to determine the length of curve and the value of r to use. Using the given grades and the approximate value of r in Eq. 63, a trial value of L is computed. This will usually be an odd value which is inconvenient to use and, therefore, the next higher, even number of stations is adopted as the length of curve to use. This value of L is then inserted in Eq. 63 and the value of r to be used is determined. This will generally be an odd value, but it must be used as computed and should be carried out to several decimal places, since any fractions that are dropped will form a cumulative error.

The second step is to determine the station numbers and elevations of the *T.C.* and the *C.T.* which is easily done since the grades, the station number and elevation of the *P.I.*, and the length of curve are known.

The last step is to determine the elevations of the stations on the curve. This is done by first computing the grade of the chord from each station to the next station, and from this new grade determining the elevation of the second station in question.

Since r is the rate of change of grade per station (100 ft.), the difference in grade between successive chords must be equal to r, hence the grade of a chord from one station to the next can be found from the grade of the chord to the preceding station by adding or subtracting r. In these computations r is considered positive, but when the curve is convex upward it is subtracted algebraicly from the preceding grade; and when the curve is concave upward it is

added algebraicly to the preceding grade. The grades G_1 and G_2 are used with their proper algebraic signs.

Starting at the *T.C.*, the first change of grade is from the tangent to a chord, instead of from a chord to a chord, and is therefore only $\frac{1}{2}r$, hence the grade of the first chord is $G_1 \pm \frac{1}{2}r$. The elevation of the first station on the curve can now be found from the elevation of the *T.C.* The grade of the second chord is $G_1 \pm \frac{1}{2}r \pm r$, and the elevation of the second station is found from the elevation of the first station. The grade of the third chord is $G_1 \pm \frac{1}{2}r \pm r \pm r$, and so on until the *C.T.* is reached. The grade of the chord to the *C.T.*, changed by $\frac{1}{2}r$, should equal G_2, and the elevation of the *C.T.* should check with that found in the second step.

This method can be adapted to other spacings of stakes than 100 ft. by making G_1 and G_2 the slope in feet in the given stake spacing and, at the same time, making r equal the change of slope in feet in the same distance. It is usually preferable, however, to use the first method in such cases.

Example. A $+0.8\%$ grade meets a -0.6% grade at Sta. 30 + 00, whose elevation is 750.50. It is desired to use a vertical curve whose rate of change will be about 0.2% per station.

From Eq. 63,

$$L \text{ (trial)} = \frac{0.8 - (-0.6)}{0.2} = \frac{1.4}{0.2} = 7.000 \text{ Sta.}$$

Making $L = 8.00$ Sta.,

$$r \text{ (to use)} = \frac{1.4}{8.0} = 0.175$$

Taking everything into consideration it is better to make $L =$ 800 ft. (400 ft. on each side of the *P.I.*) and use $r = 0.175$ than it would be to make $L = 700$ ft. and $r = 0.2$.

Sta. *T.C.* = 30 + 00 − 4 + 00 = 26 + 00

Sta. *C.T.* = 30 + 00 + 4 + 00 = 34 + 00

Elev. *T.C.* = 750.50 − 4 × 0.8 = 747.30

Elev. *C.T.* = 750.50 − 4 × 0.6 = 748.10

The computations of the chord grades and the elevations of the stations on the curve may be arranged in tabular form as follows:

Station		Computations		Profile Elevations	Remarks
		Grade	Elevations		
24			745.7000	745.70	On tangent
			$G_1 = +0.8000$		
25			746.5000	746.50	On tangent
		$+0.8000 = G_1$	$+0.8000$		
26	*T.C.*	$-0.0875 = \frac{1}{2}r$	747.3000	747.30	*T.C.*
		$+0.7125$	$+0.7125$		
27		$-0.1750 = r$	748.0125	748.01	
		$+0.5375$	$+0.5375$		
28		$-0.1750 = r$	748.5500	748.55	
		$+0.3625$	$+0.3625$		
29		-0.1750	748.9125	748.91	
		$+0.1875$	$+0.1875$		
30		-0.1750	749.1000	749.10	
		$+0.0125$	$+0.0125$		
31		-0.1750	749.1125	749.11	
		-0.1625	-0.1625		
32		-0.1750	748.9500	748.95	
		-0.3375	-0.3375		
33		$-0.1750 = r$	748.6125	748.61	
		-0.5125	-0.5125		
34	*C.T.*	$-0.0875 = \frac{1}{2}r$	748.1000	748.10	*C.T.* Check
		$-0.6000 \quad G_2 =$	-0.6000		
35			747.5000	747.50	On tangent
			$G_2 = -0.6000$		
36			746.9000	746.90	On tangent

Vertical Curve $L = 800$ ft.

190. Low Point on Vertical Curve. The low point on a vertical curve in a sag must sometimes be located for the purpose of installing storm sewer inlets or other drainage appurtenances.

In Fig. 53, L is the length of the vertical curve, G_1 and G_2 are the

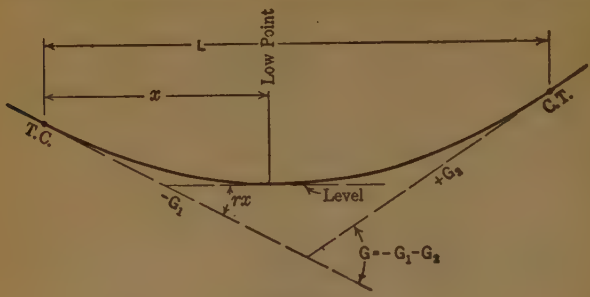

FIG. 53.

two grades, and x is the distance in stations from the $T.C.$ to the low point.

Since the tangent to the curve at the low point must be horizontal, that is, the slope must be zero,

$$G_1 - rx = 0 \quad \text{or} \quad rx = G_1$$

From Eq. 63,

$$r = \frac{G_1 - G_2}{L}$$

Substituting this value of r in the preceding expression and reducing,

$$x = \frac{LG_1}{G_1 - G_2} \tag{66}$$

in which G_1 and G_2 must be used with their proper algebraic signs.

Example. $A - 2.0\%$ grade meets a $+ 3.0\%$ grade at Sta. $10 + 00$. A vertical curve 400 ft. long is to be used, and the station of the low point is required. The station of the $T.C.$ is $10 + 00$ minus $2 + 00$, or $8 + 00$.

From Eq. 66.

$$x = \frac{4 \times (-2.0)}{-2.0 - (+3.0)} = \frac{-8.0}{-5.0} = 1.6$$

The station of the low point is $8 + 00$ plus $1 + 60$, or $9 + 60$. Eq. 66 can also be used to locate the high point on a vertical curve at a summit.

Pavement Crowns

Highway and street pavements are crowned to drain the surface water to the side ditches or the gutters. Normally the crown is convex upward, but on alleys and narrow streets an *inverted crown* with a center gutter is frequently used.

191. Form of Crown. The usual form of crown for roadways of moderate width is a vertical parabola exactly the same as a vertical curve. Some engineers specify an arc of a circle, but the difference between the two is so small that no matter which is specified the crown ordinates can always be computed by the more simple parabolic method.

With widths greater than about 40 ft., a simple parabola may result in cross slopes too steep at the sides, if the middle is crowned enough to drain properly, or the middle is too flat with suitable slopes at the edges. Several forms of crowns have been suggested to overcome these difficulties. The simplest and perhaps best form is a parabola for the middle 35 to 40 ft., flanked by side strips on uniform slopes practically tangent to the parabola.

192. Amount of Crown. The total amount of crown is a function of the width and type of surface, remembering that motor traffic desires as flat a surface as possible. Consequently the average cross slopes should be limited to values between $\frac{1}{16}$ and $\frac{3}{8}$ in. per foot for high type pavements, $\frac{1}{4}$ to $\frac{3}{4}$ in. per foot for gravel and macadam, and $\frac{3}{8}$ to 1 in. per foot for earth roads.

Suitable values of the crown for pavements up to about 40 ft. in width and conforming to present practice may be computed from the equation,

$$C = \frac{W^2}{m}$$

where C is the total crown in inches, W is the total width in feet, and m is a factor depending on the kind of surface. For high type pavements m may be taken as 400, for gravel and macadam 200, and for earth 133.

For widths above 40 ft., use values from the above equation for a middle width of 35 to 40 ft. and add uniform slopes on the side

strips of $\frac{3}{8}$ in. per foot for pavements, $\frac{3}{4}$ in. per foot for gravel and stone, and 1 in. per foot for earth. These uniform slopes are essentially tangent to the respective parabolas.

Thus the crown for a concrete pavement 20 ft. wide would be 1 in. For a width of 40 ft. it would be 4 in., and for a width of 60 ft. it would be $4 + 10 \times \frac{3}{8} = 7\frac{3}{4}$ in.

193. Constructing the Crown. In building the crown, the shape both of the subgrade and of the surface is obtained (1) by means of templates cut to the correct shape, (2) by measuring ordinates from a string stretched between the tops of the curbs or grade stakes, or (3) by similar ordinates measured from a line of sight across tees held on the curbs or grade stakes. It is necessary, therefore, to compute the offsets, first from a tangent to the parabola, and then correct these to the desired reference line.

Fig. 54 shows a normal cross-section with the edges of the pavement at the same elevation. A tangent to the parabola at the cen-

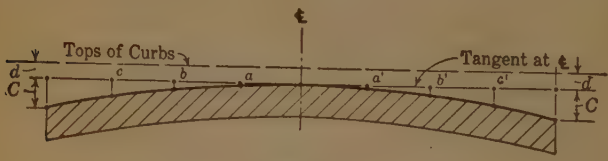

FIG. 54.

ter line passes above the edges of the pavement by the amount of the crown C, and hence C forms a known offset from the tangent at a distance $\frac{1}{2}W$ from the center line. Any other ordinates, as at a, b, c, can then be computed since they must vary as the square of the distance. By making the intervals between the ordinates equal, the various ordinates vary as the square of the digits, as explained under vertical curves. In fact the crown is merely a vertical curve with the tangent at the apex used as an axis. Since the curve is symmetrical, the ordinates at a', b', and c' are the same as at a, b, and c.

The line joining the tops of the curbs will be at a fixed distance d above (or below) the tangent at the center line, hence the ordinates from a line joining the tops of the curbs is equal to the computed ordinates plus (or minus) d. If tees are used, d becomes the distance from the crest of the crown to the line of sight across the tees.

194. Tilted Pavements. On side-hill work it may be desirable to tilt the pavement to save earthwork, reduce curb height, or to improve the appearance. The pavement is also tilted on super-elevated curves.

The most common and the simplest method of tilting is merely to set the curbs, or grade stakes, at elevations differing by the amount of the tilting, and then use the regular templet or other

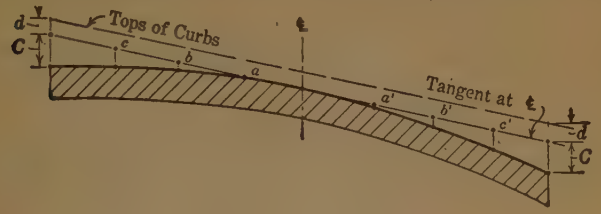

FIG. 55.

usual method of shaping the crown. This condition is illustrated in Fig. 55. The ordinates are the same, and are computed and used in the same way, as in Fig. 54. As the amount of tilting increases, the high point will move toward the upper edge, until, when the tilting is equal to or greater than four times the crown, all the surface water will drain to the low side. This must be considered in providing a sufficient number of properly located storm-water inlets.

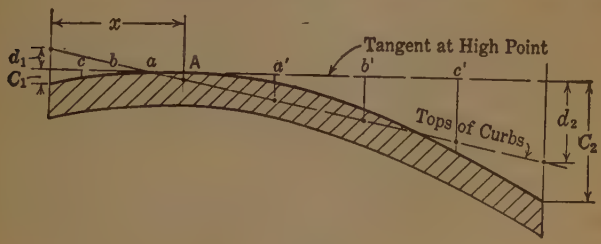

FIG. 56.

Occasionally it may be desired to make the high point of the crown near the upper edge, but provide some slope into the upper gutter. In Fig. 56, A is the high point of the crown at the distance x from the upper edge. C_1 is the amount of crown, measured from a tangent at the high point to the upper edge, and C_2 is the amount of

crown to the lower edge. The offsets a, b, and c are computed as usual, using x and C_1. The offsets a', b', and c' are not the same in this case, but are computed from the distance $W - x$ and the crown C_2. If d_1 is the distance from the tangent at the high point of the curve to the top of the curb or to the top of the grade stake adjacent to the upper edge of the pavement, and d_2 is the corresponding distance on the low side, the slope of the line of tops of curbs with respect to the tangent at the high point can be determined. The ordinates between these two lines at any points can then be found and the ordinates for use with a string or sight tees computed.

195. Subgrade Crowns. If the pavement is of uniform thickness, the subgrade crown has the same shape as the surface crown, and the ordinates to it from the top of curb are merely increased by the thickness of the pavement. If the pavement is not uniform in thickness, separate offsets for the subgrade must be computed to conform to its shape.

Horizontal Curves

The advantages of a parabola for a horizontal curve are: (1) it is not necessary to measure the intersection angle, (2) only the tape and simple sighting methods are needed to lay it out, (3) the calculations are extremely simple in the cases where the previous advantages apply, and (4) both equal and unequal tangents may be used. Theoretically it possesses slight easement properties as compared with the circle, but this is never the reason for using it. Its disadvantages are: (1) it is not easily laid out with the transit and tape by angle and line, (2) location with a tape is not as accurate as with a transit, (3) the entire area between tangents and long chord must be clear enough for field work, and (4) angles of intersection with lines crossing it are not easily measured or computed. It is used, therefore, only as a substitute for circular curves under conditions where its advantages outweigh its disadvantages. The experienced field man, however, finds many cases, especially on street work, where the parabola is exceedingly convenient.

There are two general methods of locating the parabola in the field with the tape, which may be termed the *offset method* and the *middle ordinate method.*

196. Offset Method. (*a*) *Equal tangents.* In Fig. 57, BA and AC are the two tangents, with the $P.I.$ at A, to be connected by the parabola having the tangent distance T. The tangent distance is

measured along both tangents and the $T.C.$ and the $C.T.$ are located. The long chord is then lined in, measured, and its middle point A' located. The line AA' is measured and its middle point a located. Point a is also the middle point of the curve, hence Aa is the external distance E, or the offset to the middle of the curve. The other offsets are computed as previously explained for vertical curves, since they vary as the square of the distances for chosen points on

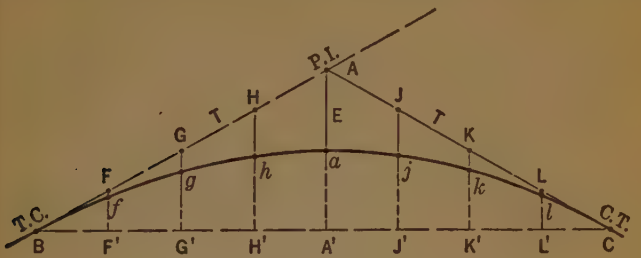

FIG. 57.

the tangents. These offsets are then measured from the tangent on lines parallel to AA'. For greater accuracy in alining the offsets BA' and $A'C$ may be divided into the same number of parts as the tangents, and the offsets directed between corresponding points, as shown in the figure.

(*b*) *Unequal Tangents.* When the tangents are unequal as shown

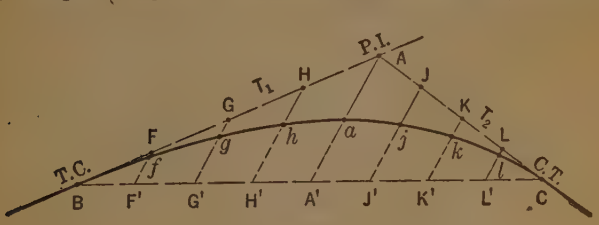

FIG. 58.

in Fig. 58, the parabola forms an excellent substitute for a compound curve. The curve is laid out in exactly the same way as for equal tangents, as explained in the preceding paragraph. In this case the dividing of the long chord to correspond with the tangents is of especial value in alining the offsets.

197. Middle Ordinate Method. In Fig. 59, the $T.C.$ and $C.T.$ and the middle point a of the curve are located as in the preceding method. $A'a$ is then the middle ordinate M. The chords Ba and aC are then lined in and their middle points G and K located. From these points perpendiculars to the chords, equal in length to $\frac{1}{4}Aa = \frac{1}{4}M$, are laid off which locate the points g and k. New chords

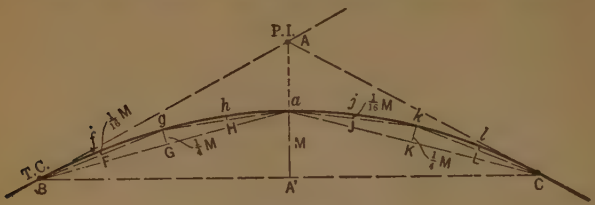

FIG. 59.

between these points may then be lined in and middle ordinates from them, equal to one-fourth the preceding middle ordinate, or $\frac{1}{16}M$, are laid off. This process of subdivision, making each new middle ordinate one-fourth of the one preceding, may be continued until enough points on the curve are located. This method can be used with either equal or unequal tangents.

CHAPTER 8

RAILROAD TURNOUTS, CONNECTIONS, AND CROSSINGS

TURNOUTS

A TURNOUT is a combination of a switch, a frog, the rails necessary to connect the switch and the frog, two guard rails—unless the frog is self-guarded—and a switch-stand for operating the switch. A turnout begins with the switch and ends with the frog. The purpose of a turnout is to permit engines and cars to pass from one track to another.

198. Switches. A switch is a device to deflect at will the wheels of a train from the track upon which they are running. Switches are of three types, the split switch, the tongue switch, and the stub switch.

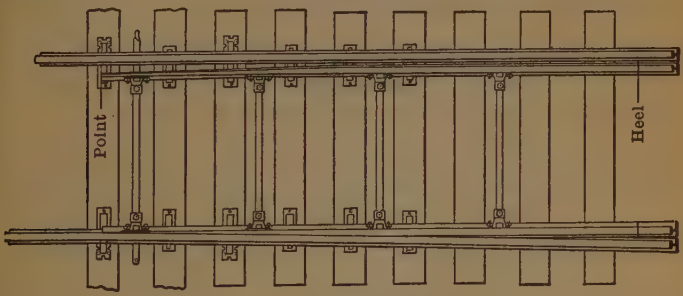

FIG. 60.—Split Switch.

The *split switch* is the standard type used by railroads. It consists of two switch, or point, rails which are connected by tie-rods and operated as a unit, as illustrated in Fig. 60. The switch rails are of full section at one end, and are tapered to a $\frac{1}{4}$-in. point at the other end. The tapered end is called the *point of switch* and the other end is called the *heel of switch*. The switch rails rest upon

metal plates which are spiked to the ties. The heel of each switch rail is connected to its lead rail by means of the usual splice bars, and the switch as a unit pivots about these connections. The point of switch moves through a distance of about 5 in. which is called the *throw*. The movement of the switch rails is controlled by a switch-stand placed outside the track on a long tie, sometimes two, called a *head-block*. The distance between the gage lines of the main track and of the turnout at the heel of the switch rails is called the *heel spread* and varies from $5\frac{1}{2}$ to $6\frac{1}{4}$ in. The angle between the gage lines of the switch rail and of the main track rail is called the *switch angle*, s, and is computed from the equation,

$$\sin s = \frac{\text{heel spread } (h) - \text{width of switch point } (p)}{\text{length of switch rail } (l)}$$

Switch rails vary in length from 10 to 33 ft., depending on the curvature of the turnout. They are cut from rails of standard length so as to have no waste. For 30-ft. rails, 10, 12, 15, 18, 20, and 30-ft. switch points are used; and for 33-ft. rails, 11, 13, $16\frac{1}{2}$, 20, 22, and 33-ft. The increasing use of the 39-ft. rail will doubtless result in the use of the $19\frac{1}{2}$-ft. length of switch rail. The American Railway Engineering Association recommends the use of the 11-ft. switch rail with the No. 6 frog, the 16.5-ft. switch rail with Nos. 7, 8, and 10 frogs, the 22-ft. switch rail with the No. 11 frog, and the 30-ft. switch rail with Nos. 16 and 20 frogs.

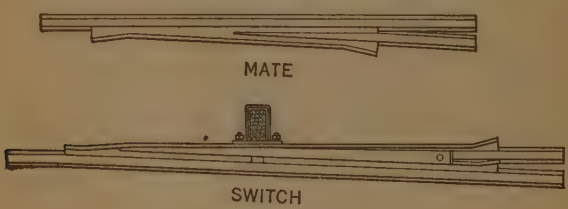

FIG. 61.—Tongue Switch.

The *tongue switch*, Fig. 61, consists of a steel wedge or tongue pivoted at one end and moving in a heavy cast-steel frame so arranged that pavement may be built around it. This is the common form on street railways, where it is generally used singly with a rigid fitting called a *mate* in the opposite rail. Steam roads use them only for turnouts in pavement, and then usually in pairs with

the tongues connected by a tie-rod, where the action is the same as in a split switch.

The *stub switch*, Fig. 62, consists of a pair of ordinary rails fastened together with tie-rods. The rails are spiked to the ties for part of their length, or else are short and are held only by the splice bars at the fixed end, the free end being thrown to match the stub ends of the lead rails. A stub switch differs from a split switch in that the toe is fixed and the heel is moved. This form of switch may be employed to advantage on construction and mine tracks,

FIG. 62.—Stub Switch.

owing to its cheapness and ease of installation. *A stub switch should never be used on a standard road.*

199. Frogs. A frog is a device at the intersection of two running rails to permit the flanges of wheels moving along one rail to cross the other rail. Turnout frogs may be classified as (1) rigid frogs, and (2) spring-rail frogs. Both types of frogs are made with straight gage lines, except those used on street railways. The main point rail is finished with a blunt point about $\frac{1}{2}$ in. wide. The distance P between this *actual* frog point and the *theoretical* point, or intersection of gage lines, equals the width of the blunt point multiplied by the frog number.

Fig. 63 shows the common form of rigid frog and the names of its principal parts. As its name implies, the rigid frog has no movable

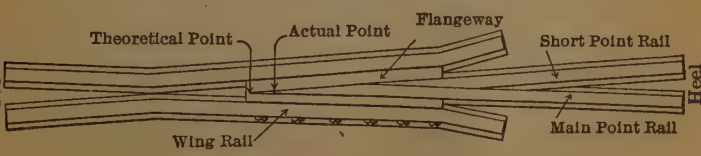

FIG. 63.—Rigid Frog.

parts. The distinguishing feature of the rigid frog is that both flangeways are open and must be jumped by passing wheel treads. This type of frog is used chiefly in turnouts from secondary tracks.

Fig. 64 shows the common form of a spring-rail frog. In this type one wing rail, and sometimes both, is made movable. The

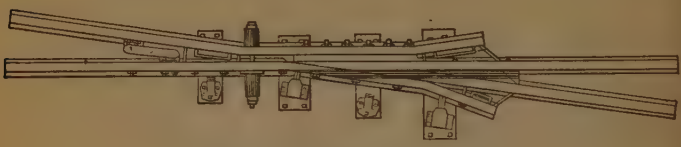

FIG. 64.—Spring-rail Frog.

wing rail is normally held against the point rail by springs. A wheel moving along the main track, therefore, has an open flangeway and a continuous support for the tread, while a wheel moving along the turnout track must spring the wing-rail open to allow the

FIG. 65.—Self-guarded Frog.

flange to pass, and must jump the open flangeway as in the case of the rigid frog. Guard rails are essential to the safe operation of a spring-rail frog. This type of frog is generally used with turnouts from main-line track.

200. Guard Rails. Guard rails are short pieces of rails, with flared ends, spiked, bolted, or clamped against the outside rails of both main and turnout tracks opposite the frog, so as to leave a flangeway of $1\frac{3}{4}$ in. The function of the guard rail is to force the flange of a wheel on the outside rail against the head of that rail and thus to prevent the flange on the inside wheel from striking the point of the frog. Guard rails in place are illustrated in Fig. 66.

Self-guarded frogs are used extensively. This type of frog eliminates the separate guard rails, as it has, as an integral part of its structure, raised guards on the wing rails which bear against the face of the wheel and thus guide the wheel flanges past the frog point. A self-guarded frog is shown in Fig. 65.

201. Frog Angle, *F*. The frog angle is the angle between the gage lines of the frog at their point of intersection. Obviously, the frog angle may have any value, but only values between 2° and 8° are practical for railroad turnouts. Larger values may be used on mine, industrial, and street-railway tracks. A few standard values which meet all practical requirements have been chosen by the railroads to

FIG. 66.—Crossover.

agree with simple values of the frog number, which is a more convenient method of designating frogs. The frog number is defined in the following section. The values of standard frog angles used are given in column 2, Table 8.

202. Frog Number, N. The frog number is the ratio of the axial length to the spread, that is, it is the distance, measured along the bisector of the frog angle, in which the gage lines diverge a unit distance. Expressed as a trigonometric function,

$$N = \tfrac{1}{2} \cot \tfrac{1}{2}F \qquad (67)$$

Values of N from 7 to 16 are commonly used, but for special purposes values from 3 to 27 are occasionally used. The American Railway Engineering Association recommends the use of Nos. 16 and 20 frogs for main-line, high-speed movements, Nos. 10 and 12 for main-line, low-speed movements, and No. 8 for yards and sidings.

203. Location of Turnouts. The first step in staking out a turnout is to establish the position of the frog. The heel or the toe of the frog should come at a regular rail joint in order to avoid short pieces of rail in the track. Knowing the length of the frog to be used, the position of the point of frog, *P.F.*, can be fixed. The next step is to mark the position of the point of switch, *P.S.* This is done by measuring along the main track a distance which is called the *lead*, L. If this brings the point of switch too near a rail joint, the position of the frog must be changed, making the toe of the frog come at a rail joint instead of the heel, or vice versa. Having satisfactorily established the positions of the point of frog and the point of switch, the outside rail of the turnout curve is then located. This is done by measuring offsets from the gage line of the outside main rail at the middle and quarter points of the turnout curve. The inner turnout rail is set by gage from the outer rail.

204. Turnouts from Straight Track. Since the frog and the switch rails are straight, the alinement of a turnout is not a simple curve throughout, but consists of two short pieces of tangent connected by a simple curve.

The solution of the turnout curve is as follows:

In triangle 1, Fig. 67, the hypotenuse is the distance from toe to point of frog, and the angles are known. Solve the triangle for the base m and the altitude n.

In triangle 2, the altitude, $q = g - h - n$, and the angles are known. Solve the triangle for the base j and the hypotenuse c. c is also the long chord of the outside rail of the turnout curve, whose central

angle is $F - s$; hence, the radius of the turnout curve can be computed from the equation,

$$R + \tfrac{1}{2}g = \frac{l\ c}{2 \sin \tfrac{1}{2}(F - s)} \qquad (68)$$

The distance L along the straight main track from the *P.S.* to the *P.F.* is

$$L = l + j + m + P \qquad (69)$$

To compute the offsets from the main track rail to the outside rail of the turnout curve, extend the turnout curve through the switch angle s until its tangent is parallel to the main rail. The distance between these tangents is

$$e = h - (R + \tfrac{1}{2}g) \text{ vers } s \qquad (70)$$

The offset from the gage line of the main track rail to the gage line of the turnout curve at its quarter point is then,

$$y_1 = e + (R + \tfrac{1}{2}g) \text{ vers } [s + \tfrac{1}{4}(F - s)] \quad \text{(Slide Rule)}$$

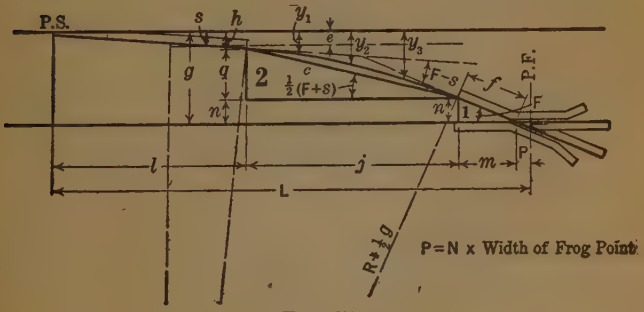

FIG. 67.

For the middle point,

$$y_2 = e + (R + \tfrac{1}{2}g) \text{ vers } [s + \tfrac{1}{2}(F - s)] \quad \text{(Slide Rule)}$$

And for the three-quarters point,

$$y_3 = e + (R + \tfrac{1}{2}g) \text{ vers } [s + \tfrac{3}{4}(F - s)] \quad \text{(Slide Rule)}$$

205. Turnouts from Curved Track. It can be shown—the solution is too involved to be included here—that the lead of a turnout from a curved track is practically the same as the lead of a turnout from a straight track with the same frog number. Also, the degree of the turnout curve is equal to the degree of curve of the turnout from a straight track increased or diminished by the degree of the main track curve, depending on whether the turnout is on the inside or on the outside of the main track curve. A turnout from a curved track, therefore, is staked out with the same dimensions as one with the same frog number from a straight track.

206. Double Turnouts. Occasionally two turnouts to opposite sides of the main track are located at the same point. This involves the use of two regular frogs in the main rails, a third or *crotch frog* at the intersection of the lead rails, and a *three-throw switch.* Such a switch is structurally weak and should never be used in a main track. And further, the conditions under which such arrangements are absolutely necessary are extremely rare. It is better practice to use two separate turnouts, placing one switch ahead of the other and, if necessary, using a sharper turnout to save distance, than to use a three-throw switch. Double turnouts will therefore not be further considered.

207. Practical Leads. Since it is undesirable to use short pieces or odd lengths of rails, it is the universal practice to modify the theoretical leads as computed in Eq. 69 so as to use more convenient lengths of lead rails. The difference between the theoretical lead and the practical lead is never very great, and the turnout is located as previously explained except for this modification in the lead. Changing the lead has the effect of changing the degree of the turnout curve.

Every railroad has its standard turnouts, all dimensions of which are worked out and tabulated. The turnouts are then installed by the trackmen, the only duty required of the engineer being to locate the *P.F.* and occasionally the *P.S.* The engineer, therefore, has little occasion to use the functions of the turnout proper. His problem is to connect the turnout to the required track.

Table 8 gives the practical turnout leads and length of switchrail recommended by the American Railway Engineering Association; also, the rectangular coordinates to the quarter points on the gage side of the outside turnout rail, referred to the point of switch as origin.

TABLE 8

Practical Turnout Leads as Recommended by the American Railway Engineering Association

Frog Number	Frog Angle	Length of Switch Rail*	Radius of Center Line	Degree of Lead Curve	Lead-Distance from Actual P.S. to Actual P.F.	Rectangular Coordinates to the Quarter and Center Points on Gage Side of Curved Rail, Referred to Point of Switch as Origin					
						x	x_1	x_2	y	y_1	y_2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4	14° 15′ 00″	11.0 ft.	109.15 ft.	54° 31′ 38″	36.96 ft.	16.56 ft.	22.12 ft.	27.69 ft.	0.92 ft.	1.59 ft.	2.55 ft.
5	11 25 16	11.0	180.49	32 09 56	42.83	18.00	25.00	32.00	0.97	1.70	2.78
6	9 31 38	11.0	269.54	21 22 52	48.29	19.19	27.37	35.56	1.01	1.75	2.74
7	8 10 16	16.5	342.85	16 46 16	61.42	26.19	35.88	45.56	0.95	1.64	2.61
8	7 09 10	16.5	468.64	12 14 58	67.08	27.60	38.71	49.81	0.99	1.72	2.80
9	6 21 35	16.5	598.33	9 35 14	72.08	28.85	41.21	53.56	1.02	1.78	2.79
10	5 43 29	16.5	769.40	7 27 07	77.42	29.98	43.46	56.94	1.04	1.81	2.81
11	5 12 18	22.0	914.37	6 19 09	91.85	37.71	53.42	69.12	1.02	1.78	2.81
12	4 46 19	22.0	1087.30	5 16 17	96.67	38.71	55.42	72.12	1.03	1.80	2.82
14	4 05 27	22.0	1560.07	3 40 24	107.06	41.10	60.21	79.31	1.07	1.86	2.87
15	3 49 06	30.0	1721.17	3 19 46	126.38	51.75	73.50	95.25	1.01	1.77	2.81
16	3 34 47	30.0	2005.33	2 51 27	131.33	53.00	76.00	99.00	1.03	1.82	2.86
18	3 10 56	30.0	2561.77	2 14 12	140.96	55.00	80.00	105.00	1.06	1.84	2.87
20	2 51 51	30.0	3290.64	1 44 28	151.96	57.75	85.50	113.25	1.09	1.90	2.93

* Thickness of Point = $\frac{1}{4}$ in. Heel Spread = $6\frac{1}{2}$ in.

CONNECTIONS

Connections are the tracks used to connect a turnout from a main track with (1) a diverging track, (2) a parallel siding, or (3) a turnout on another track. These three classifications are treated separately in the following sections.

Diverging Tracks

208. Case 1. From Straight Track.

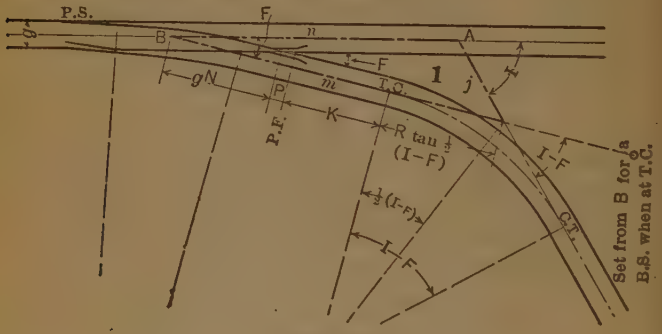

FIG. 68.

The solution of this problem requires that I, N, R, K, and P be known. I is the intersection angle between the diverging track and the straight main track. It is measured in the field after the point of intersection, A, has been determined. The angle should be measured to the nearest one-half minute.

N is the frog number chosen for the turnout. The value of N is determined by the standard practice of the individual railroad.

R is the radius chosen for the connecting curve. The factors which affect the choice of R and D are (1) the degree of the turnout curve, (2) the topography, and (3) special space limitations. Of these factors, the first is generally the one which determines the value of R. In general, an even value of D is chosen which is slightly less than the degree of the turnout curve. The degrees of the turnout curves for the various frog numbers are given in column 5, Table 8.

K is the frog tangent, or the length of tangent between the actual point of frog and the $T.C.$ of the connecting curve. The minimum

value of K is the distance from the actual point to the heel of the frog. Generally, values of K from 20 to 40 ft. are chosen, the larger values being adapted to the sharper curves.

P is the distance from the actual to the theoretical point of frog. It is equal to the width of the blunt point of the frog multiplied by the frog number. This width is usually $\frac{1}{2}$ in.

The solution of this problem requires the computation of the data necessary to locate (1) the *P.F.*, (2) the *T.C.*, and (3) the *C.T.*

In triangle 1, Fig. 68, all the angles and the side, $m = gN + P + K + R \tan \frac{1}{2}(I - F)$, are known, whence the sides n and j can be computed.

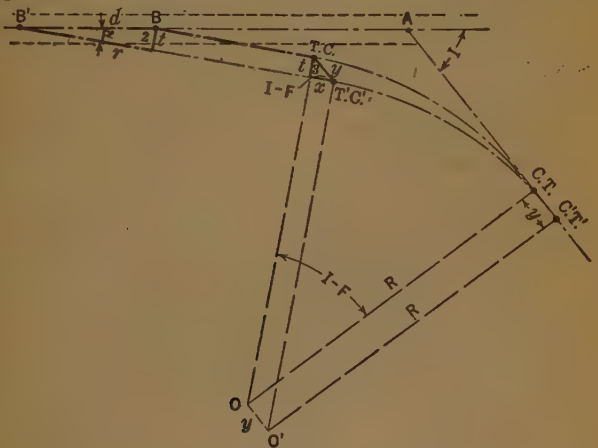

FIG. 69.

The point B and the *C.T.* of the connecting curve are located from point A by the distances n and $j + R \tan \frac{1}{2}(I - F)$, respectively. The *P.F.* is located from B by the coordinates $gN + P$ and $\frac{1}{2}g$. The *T.C.* of the connecting curve is located from B by the angle F and the distance $gN + P + K$. For standard gage, $g = 4.708$ ft.

The frog, as located in the preceding paragraph, will rarely come at an existing rail joint in the main track, and it must be moved forward or backward until it does. This involves a change in the location of point B and of the *C.T.*, and in the length of the frog tangent K. The solution is as follows:

In Fig. 69, the points A, B, *T.C.*, and *C.T.*, have the same positions as shown in Fig. 68. In order to make the frog come at a rail

joint in the main track, it is necessary to move it backward the distance d. The point B is moved through the same distance and now occupies the position B'. The $T.C.$ and the $C.T.$ take the positions $T'.C'.$ and $C'.T'.$, respectively.

In triangle 2, Fig. 69, all the angles and the hypotenuse d are known; solve the triangle for r and t. In triangle 3, all the angles and the side t are known; solve for x and y. The change in the length of the frog tangent K is $r + x$. The $T'.C'.$ can now be located from B'. The $C'.T'.$ is located from the $C.T.$ by the distance y measured along the tangent.

In case the frog is moved *forward* the distance d, the frog tangent is decreased by an amount equal to $r + x$, and the $C.T.$ is moved backward the distance y instead of forward.

209. Case 2. Turnout from the Inside of a Curved Main Track.

(*a*) *Points A and H on circular curve.*

The data and the requirements of this problem are the same as in the preceding problem, except that the main track is on a $D°$ curve.

In triangle 1, Fig. 70, the hypotenuse and the angles are known, and the other two sides are computed as shown in the figure.

In triangle 2, the base and the altitude can be found by arithmetic. Solve the triangle for the hypotenuse and the angle a.

In triangle 3, the hypotenuse and the angles are known, and the other two sides are computed as shown in the figure.

In triangle 4, the hypotenuse OO_1 is known from triangle 2, and the side O_1J can be found by arithmetic. Solve the triangle for the side OJ and the angle b.

Then

$$x = a - b + (90 - I) - (90 - F)$$

$$= a - b - I + F \qquad (71)$$

And

$$y = a - b \qquad (72)$$

The point H opposite the theoretical point of frog is located from point A by measuring along the center line the distance AH (in feet) $= \frac{x}{D}100$. The $C.T.$ is located from point A by the distance $AE = AG - EG = AG - JO$. The $T.C.$ is best located as follows: Set-up over H; back-sight on A; turn off $\frac{1}{2}x$ to get on tangent; locate M for a temporary back-sight; plunge telescope and set point B a dis-

tance gN from H; set-up at B and back-sight on M; turn off the frog angle and locate the $T.C.$ on this line a distance $gN + P + K$ from B; set point Q for a temporary back-sight; set-up at the $T.C.$, back-sight on Q, and run in the connecting curve to point E.

It is to be especially noted that, when $R \cos I$ is less than R_1, the line OJ, Fig. 70, falls on the other side of the line OO_1, and the *sign of the angle b in Eqs.* 71 *and* 72 *becomes plus.*

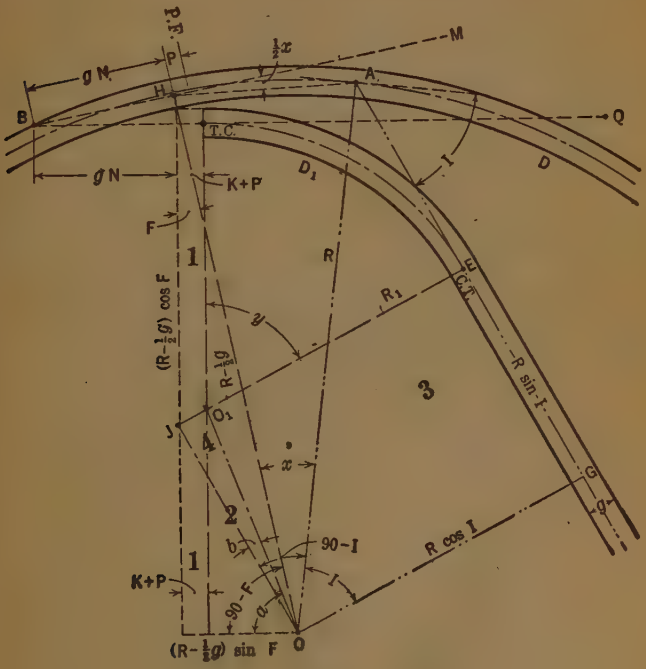

FIG. 70.

The frog as located opposite H in Fig. 70 will rarely come at an existing rail joint in the main track, and it must be moved forward or backward until it does. This involves a change in the location of the point B and of the $C.T.$ and a change in the length of the frog tangent K. The solution is as follows: In Fig. 71, the points A, H, B, $T.C.$, and $C.T.$ have the same positions as in Fig. 70. It

is necessary to move the *P.F.* forward, as shown, a measured distance d to a point opposite H'. The angle z through which the frog is moved is found from the equation,

$$z \text{ (in minutes)} = 0.6dD \qquad (73)$$

The moving of the *P.F.* makes the following changes in the track layout:

1. The direction of the frog tangent is changed an amount equal to the angle z.
2. The length and the position of the frog tangent are changed.
3. The central angle y of the connecting curve is decreased by the angle z, thereby decreasing the length of the connecting curve.
4. The entire connecting curve is moved parallel to the tangent *AE*.

It is required to find the length of the new frog tangent and the distance the *C.T.* is shifted.

In triangle 1, Fig. 71, the angles and the hypotenuse are known.

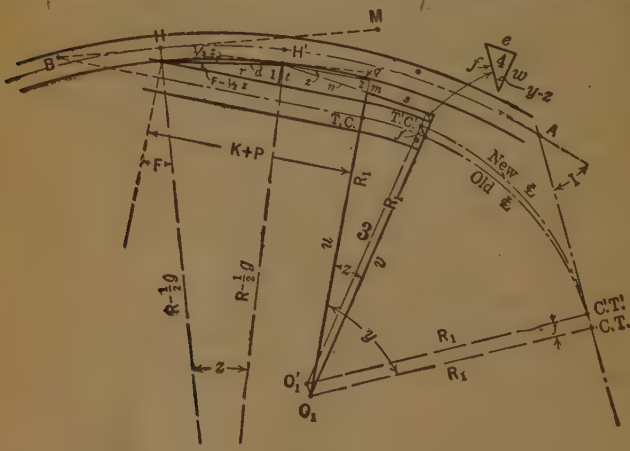

FIG. 71.

Solve the triangle for the base r and the altitude t. This triangle and those that follow may be solved on the slide rule.

In triangle 2, the base, $q = K + P - r$, and the angles are known. Solve the triangle for altitude m and the hypotenuse n.

In triangle 3, the hypotenuse, $u = R_1 + \frac{1}{2}g + t - m$, and the angles are known. Solve the triangle for the base v and the altitude s.

In triangle 4, the base, $w = v - R_1 - \frac{1}{2}g$, and the angles are known. Solve the triangle for the altitude e and the hypotenuse f. Then

$$\text{The new } K + P = n + s - e$$

And

$$\text{The change in the } C.T. = f$$

If the frog is moved backward, another figure is required for an accurate solution. Although this figure will have a somewhat different appearance, the four triangles necessary for the solution are constructed in the same manner as those shown in Fig. 71. For example, triangle 1 is constructed with a hypotenuse equal to d and with a base and altitude formed by dropping a perpendicular from the old $P.F.$ to a line through the new frog point parallel to the old frog tangent. The angle at H', however, now becomes $F + \frac{1}{2}z$ instead of $F - \frac{1}{2}z$. The student should construct this figure as an exercise. The change in the length of the frog tangent and the change in the position of the $C.T.$, however, are approximately the same as when the frog is moved forward, but with opposite sign. Compare the answers to problems 5 and 6, page 216.

The solution of this problem, as given in Figs. 70 and 71, is based on the assumption that the alinement of the main track is perfect. In many instances, however, the main line curve will be found to be more or less out of correct alinement. In these cases, the problem is best solved as follows:

1. Solve the triangles in Fig. 70 approximately only, using a slide rule or three-place tables. Measure from A the computed distance AH, and locate point H.

2. Locate the point of frog so that the heel or the toe of the frog will come at the rail joint nearest to the point H.

3. Set-up the transit in the center of the track opposite the selected point of frog, and establish the tangent to the curve at this point by sighting at a point in the center of the track 100 ft. ahead of the instrument, plunging the telescope, measuring the deflection angle to a point in the center of the track 100 ft. back of the instrument, and bisecting this deflection angle.

4. Measure back along the tangent thus established the dis-

tance gN, and locate a point which corresponds to point B in Fig. 70.

5. Set-up at point B, turn off the frog angle from the tangent BH, and produce this line to an intersection with the tangent of the diverging track.

6. Set-up at this point of intersection and measure the intersection angle, which is the central angle of the connecting curve.

7. From the $P.I.$ established in step 5, lay off the tangent distances for an R_1 curve, and locate the $T.C.$ and the $C.T.$ of the connecting curve.

(*b*) *Point A on curve, but point H on tangent.*

When the distance from the $T.C.$ of the main line curve to the point A is so short that it is evident from inspection that the frog point H will come on the tangent, the procedure is somewhat different. In this case, point A is taken at the intersection of the tangent of the diverging track and the tangent of the main track prolonged. The problem then becomes the same as that illustrated in Fig. 68.

(*c*) *Point A on tangent and point H on curve.*

In Fig. 72, A is the intersection of the diverging track tangent and the main track tangent. A' is the $C.T.$ of the main track curve. The distance AA' is measured in the field.

Triangles 1, 2, and 3 are exactly the same as in Fig. 70 and are solved in the same manner.

In triangle 4, Fig. 72, the hypotenuse AA' and the angles are known. Solve the triangle for the base e and the altitude f.

In triangle 5, which corresponds to triangle 4 in Fig. 70, the hypotenuse OO_1 is known from triangle 2 and the side O_1J is equal to $R_1 - OG' - EE'$, all of which are known. If O_1J comes out a minus quantity, then J falls on the left of O_1, as shown in Fig. 70. Solve the triangle for the side OJ and the angle b.

Then

$$x = a + b^\circ - I + F \tag{74}$$

$$y = a + b^\circ \tag{75}$$

$$AH = AA' + A'H = AA' + \frac{x}{D}100$$

$$AE = A'G' - EG - A'L$$

If the *C.T.* of the main track curve is not monumented, it will be impossible to locate it by inspection closer than about 10 ft. In this case, the solution given on page 181 should be used.

(*d*) *Same as* (*c*), *except that the main track curve is spiraled.*

In Fig. 73, triangles 1, 2, and 3 are the same as in Fig. 72.

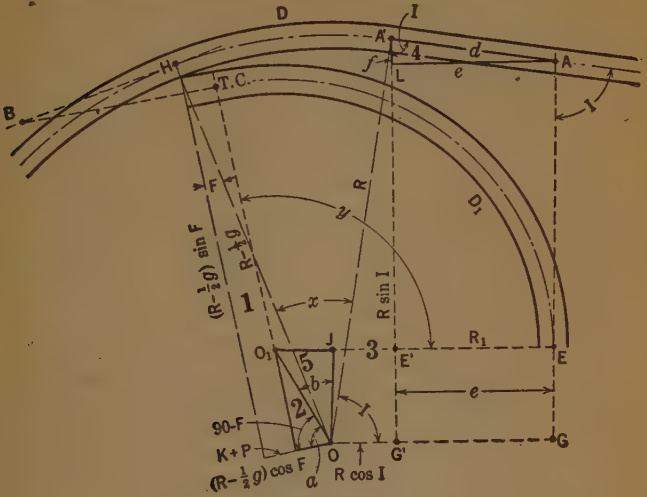

Fig. 72.

In triangle 4, the angles and the altitude, o = spiral offset, are known. Solve the triangle for the base m and the hypotenuse n.

In triangle 5, the angles and the hypotenuse, $A_1A' = t + d + m$, are known. Solve the triangle for the base e and the altitude f.

In triangle 6, the hypotenuse OO_1 is known from triangle 2, and the side $O_1J = R_1 - R\cos I - e$. Solve the triangle for the side OJ and the angle b.

The points H and E are established as in the preceding problem.

(*e*) *Point A falls on spiral.*

When the point A falls on the spiral, the problem is slightly different with respect to the solution of triangles 3, 4, and 5. After the point A is established in the center of the main track, the distance to the *S.T.*, s, and the intersection angle at A must be meas-

* Angle b is minus when OG' is greater than O_1E'.

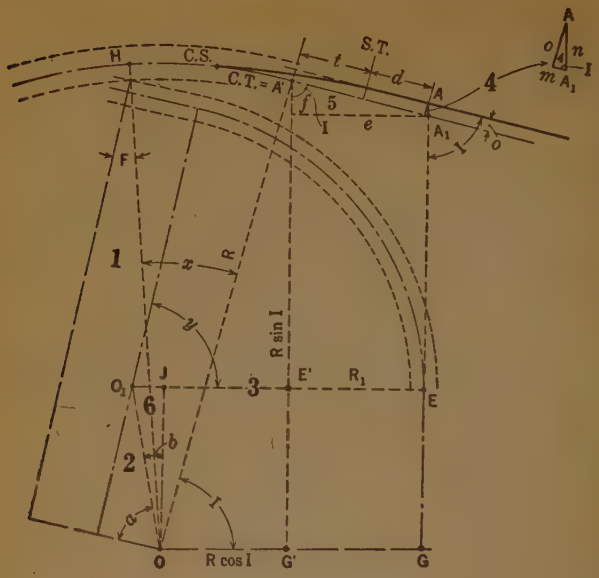

FIG. 73.

ured. To get on tangent at point A, the transit is sighted at the $S.T.$ and the angle $2a = 2(10ks^2)$ turned off.

Triangle 4, Fig. 74, is constructed by dropping a perpendicular from A to a tangent through the $C.T.$ The distance between this tangent and the main track tangent is o, the spiral offset. The altitude of triangle 4 is equal to $o - 0.291ks^3$. Since the angle at A_1 is $I - \Delta$, which is known, the triangle can be solved for the base n and the hypotenuse m.

In triangle 3, the angle at O now becomes $I - \Delta$ instead of I.

In triangle 5, the hypotenuse is equal to $t - s + n$, all of which are known, and the angle at A' is $I - \Delta$. Solve the triangle for the base e and the altitude f.

Triangle 6 (not shown in Fig. 74) is the same as triangle 6 in Fig. 73. The problem from this stage is the same as the preceding one.

The point A in Fig. 74 is between the $C.T.$ and the $S.T.$ The point A, however, is just as likely to come between the $C.T.$ and the $C.S.$

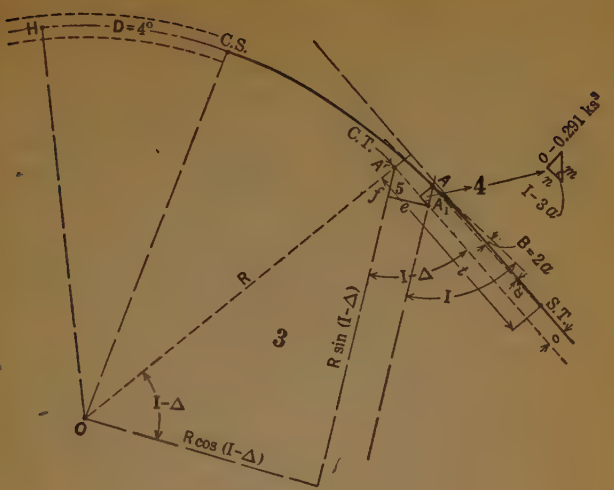

FIG. 74.

In this case, triangles 4 and 5 shift to the other side of the *C.T.*, and the hypotenuse of triangle $5 = s - t - n$. It is suggested that the student draw the figure for this problem.

210. Case 3. Turnout from the Outside of a Curved Main Track.

(*a*) *Points A and H on circular curve.*

In triangle 1, Fig. 75, the angle F and the hypotenuse $R + \frac{1}{2}g$ are known. Solve the triangle for the other two sides, as shown in the figure.

In triangle 2, the base is equal to $(R + \frac{1}{2}g) \sin F + K + P$, and the altitude is equal to $(R + \frac{1}{2}g) \cos F + \frac{1}{2}g + R_1$. Solve the triangle for the hypotenuse OO_1 and the angle a.

In triangle 3, the hypotenuse is R and the angle at O is I. Solve the triangle for the other two sides, as shown in the figure.

In triangle 4, the hypotenuse OO_1 is known from triangle 2, and the base is $R \cos I + R_1$. Solve for the altitude and the angle b.

Then

$$x = (90° - F) - (90° - I) - (a - b)$$

$$= b + I - a - F \qquad (76)$$

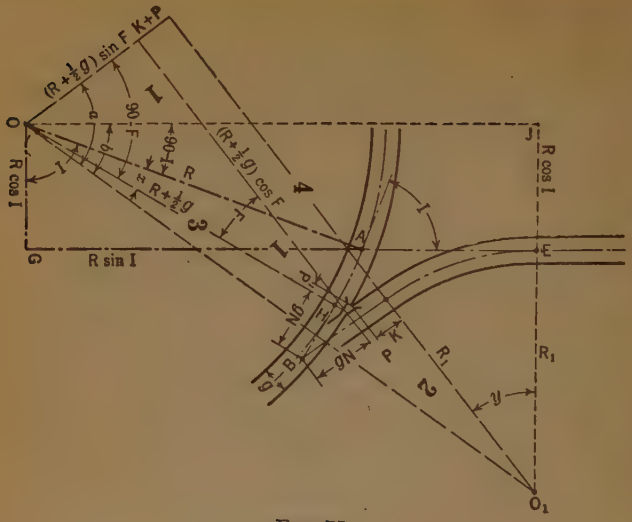

Fig. 75.

And

$$y = a - b \tag{77}$$

$$AH = \frac{x}{D}\,100 \text{ (in feet)}$$

$$AE = OJ - R \sin I$$

Fig. 75 and the solution outlined above are correct up to a value of $I = 90°$. As I approaches 90°, J approaches E and G approaches O. When $I = 90°$, J and E coincide and also G and O. That is, triangle 3 becomes a line. When I is greater than 90°, J falls between E and O_1, and G falls on the other side of O. When I is greater than 90°, however, the connecting track should be located on the other side of the diverging tangent, thus using a value of I less than 90°.

The field work in this problem is the same as explained for the corresponding problem under Case 2.

The point of frog as located opposite H in Fig. 75 will rarely be

such that the heel or the toe of the frog will come at an existing rail joint, and the frog must be moved until it does. This involves a change in the location of the point B and of the $C.T.$ and a change in the length of the frog tangent. Fig. 76 shows the solution of this problem when the frog is moved forward through the measured distance d.

In triangle 1, d is the hypotenuse and the angle at H is $F + \frac{1}{2}z$.

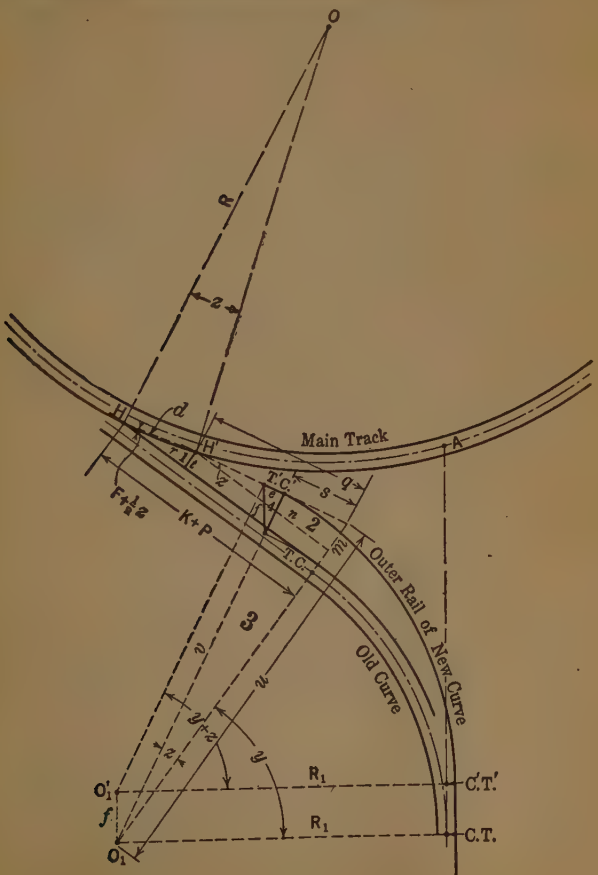

FIG. 76.

The angle z is equal to $0.6dD$. Solve the triangle for the base r and the altitude t.

In triangle 2, the angle z and the base, $n = K + P - r$ are known. Solve the triangle for the altitude m and the hypotenuse q.

In triangle 3, the angle z is known and the hypotenuse u is equal to $R_1 + \frac{1}{2}g + t + m$. Solve the triangle for v and s.

In triangle 4, one side equals $v - R_1 - \frac{1}{2}g$ and one angle equals $y + z$. Solve the triangle for the hypotenuse f and the other side e. Then

$$\text{New} \quad K + P = q - s - e$$

And

$$\text{Change in } C.T. = f$$

The solution of this problem as given in Figs. 75 and 76 is based on the assumption that the alinement of the main track is perfect. In most instances this curve will be found to be more or less out of correct alinement. Should the error of alinement at H be more than about 0.3 ft., the solution outlined on page 181 should be used.

(*b*) *Point A on curve, but point H on tangent.*

When the distance from the $T.C.$ of the main line curve to the point A is so short that it is evident by inspection that the frog point H will come on the tangent, the procedure is somewhat different. In this case, point A is taken at the intersection of the tangent of the diverging track and the tangent of the main track prolonged. The problem then becomes the same as that illustrated in Fig. 68.

(*c*) *Point A on tangent and point H on curve.*

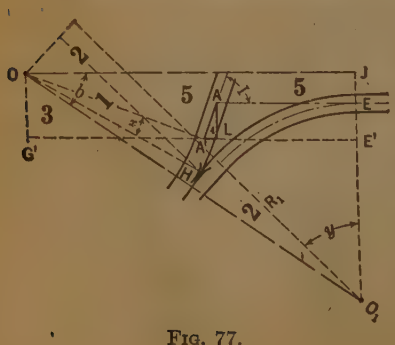

Fig. 77.

In Fig. 77, A is the intersection of the diverging track and the main track. A' is the $C.T.$ of the main track curve. The distance AA' is measured in the field.

Triangles 1, 2, and 3 are exactly the same as in Fig. 75 and are solved in the same manner.

In triangle 4, Fig. 77 the hypotenuse AA' and

the angles are known. Solve the triangle for the base AL and the altitude $A'L$.

In triangle 5, which corresponds to triangle 4 in Fig. 75, the hypotenuse is known from triangle 2, and the side $O_1J = R_1 - EE' + OG'$, all of which are known. If I is greater than 90°, J falls on the radius between O_1 and E', and $O_1J = R_1 - EE' - OG'$. Solve triangle 5 for the side OJ and the angle b.

Angles x and y and distances AH and AE are computed as in Case 3 (a).

If the $C.T.$ of the main track curve is not monumented, it will be impossible to locate it by inspection closer than about 10 ft. In this case, the solution on page 181 should be used.

(*d*) *Same as* (*c*) *except that the main track curve is spiraled:*

No new figure is necessary for the solution of this case. Triangles 1, 2, and 3 are exactly the same as in Figs. 75 and 77. Triangle 4 is the same as in Fig. 73. In triangle 5, Fig. 73, the hypotenuse now becomes $t + d - m$, instead of $t + d + m$, and the sides e and f are computed. These two sides correspond to AL and $A'L$ in triangle 5, Fig. 77.

Then

$$x = b + I - A - F \tag{78}$$

And

$$y = a - b \tag{79}$$

$$A'H = \frac{x}{D} 100 \text{ (in feet)}$$

$$AH = A'H + t + d$$

$$AE = OJ - R \sin I - f - n$$

Parallel Tracks. Sidings

211. Case 1. Straight Tracks.

The best way to connect a turnout to a parallel siding is to make the frog tangent of such a length that it can be connected to the siding by a simple curve of about the same radius as that of the lead rails. A connecting track of this kind is most easily located by coordinates from the actual point of frog.

In Fig. 78, x_1, x_2, and x_3 are distances to points on the main track opposite the *T.C.*, the middle point of the curve, and the *C.T.*, respectively, and y_1, y_2, and y_3 are the corresponding offsets from the center line of the main track. From the figure,

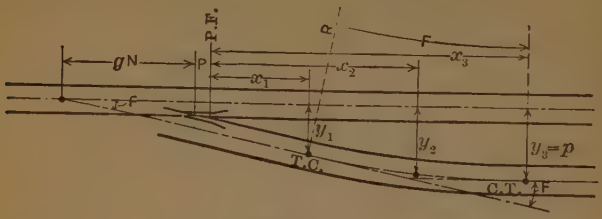

FIG. 78.

$$\left.\begin{aligned} y_1 &= p - R \operatorname{vers} F \quad \text{and} \quad x_1 = y_1 \cot F - gN - P \\ y_2 &= p - R \operatorname{vers} \tfrac{1}{2}F \quad \text{“} \quad x_2 = x_1 + R(\sin F - \sin \tfrac{1}{2}F) \\ y_3 &= p \quad \text{“} \quad x_3 = x_1 + R \sin F \end{aligned}\right\} \quad (80)$$

Table 9 gives the values of x and y for track centers from 13 to 18 ft. and for frog numbers from 7 to 16.

Sometimes the connecting curve is commenced at the heel of the frog or a fixed distance beyond it. In this case x_1 and y_1 are known, and the radius of the connecting curve must be computed. The other dimensions are then computed as before. Although this method flattens the connecting curve, it increases the distance from the *P.F.* to the *T.C.* There is nothing gained by making the connecting curve flatter than that of the turnout itself, and also the greater length of tangent is advantageous.

Sometimes the connection is made in the form of a reversed curve with radii about the same as the radius of the turnout curve under the mistaken idea that this saves distance between the *P.F.* and the *T.C.* The actual difference in x_3 for such a connection and for one of the form first given for a No. 10 turnout is only about 1.1 ft., which is an inappreciable amount when the relative riding qualities of the two layouts for heavy road engines are considered.

212. Case 2. Curved Tracks.

A turnout and a connection to a parallel track on a curve may be staked out by the coordinates given in Table 9. Such a layout will have the following characteristics:

TABLE 9

COORDINATES FOR LOCATING PARALLEL SIDINGS WITH TAPE

Frog Number, N	Track Centers, $p = y_3$	x_1	x_2	x_3	y_1	y_2	Connecting Curve, D	Frog Angle, F
7	13.0	30.28	57.42	84.70	9.12	12.03	15° 00′	8° 10′ 16″
	13.5	33.76	60.90	88.18	9.62	12.53		
	14.0	37.24	64.38	91.66	10.12	13.03		
	15.0	44.21	71.35	98.63	11.12	14.03		
	16.0	51.18	78.32	105.60	12.12	15.03		
	17.0	58.15	85.29	112.57	13.12	16.03		
	18.0	65.12	92.26	119.54	14.12	17.03		
8	13.0	33.31	65.71	98.24	8.94	11.98	11° 00′	7° 09′ 10″
	13.5	37.29	69.69	102.22	9.44	12.48		
	14.0	41.28	73.68	106.21	9.94	12.98		
	15.0	49.25	81.65	114.18	10.94	13.98		
	16.0	57.22	89.62	122.15	11.94	14.98		
	17.0	65.19	97.59	130.12	12.94	15.98		
	18.0	73.16	105.56	138.09	13.94	16.98		
9	13.0	38.55	73.83	109.22	9.07	12.02	9° 00′	6° 21′ 35″
	13.5	43.03	78.31	113.70	9.57	12.52		
	14.0	47.51	82.79	118.18	10.07	13.02		
	15.0	56.47	91.75	127.14	11.07	14.02		
	16.0	65.43	100.71	136.10	12.07	15.02		
	17.0	74.39	109.67	145.06	13.07	16.02		
	18.0	83.35	118.63	154.02	14.07	17.02		
10	13.0	41.19	82.04	123.00	8.90	11.98	7° 00′	5° 43′ 29″
	13.5	46.17	87.02	127.98	9.40	12.48		
	14.0	51.15	92.00	132.96	9.90	12.98		
	15.0	61.11	101.96	142.92	10.90	13.98		
	16.0	71.07	111.92	152.88	11.90	14.98		
	17.0	81.03	121.88	162.84	12.90	15.98		
	18.0	91.00	131.84	172.80	13.90	16.98		
11	13.0	47.16	90.46	133.83	9.05	12.01	6° 00′	5° 12′ 18″
	13.5	52.65	95.95	139.32	9.55	12.51		
	14.0	58.14	101.44	144.81	10.05	13.01		
	15.0	69.12	112.42	155.79	11.05	14.01		
	16.0	80.09	123.39	166.76	12.05	15.01		
	17.0	91.07	134.37	177.74	13.05	16.01		
	18.0	102.05	145.35	188.72	14.05	17.01		

TABLE 9.—(*Continued*)

COORDINATES FOR LOCATING PARALLEL SIDINGS WITH TAPE

Frog Number, N	Track Centers, $p = y_3$	x_1	x_2	x_3	y_1	y_2	Connecting Curve, D	Frog Angle, F
12	13.0	51.38	98.96	146.63	9.03	12.00	5° 00′	4° 46′ 19″
	13.5	57.37	104.95	152.62	9.53	12.50		
	14.0	63.37	110.95	158.62	10.03	13.00		
	15.0	75.36	122.94	170.61	11.03	14.00		
	16.0	87.35	134.93	182.60	12.03	15.00		
	17.0	99.34	146.92	194.59	13.03	16.00		
	18.0	111.33	158.91	206.58	14.03	17.00		
14	13.0	57.78	116.15	174.59	8.83	11.95	3° 30′	4° 05′ 27″
	13.5	64.77	122.14	181.58	9.33	12.45		
	14.0	71.77	130.14	188.58	9.83	12.95		
	15.0	85.75	144.12	202.56	10.83	13.95		
	16.0	99.73	158.10	216.54	11.83	14.95		
	17.0	113.71	172.08	230.52	12.83	15.95		
	18.0	127.69	186.06	244.50	13.83	16.95		
16	13.0	60.16	131.68	203.28	8.52	11.90	2° 30′	3° 34′ 47″
	13.5	68.16	139.67	211.27	9.02	12.40		
	14.0	76.14	147.66	219.26	9.52	12.90		
	15.0	92.13	163.64	235.24	10.52	13.90		
	16.0	108.11	179.62	251.22	11.52	14.90		
	17.0	124.10	195.61	267.21	12.52	15.90		
	18.0	140.08	211.59	283.19	13.52	16.90		

1. The degree of curve of the turnout will be increased or diminished by the degree of the main curve, depending on whether the siding is inside or outside of the main track curve.

2. The degree of the connecting curve will be diminished or increased by the degree of the main curve (approximately), depending on whether the siding is inside or outside of the main track curve.

3. The track between the frog point and the beginning of the connecting curve will become a curve of the same degree (approximately) as the main curve.

On curves of about 1° this method is both satisfactory and convenient, but on sharper curves, or in case it is desired to maintain a frog tangent, it will be necessary to compute the connection for the particular case. Two general cases arise:

(*a*) *Sidings on the outside of the main curve.*

In Fig. 79

$$OA = (R + \tfrac{1}{2}g)\cos F$$

$$AB = (R + \tfrac{1}{2}g)\sin F$$

$$AC = AB + P + K = O_1H$$

$$OO_1 = R + p - R_1$$

$$OH = OA - (R_1 - \tfrac{1}{2}g)$$

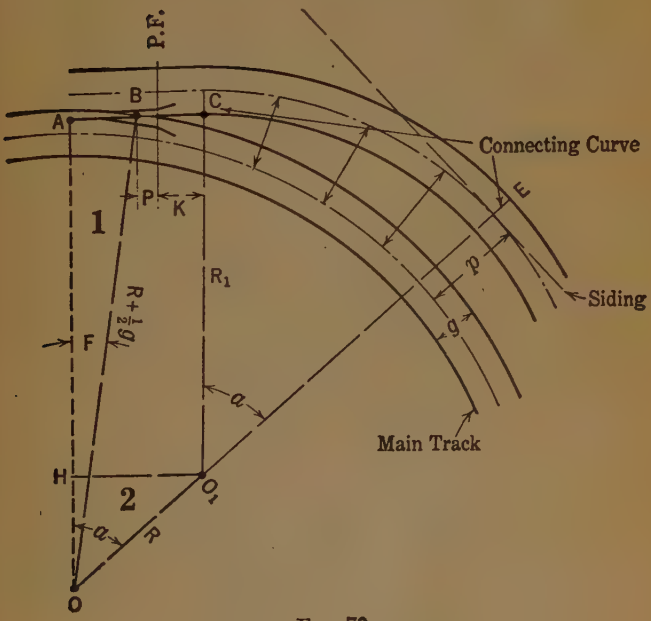

FIG. 79.

In triangle 2, $(OO_1)^2 = (O_1H)^2 + (OH)^2$

$$(R + p - R_1)^2 = (AC)^2 + (OA + \tfrac{1}{2}g - R_1)^2$$

expanding and reducing,

$$R_1 = \frac{(AC)^2}{2(OA + \frac{1}{2}g - R - p)} + \frac{OA + \frac{1}{2}g + R + p}{2} \qquad (81)$$

Then

$$\sin a = \frac{AC}{OO_1}$$

Since the central angle a and the radius R_1 are now known, the length of the connecting curve can be computed.

The curve may be staked out either by deflection angles from either end (orient how?) or by offsets from the main track. The latter method is the more rapid and convenient.

Since the connecting curve and the siding curve have a common tangent at E and since the angles are small, the offset between them at any point is equal to the difference of their tangent offsets. Then the offset from the center line of the main track to the center line of the connecting curve at any point is

$$p = \frac{7}{8} n^2(D_1 - D_s),$$

in which D_1 and D_s are the degrees of curve of the connecting track and the siding, respectively, and n is the distance in stations along the connecting curve from the point where it joins the siding. These offsets should be measured radially to the main curve and the distances should be measured along the connecting curve.

(*b*) *Sidings on the inside of the main curve.*

In Fig. 80a,

$$OA = (R - \tfrac{1}{2}g)\cos F$$
$$AB = (R - \tfrac{1}{2}g)\sin F$$
$$AC = AB - K - P = OH$$
$$OO_1 = R_1 + p - R$$
$$O_1H = R_1 + \tfrac{1}{2}g - OA$$

In triangle 2,

$$(OO_1)^2 = (OH)^2 + (O_1H)^2$$
$$(R_1 + p - R)^2 = (AC)^2 + (R_1 + \tfrac{1}{2}g - OA)^2$$

Expanding and reducing,

$$R_1 = \frac{(AC)^2}{2(OA - \tfrac{1}{2}g - R + p)} + \frac{OA - \tfrac{1}{2}g + R - p}{2} \qquad (82)$$

Then

$$\sin a = \frac{AC}{OO_1}$$

If Eq. 82 gives a positive value of R_1, the layout is as shown in Fig. 80a.

If a negative value of R_1 is obtained, the layout is as shown in Fig. 80b and the value of R_1 is numerically correct.

If $(OA - \frac{1}{2}g) = R - p$, Eq. 82 gives R_1 = infinity, and the frog tangent produced is tangent to the siding as shown in Fig. 80c.

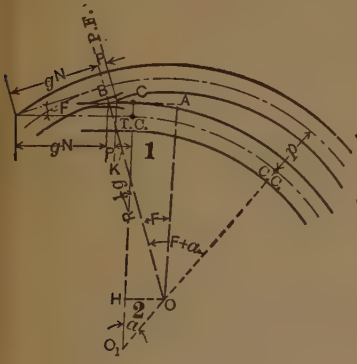

Fig. 80a.

Fig. 80b.

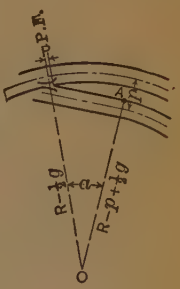

Fig. 80c.

These connections can be staked out by offsets as explained under case (*a*). It is to be noted, however, that if the layout is as shown in Fig. 80b, the *sum* of the tangent offsets must be used.

Parallel Tracks. Crossovers

Since a crossover is equivalent to an entry to a siding, and therefore both turnouts will be run over at practically the same speed, there is no valid reason for making one turnout flatter than the other, even when the crossover is between a main track and a siding. A crossover should therefore always have frogs of the same number at both ends.

213. Case 1. Straight Tracks.

(*a*) *Straight track between frogs of the same number.*

From Fig. 81,

$$AC = p \cot F$$

Then

$$x = AC - 2gN - 2P \qquad (83)$$

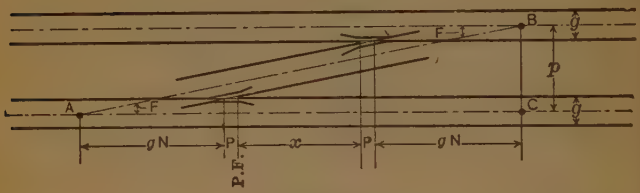

FIG. 81.

To locate the crossover one *P.F.* is set so that the heel or toe will be at a regular rail joint. The other *P.F.* is fixed by the distance x. These are the only stakes ordinarily required.

(*b*) *Simple curve between frogs of different number.*

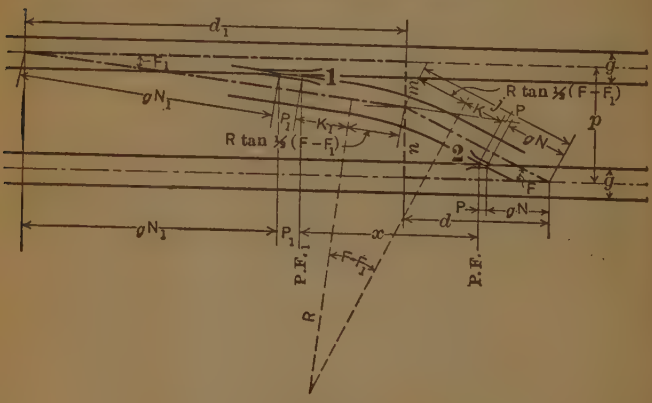

FIG. 82.

Sometimes unequal frogs are used on crossovers, and in this case the best form of layout to use is a connecting curve of about the

same degree as the *sharper* turnout, tangent at the heel of the *flatter* frog. This will place a short piece of tangent between the curve and the heel of the sharper frog.

In Fig. 82, F, F_1, p, g, and K_1 are known. R is chosen to agree with the turnout curve of F. (From Table 9.)

In triangle 1, the hypotenuse and angle F_1 are known. Solve for the sides d_1 and m.

In triangle 2, the altitude $n = p - m$ and the angle F are known. Solve for the sides d and j. Then

$$K = j - gN - R \tan \tfrac{1}{2}(F - F_1) - P$$

The distance along the main tracks between frog points is

$$x = (d + d_1) - (gN + P + gN_1 + P_1) \qquad (84)$$

214. Case 2. Curved Tracks.

It can be shown that, if a crossover is located between curved tracks with the same frog numbers and a simple curve tangent to the heels of the frogs, the distance along the main tracks between frog points is practically the same as for a straight crossover between straight tracks; and also that the degree of curve of the connection is practically the same as that of the main tracks.

Therefore, a crossover on a curve is located with the same dimensions as one on tangent. It may be desirable, however, to locate points on the connecting curve. Usually one point at the middle is sufficient, and it will be seen that this point is midway between the frog points and on the center line between tracks.

Crossovers should be avoided on curves.

215. LADDER TRACKS. The series of parallel tracks in a railroad terminal yard upon which freight or passenger cars are temporarily stored are called "body tracks." At one or both ends of the body tracks there is a diagonal track, called a "ladder track," which connects the body tracks to the main track, or running track. Fig. 83 shows a main track, a ladder track, and four body tracks.

The angle between the ladder track and the main track is generally equal to the frog angle used for the turnouts from the main track and the body tracks, as shown in Case 1, Fig. 83. Sometimes, however, the ladder track makes a slightly larger angle than the frog angle. This necessitates a short curve beyond the frog of the main track turnout and a curve at the beginning of each body

track. This condition is illustrated in Case 2, Fig. 84. Again, the angle which the ladder track makes with the main track may be twice the frog angle. In this case, only alternate body tracks are connected to the ladder track and the other body tracks are connected to the preceding body track. Since this is rather an unusual case, its solution is not given; but, if the student understands thoroughly the two cases given, he should have no difficulty in solving the more unusual cases which may arise.

216. Case 1. Angle of Ladder Track Same as Frog Angle. In triangle 1, Fig. 83, the frog angle and the altitude p, or distance between body tracks, are known. Solve for the hypotenuse

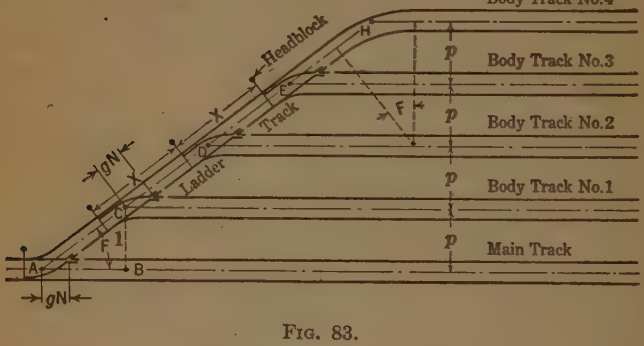

FIG. 83.

AC. The distance AC equals CD, DE, and EH, and also equals the distance between frog points and the distance between switch points, X. The minimum distance which should be used between the frog point of one turnout and the switch point of the succeeding turnout is about 13 ft., hence AC must be equal to or greater than the lead of the turnout plus 13 ft. The distance, p, between body tracks must be large enough to satisfy this condition.

The ladder track and body tracks are staked as follows: The approximate location of the frog in the main track is known, being determined by the location of other tracks and connections not shown in the figure. Locate the frog so that it will abut the end of a rail. Measure the distance gN from the theoretical point of frog, and locate point A. Set-up at A and turn off the frog angle F. The line of sight is now along the line AH. On this line locate the points C, D, and E which are the distance AC apart. Measure for-

ward from each of these points the distance gN and locate the successive frog points. The body tracks are best located by perpendicular offsets from the main track.

217. Case 2. Angle of Ladder Track Larger than Frog Angle. In this problem F, F_1, a, p, K, K_1, P, P_1, R, and R_1 are known. In triangle 1, Fig. 84, all of the angles are known and the longest side is equal to $gN + K + P + R \tan \frac{a}{2}$. Solve for the other two sides.

Triangle 2 is solved in the same manner. If the angles F and F_1

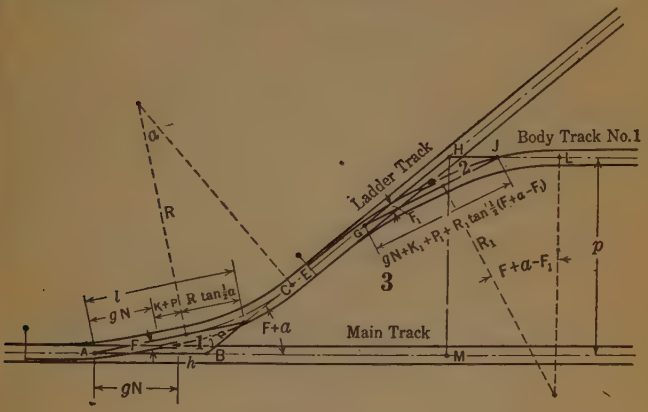

FIG. 84.

are equal and the radii R and R_1 are equal, triangle 2 is identical to triangle 1.

In triangle 3, the angles and the altitude p are known. Solve for the hypotenuse BH.

In the field, point A is located as in the preceding problem. The distance AB locates point B. Generally B will fall between A and the $P.F.$ Set-up at B and turn off the angle $F + a$. The line of sight is now along the ladder track. The point of frog of the turnout to body track No. 1 is located from point B by the distance $BH - GH + gN$. The other body tracks, not shown in the figure, are located in the same way. In this case, as in the preceding one, the clearance between the frog point of one turnout and the switch point of the next turnout should be at least 13.0 ft.

If $F + a$ in Fig. 84 equals F_1, as frequently happens, no curve is necessary to connect the body tracks to the ladder track, and the solution given in Fig. 83 is used to determine the distances between the frog points of the turnouts to the body tracks.

Sometimes it is considered desirable to group the body tracks in pairs and to use a greater distance between groups than between the two tracks in each group. This involves the solution of two triangles in Figs. 83 and 84 instead of one, using p as the altitude of one triangle and p' as the altitude of the other.

Intersecting Tracks

218. Case 1. Both Tracks Straight.

(*a*) *Connection in form of simple curve.*

Given in Fig. 85, the intersection angle I, the frog angles F and F_1 the radius of the connecting curve, and K and K_1.

Required: The distances, A and B, from the intersection of the center lines to points opposite the frog points.

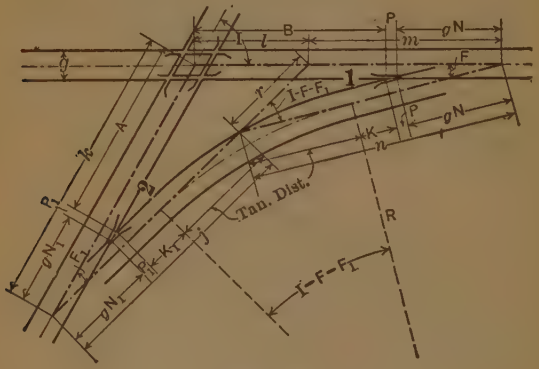

FIG. 85.

In triangle 1, the angles are known and one side, n, is readily obtainable. Solve for r and m.

In triangle 2, one side, $j + r$, and the angles are now known, whence k and l can be computed.

Then

$$\left.\begin{aligned} A &= k - gN_1 - P_1 \\ B &= l + m - gN - P \end{aligned}\right\} \qquad (85)$$

When the distances A and B, as computed from Eq. 85, are measured off in the field, it will generally be found that the frog points thus located will not be such that the heel or the toe of the frog will come at rail joints. The points, therefore, will have to be moved a short distance forward, or backward, until the frogs will abut rail joints. This will change the lengths of both frog tangents; and before the connecting track can be located, the new frog tangents must be computed. The solution for the case where both frogs are moved forward is shown in Fig. 86.

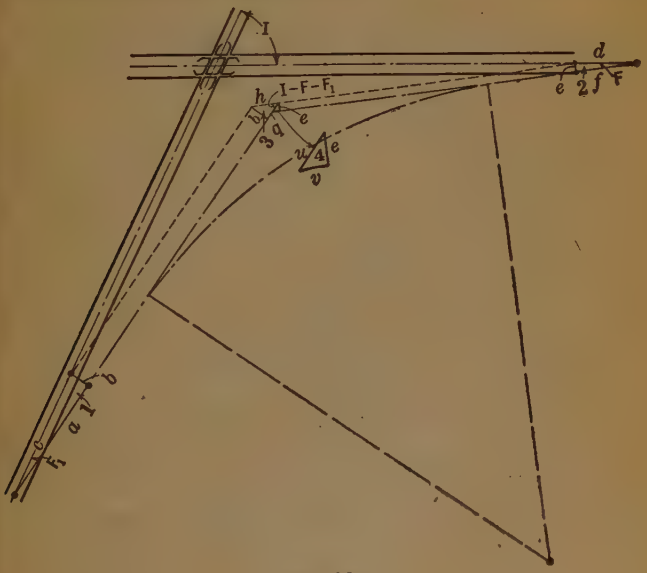

FIG. 86.

In triangle 1, the hypotenuse c is the measured distance the N_1 frog is moved, and the small angle is F_1. Solve the triangle for the sides a and b.

In triangle 2, the hypotenuse d is the measured distance the N frog is moved, and the small angle is F. Solve the triangle for the sides e and f.

In triangle 3, the altitude b and the angle, $I - F - F_1$, are known. Solve the triangle for h and q.

In triangle 4, the altitude e and the angle, $I - F - F_1$, are known. Solve the triangle for u and v.

Then

$$\left.\begin{aligned} \text{New } K_1 &= \text{old } K_1 - a - q + u \\ \text{New } K &= \text{old } K - f + h - v \end{aligned}\right\} \tag{86}$$

When both frogs are moved backward, the new frog tangents become,

$$\left.\begin{aligned} \text{New } K_1 &= \text{old } K_1 + a + q - u \\ \text{New } K &= \text{old } K + f - h + v \end{aligned}\right\} \tag{87}$$

When the N_1 frog is moved forward and the N frog is moved backward, the new frog tangents become,

$$\left.\begin{aligned} \text{New } K_1 &= \text{old } K_1 - a - q - u \\ \text{New } K &= \text{old } K + f + h + v \end{aligned}\right\} \tag{88}$$

(*b*) *Connection in form of reversed curve.*

Given in Fig. 87, the intersection angle I; the frog angles F and F_1; the radii of the reversed curve, R and R_1; and the location of one frog point, which is the distance T from the $P.I.$ at A.

Required: The central angles a and b of the reversed curve, the distance T_1 from the $P.I.$ to the unknown frog point, and the distance x between frog points measured along the straight track.

In triangle 1 (CDB), CD and angle F are given. Solve for CB and DB, whence AB and OB can be found.

In triangle 2 (ABE), AB and the angles are known. Solve for BE and AE.

In triangle 3 (OEG), OE and the angles are known. Solve for OG and GE.

In triangle 4 (LHQ), LH and the angles are known. Solve for HQ and $LQ = JM$.

In triangle 5 (HJO_1) HO_1 and the angle F_1 are known. Solve for O_1J and HJ.

In triangle 6 (O_1OS), $O_1O = R + R_1$ and $O_1S = O_1J + JM + GO$ are known. Solve for angles c and d, whence angles a and b are readily determined.

$$T_1 = AE + EG - OS - LM + gN_1 + P_1 \tag{89}$$

$$x = T - AL \cos I - gN_1 - P_1 \tag{90}$$

If T_1 instead of T is known, the plan of solution is the same, but another figure is necessary, in which triangle 1 comes at the other

frog. If the same letters are used to designate corresponding points, the solution as outlined above for Fig. 87 can be used.

If the tracks do not run to an intersection or if it is inadvisable to run out the tangents and determine I, locate the point D, set-up

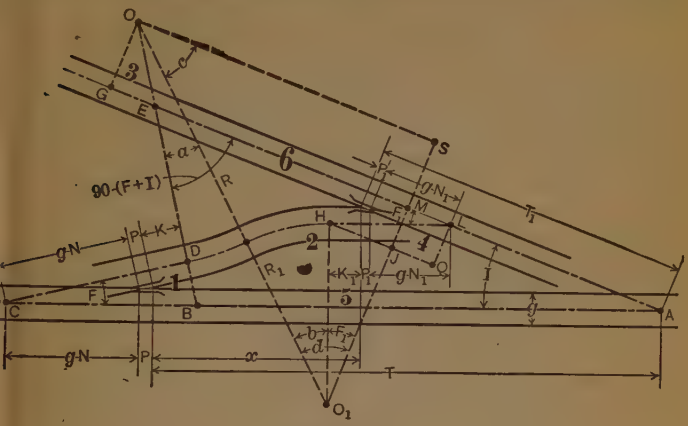

FIG. 87.

and turn off 90° from the frog tangent, and measure DE. Then set-up at E and measure DEL. With these data the problem can be solved in the same manner as before.

219. Case 2. One Straight and One Curved Track.

Solution: From Fig. 88.

In triangle 1, the hypotenuse and the angle F are known. Solve for the two sides. Whence,

In triangle 2, the two sides can be obtained by arithmetic. Solve for the hypotenuse and the angle a.

In triangle 3, the hypotenuse and the angle I are known. Solve for the two sides.

In triangle 4, the altitude, $R_1 + \frac{1}{2}g$, and the angle F_1 are known. Solve for the hypotenuse and the base.

In triangle 5, the hypotenuse and the angle F_1 are known. Solve for the sides l and m. Whence,

In triangle 6, the altitude is found by arithmetic and the hypotenuse is known from triangle 2. Solve for the angle b.

Then

$$x = a - b + (90° - I) - (90° - F)$$
$$= a - b - I + F \qquad (91)$$

And

$$\text{(curve) } AH \text{ (in feet)} = \frac{100x}{D}$$

The $P.F.$ is set at the distance $AH - P$ from A.

From triangles 3, 5, and 6, the distances AG, m, and OJ are known. Then AC is found by arithmetic.

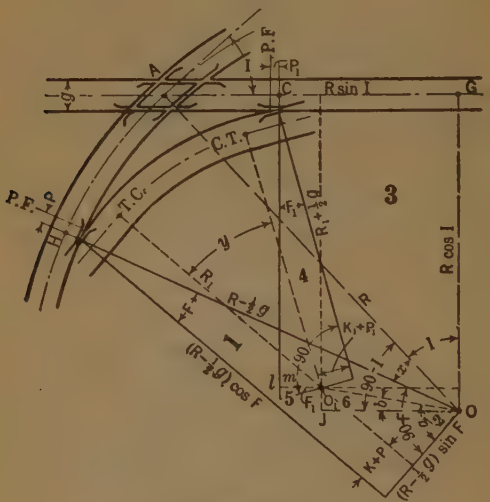

FIG. 88.

The $P.F._1$ is set at a distance $AC - P_1$ from A. The central angle of the connecting curve is

$$y = I + x - F_1 - F$$

Obviously a connecting track can be placed in any of the four quadrants formed by the intersection of the two main tracks, but the plan of solution for any quadrant is identical with that given above; and if the same letters are used to designate the correspond-

ing points and the triangles are constructed in the same manner, no difficulty should arise in the solution of the other three cases.

When the frog points H and C are located in the field, it will be found generally that the frogs as thus located will not abut existing rail joints. Both frogs will therefore have to be moved a short distance backward or forward until they do abut existing rail joints. Both frog tangents will thus be changed in length, and the direction of the frog tangent from the curved main track will be changed by an amount equal to the angle z. Angle z, in minutes, equals $0.6dD$, where d is the distance the N frog is moved and D the degree of the main track curve. The solution for the case where both frogs are moved toward A is as follows. The slide rule, or three-place functions, may be used in all solutions, except that for v.

$t = d \sin (F - \frac{1}{2}z)$ (See triangle 1, Fig. 71)
$r = d \cos (F - \frac{1}{2}z) = d$ (approx.)
$m = (K + P - r) \tan z$ (See triangle 2, Fig. 71)
$n = (K + P - r) \sec z$
$u = R_1 + \frac{1}{2}g + t - m$
$s = u \sin z$ (See triangle 3, Fig. 71)
$v = u \cos z$
$w = v - (R_1 + \frac{1}{2}g)$
$b = w \cos (y - z)$
$c = w \sin (y - z)$
$e = d' \cos F_1$ (d' equals distance N_1 frog is moved)
$f = d' \sin F_1$
$h = f - b$

$$i = \frac{h}{\sin (I + x - F - F_1 - z)}$$

$j = h \cot (I + x - F - F_1 - z)$

Then

New Frog Tangent, $K' = n + s + i$

And

New Frog Tangent, $K'_1 = K_1 + P_1 - e + c - j$

It is to be noted that if b is greater than f in the above equations, h, i, and j become minus.

Solution 2: Due to the length of the above solution, and also to the fact that the main line curve may be somewhat out of alinement, it may be best to solve this problem in another way. First, solve

for AH and AC, Fig. 88, approximately only. Then measure these approximate distances and locate the frogs at the nearest rail joints. Set-up, in turn, at the frog points thus established, get on tangent, and measure back—toward the switch points—the distances $gN + P$ and $gN_1 + P_1$, respectively, to locate points corresponding to point B in Fig. 70. Set-up the instrument at these points and turn off the respective frog angles, thus obtaining the tangents for the connecting curve. Produce these tangents to an intersection, and measure the intersection angle, which is also the central angle of the connecting curve. For this angle and for the degree of the connecting curve, compute the tangent distances. By laying off these tangent distances from the $P.I.$ of the frog tangents produced, the $T.C.$ and the $C.T.$ of the connecting curve are established.

220. Case 3. Both Tracks Curved.

From Fig. 89,

In triangle 1(HEO), the hypotenuse and the angle F are known. Solve for the sides HE and EO.

In triangle 2 (OO_2G), the two sides can now be found by arithmetic. Solve for the angle a and the side OO_2.

In triangle 3 (AOO_1), the two sides, R and R_1, and the included angle I are known. Solve for the side OO_1 and the angles b and c.

In triangle 4 (O_1CM), the hypotenuse and the angle F_1 are known. Solve for the sides CM and O_1M.

In triangle 5(O_1O_2J), the base and the altitude can now be found by arithmetic. Solve for the hypotenuse O_1O_2 and the angle d.

In triangle 6 (OO_1O_2) the three sides are now known. Solve for the angles e and f..

Then

$$\left.\begin{aligned} x &= a + b - f - (90 - F) \\ &= a + b + F - f - 90 \end{aligned}\right\} \qquad (92)$$

And

$$z = c + d - e + F_1 - 90$$

$$\text{arc } AH = \frac{x}{D} 100 \text{ (in feet)}$$

$$\text{arc } AC = \frac{z}{D_1} 100 \text{ (in feet)}$$

The central angle of the connecting curve is

$$y = I + x + z - F - F_1 \qquad (93)$$

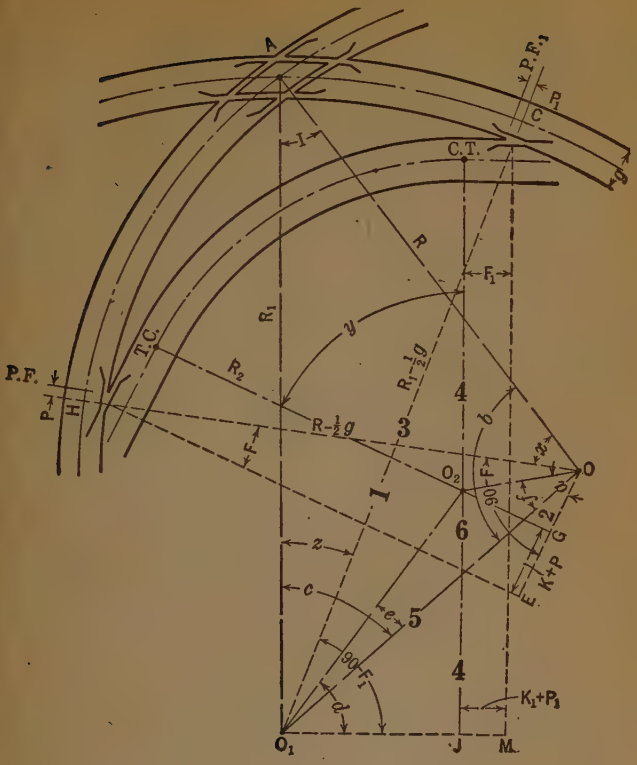

FIG. 89.

In case a connection is placed in any other quadrant the plan of solution is identical with that above, and if followed exactly no difficulty should arise.

221. WYE TRACKS. A wye is an arrangement of tracks, in the shape of the letter "Y," connected to a main track to make it possible to turn an engine or cars so that they will head in the opposite direction. A wye performs the same function as a turntable. Fig. 90 shows a wye connected to a straight main track, and Fig. 91 shows a wye for a curved main track with the wye on the outside of the curve. The third case, where the wye is inside of a curved main

track, is not solved, but its general method of solution is the same as that in case 2, although a new figure, with all the centers of curves on the same side of the main track, is required.

221. Case 1. Straight Main Track.

In Fig. 90, F_1, F_2, F_3, N_1, N_2, N_3, K_1, K_2, K_3, P_1, P_2, P_3, R_1, and R_2 are given. It is required to find the distance along the main track between frog points, and the central angles x and y of the wye curves.

In triangle 1, the angles and the base are known. Solve for altitude m and hypotenuse l.

In triangle 2, the angles and the hypotenuse, $R_1 + m$, are known. Solve for the altitude n and the base p.

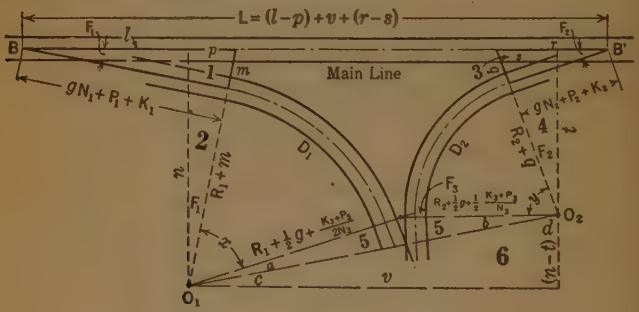

FIG. 90.

In triangle 3, the angles and the base are known. Solve for the altitude q and the hypotenuse r.

In triangle 4, the angles and the hypotenuse, $R_2 + q$, are known. Solve for the altitude t and the base s.

In triangle 5, two sides and the included angle are known, as shown in the figure. Solve for the third side O_1O_2 and for the angles a and b.

In triangle 6, the hypotenuse O_1O_2 and the altitude $n - t$, are known. Solve for the base v and the angles c and d.

Then

$$\left.\begin{aligned} x &= 90 - (F_1 + a + c) \\ y &= 180 - (F_2 + b + d) \end{aligned}\right\} \qquad (94)$$

$$L = (l - p) + v + (r - s)$$

Distance between frog points $= L - gN_1 - gN_2$.

223. Case 2. Curved Main Track.

In Fig. 91, R, R_1, R_2, F_1, F_2, F_3, K_1, K_2, K_3, P_1, P_2, and P_3 are given. It is required to find angles x, y, and z and the distance along the main line track between frog points.

In triangle 1, the angles and the base, $K_1 + P_1$, are known. Solve for the altitude m and the hypotenuse l.

In triangle 2, two sides and the included angle F_1 are known. Solve for the other side OO_1 and the angles a and b.

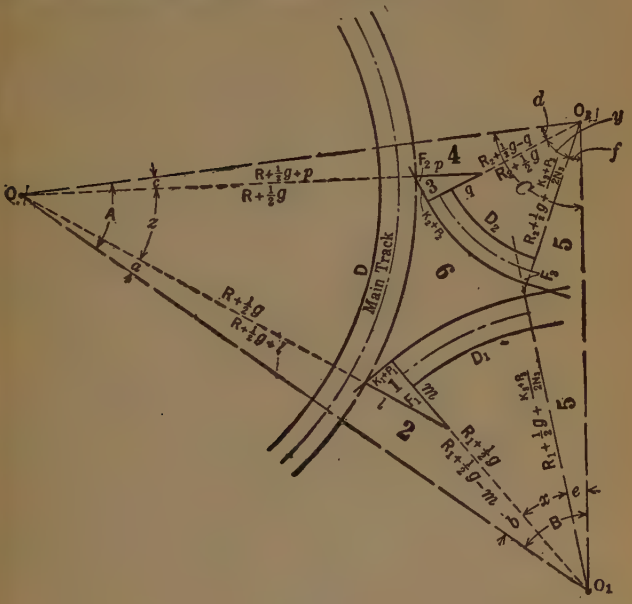

FIG. 91.

In triangle 3, the angles and the altitude, $K_2 + P_2$, are known. Solve for the base q and the hypotenuse p.

In triangle 4, two sides and the included angle F_2 are known. Solve for the third side OO_2 and the angles c and d.

In triangle 5, two sides and the included angle F_3 are known. Solve for the third side O_1O_2 and the angles e and f.

In triangle 6, OO_1O_2, the three sides are known. Solve for the angles A, B, and C.

Then

$$\left.\begin{aligned} x &= B - (b + e) \\ y &= C - (d + f) \\ z &= A - (a + c) \end{aligned}\right\} \qquad (95)$$

The distance between frog points on main track is equal to $\frac{z}{D}100$.

In the two cases illustrated in Figs. 90 and 91, the wye tracks beyond the N_3 frog are not shown. Of course it is necessary to continue one of the tracks until it is long enough to contain a full train. As far as the solution for the locations of the three frogs is concerned, it is immaterial what arrangement of tracks is used beyond the N_3 frog. For example, either branch of the wye can be considered as the primary branch and continued as far as desired either as a curved track or as a straight track, and the other branch connected to it with an N_3 turnout. Another arrangement is to give equal importance to the two branches and use the "diamond" turnout as explained in the following section.

224. Diamond Turnouts. When both tracks through a turnout are of equal importance, as where a single track changes to double

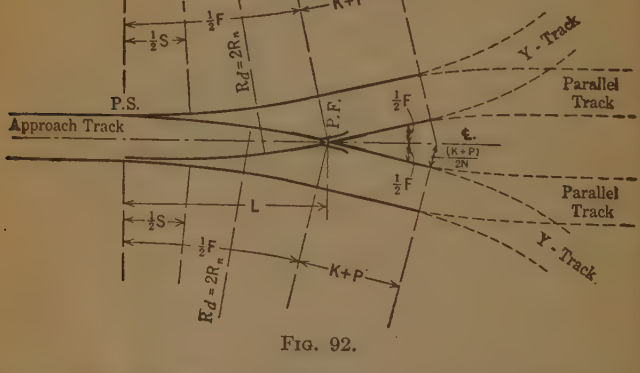

FIG. 92.

track or where the two branches of a wye unite, it is often desirable to insert the turnout as shown in Fig 92. This is termed a diamond turnout.

The switch is so installed that the points deflect one-half the switch angle ($\frac{1}{2}s$) to each side instead of the full angle s to one side as in the normal turnout. The frog is placed with its axis coin-

ciding with the center line of the approach track. The lead L is unchanged by this arrangement but the radius of the lead rails becomes twice the radius in a normal turnout, and the total deflection of the train at the frog point is only $\frac{1}{2}F$. It is therefore evident that the diamond turnout subjects both track and rolling stock to less shock as a train passes through it and consequently trains can be operated faster if desired. This is the especial point of merit of the diamond turnout.

If the approach track is on a curve the turnout is installed in the same way with respect to the center line as on straight track. The relative positions of the parts are unchanged but the radii of the lead rails are different as explained in section 205.

CROSSINGS

225. A crossing is a device used at the intersection of two tracks. It consists of four frogs and the necessary connecting rails. Any one of the frogs is a *crossing frog*. The *crossing angle* is the angle between the center lines of the tracks at their point of intersection.

Crossings are designated as *single curve*, *double curve*, or *straight*, according as one, both, or neither of the tracks are curved.

Crossings are usually made of rolled rails fitted together. When the crossing angle is greater than about 25°, the various pieces are cut to fit against each other and are united by filling blocks and heavy straps well bolted. This is frequently termed *solid construction*. For angles under about 25°, regular frog point construction is used, and such crossings are termed *frog crossings*. Distinguish between *frog crossing* and *crossing frog*.

On street railway work hard steel (manganese) centers for frogs have been used for many years, and since 1905 such construction has been growing in use on steam roads both for standard frogs and for crossings. (This type is usually designated as "manganese frogs or crossings.")

There have been numerous attempts to construct crossings with a revolving section or turntable at each frog so that the wheels will have no flangeway to jump; but only one of these seems to give much promise of success.

The end frogs of a frog crossing are similar to a standard rigid frog in that there is a *single point* on which the wheels run. The middle frogs, however, have *two running points* and are therefore frequently termed "double-pointed frogs."

When "slip switches" are used, the crossing is made to a standard frog number, and if located at an interlocking plant the middle frogs are frequently made with *movable points*. That is, with movable points joined in pairs and moving together similar to a split switch in such a way that the wheels have a solid bearing and no flangeway to jump.

A "slip switch" or "combination crossing" is a combination of a small angle crossing with a pair of connecting tracks placed entirely within the limits of the crossing. They are used in large yards and terminals and are usually made to some standard frog number. A double slip switch is illustrated in Fig. 93.

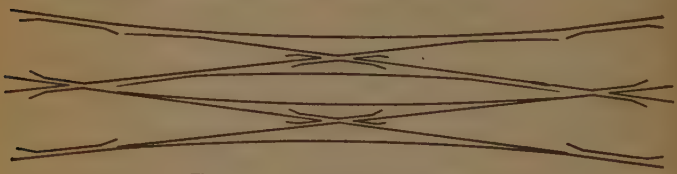

FIG. 93.—Double Slip Switch.

226. Crossing Data. Very few railroads construct their own crossings, but have them built by manufacturers who make a specialty of such work. The field engineer is rarely called on to compute the dimensions of a crossing, and to do so is a waste of time if the crossing is ordered from a manufacturer. It is far more important that the makers have *all* the data, and the field engineer is frequently required to furnish the data. The information required is:

1. The crossing angle.
2. The gage of each track.
3. The curvature—degree of curve, radii, or the equivalent.
4. The direction of curvature.
5. The length along each gage line from one gage line intersection (theoretical *P.F.*) to the nearest rail joint.
6. Length over all along each gage line.
7. The height, weight, and style of rail of which the crossing is to be made.
8. The height, weight, and style of rail in intersecting track if offset splices are to be furnished.
9. The spacing and size of holes for splice bars.
10. The type of crossing, etc., unless covered by general specifications.

This information can best be given by means of a small sketch. Field dimensions should be taken to the nearest $\frac{1}{8}$ in. (0.01 ft.)

Occasionally the field engineer is called on to compute the dimensions of a crossing. The values required are the frog angles, F_1, F_2, F_3, F_4, the length of sides along the gage lines, and the two diagonals. The computations should be made with sufficient accuracy to give results which are correct to the nearest $\frac{1}{16}$ in., which is the working limit of the manufacturers.

The solutions of the three kinds of crossings are outlined as follows:

227. Straight Crossings.

(*a*) *Unequal Gages.* In Fig. 94, g_1, g_2, and I are given, and the sides and diagonals are required.

In triangle 1, the altitude g_1 and the angle I are known. Solve for s_1 and b_1.

Similarly solve for s_2 and b_2 in triangle 2.

In triangle 3, the altitude g_1 and the base $s_2 + b_1$ are known. Solve for the hypotenuse, which is the long diagonal of the crossing.

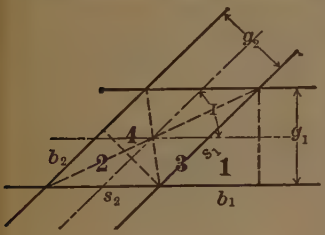

FIG. 94.

In triangle 4, the altitude g_2 and the base $s_1 - b_2$ are known. Solve for the hypotenuse, which is the short diagonal of the crossing.

(*b*) *Equal Gages.* In this case, triangles 1 and 2 are identical and all four sides are equal. The angle between the hypotenuse and the base of triangle 3 becomes $\frac{1}{2}I$, and of triangle 4 becomes $90° - \frac{1}{2}I$, which simplifies the solution.

228. Single-Curve Crossings.

Given, in Fig. 95, g_1, g_2, and I to find the frog angles, length of sides, and both diagonals.

Project the radius on a perpendicular to the straight track.

Then $OB = R \cos I$, whence OE and OC are known.

In triangle F_1CO, the base and hypotenuse are known. Solve for F_1C and the angle F_1.

Similarly solve for F_2 and F_2C, F_3 and F_3E, and F_4 and F_4E from corresponding triangles. Then the straight sides F_1F_2 and F_3F_4 can be found by subtraction. The curved sides F_1F_4 and F_2F_3 can be

determined since their radii are known and their central angles can be found by subtraction.

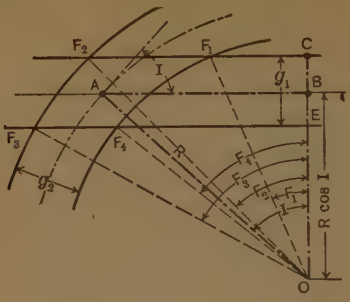

FIG. 95.

The long diagonal is the hypotenuse of a triangle whose altitude is g_1 and whose base is $F_3E - F_1C$, and the short diagonal is the hypotenuse of a triangle whose altitude is g_1 and whose base is $F_2C - F_4E$.

229. Double-Curve Crossings.

In Fig. 96, R_1, R_2, g_1, g_2, and I are given, and it is required to find the frog angles, length of sides, and both diagonals.

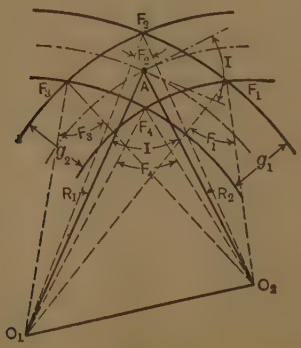

FIG. 96.

In triangle AO_1O_2, two sides and the included angle are known. Solve for the distance between centers, O_1O_2.

Then in the four triangles formed on O_1O_2 by the radii from F_1, F_2, F_3, F_4 the three sides of each are known, and the angles can be computed. From the angles at O_1 and O_2 determine the central angles of the arcs, F_1F_2, F_2F_3, F_3F_4, and F_4F_1, from which the lengths of the arcs can be computed since their radii are known. The long diagonal is found from the triangles $F_1O_1F_3$ or $F_1O_2F_3$, and the short diagonal from the triangles $F_2O_1F_4$ or $F_2O_2F_4$.

Seven-place tables should be used when I is less than about 30° and when D is less than about 15°.

16

Problems

1. The intersection angle between a diverging track extended and a straight main track is 50° 24′ 30″. It is desired to connect a No. 10 turnout from the main track with the diverging track, using a 6° curve and a frog tangent of 30 ft. Find the distances from the point of intersection of tracks to (1) the actual point of frog, and (2) the *C.T.* of the connecting curve.

Answer. (1) 381.37 ft.; (2) 453.32 ft.

2. When the *P.F.* of the turnout in Problem 1 is located in the field, it is found that the heel of the frog is 7.54 ft. from a rail joint. The *P.F.* must then be moved *forward* 7.54 ft. Find (1) the length of the new frog tangent, and (2) the change in the position of the *C.T*

Answer. (1) 21.74 ft.; (2) 1.07 ft.

3. The intersection angle between a diverging track extended and a 3° curved main track is 60° 56′ 30″. It is desired to connect a No. 10 turnout from the *inside* of the main track with the diverging track, using a 5° connecting curve and a frog tangent of 40 ft. Find the distances from the point of intersection of tracks to (1) the actual point of frog—distance measured along center line of curved main track; and (2) the *C.T.* of the connecting curve; also find (3) the central angle of the connecting curve.

Answer. (1) 1335.21 ft.; (2) 936.78 ft.; (3) 95° 17.1′

4. Solve Problem 3 with the following changes: $I = 60° 04'$, $D = 1° 30'$, and $D_1 = 6° 00'$.

Answer. (1) 627.2 ft.; (2) 611.5 ft.; (3) 63° 45′.3.

5. When the *P.F.* of the turnout in Problem 4 is located in the field, it is found that the heel of the frog is 8.90 ft. from a rail joint. The *P.F.* must then be moved *forward* 8.90 ft. Find (1) the length of the new frog tangent, and (2) the change in the position of the *C.T.*

Answer. (1) 33.37 ft.; (2) 0.90 ft.

6. Solve Problem 5 with the change that the *P.F.* is moved *backward* 8.90 ft.

Answer. (1) 47.41 ft.; (2) 0.88 ft.

7. The center line of a diverging track, extended, intersects a straight main track at point *A*, making an intersection angle of 60° 56′ 30″. The point *A* is only 295.60 ft. from the *C.T.* of a 3° curve on the main track, so it is evident that the turnout will come on the curve. It is desired to connect a No. 10 turnout from the *inside* of the curved main track with the diverging track, using a 5° connecting curve and a frog tangent of 40 ft. Find (1) the distance from the point *A*—measured along center line of main track—to the actual point of frog; (2) the distance from *A* to the *C.T.* of the connecting curve; and (3) the central angle of the connecting curve.

Answer. (1) 705.1 ft.; (2) 762.5 ft.; (3) 75° 41.4′.

8. Use the same data as in Problem 7, except that the 3° main line curve ends in a spiral whose *k* is 1°. The point of intersection *A* is 295.6 ft. from the *S.T.* Find (1) the distance from point *A* to the actual point of frog; (2) the distance from *A* to the *C.T.* of the connecting curve; and (3) the central angle of the connecting curve.

Answer. (1) 654.9 ft.; (2) 709.9 ft.; (3) 65° 41.1′.

9. In Fig. 74, $D = 4°$, $D_1 = 8°$, $I = 64° 27' 30''$, $N = 10$, $K = 40$ ft., $P = 0.40$ ft., $k = 1°$, A to $C.S. = 160.0$ ft. Required: angles x and y, and distances AH and AE.

Answer. $x = 31° 28.8'$; $y = 87° 20.0'$; $AH = 747.0$ ft.; $AE = 571.0$ ft.

10. The intersection angle between a diverging track, extended, and a 3° main track is 60° 56′ 30″. It is desired to connect a No. 10 turnout from the *outside* of the main track with the diverging track, using a 5° connecting curve and a frog tangent of 40 ft. Find the distances from the point of intersection, *A*, to (1) the actual point

of frog, and (2) the *C.T.* of the connecting curve; also find (3) the central angle of the connecting curve.

Answer. (1) 407.1 ft.; (2) 580.6 ft.; (3) 43° 00.2′.

11. When the distance $AH = 407.1$ ft., solved for in Problem 10, is measured off in the field, it is found that the point H must be moved forward—toward A—a distance of 8.90 ft. in order that the heel of the frog may abut an existing rail joint. Find (1) the length of the new frog tangent; and (2) the change in the position of the *C.T.*

Answer. (1) 25.07 ft.; (2) 1.53 ft.

12. Use the same data as in Problem 10, except that the point A falls on the tangent at a distance of 295.6 ft. from the *C.T.* of the main track curve.

Answer. (1) 493.52 ft.; (2) 650.11 ft.; (3) 49° 16.7′.

13. Use the same data as in Problem 12, with the exception that the main track curve ends in a spiral whose k is 1°. The point of intersection, A, falls on the tangent at a distance of 295.6 ft. from the *S.T.*

Answer. (1) 544.4 ft.; (2) 666.4 ft.; (3) 52° 15.0′.

14. It is desired to lay out a siding on the *outside* of a 4° main track. Track centers are to be 13.5 ft. A No. 12 turnout is to be used and a frog tangent of 15.0 ft. Find (1) the radius of the connecting curve; (2) the central angle of the connecting curve; and (3) the coordinates of the quarter points on the connecting curve with respect to the main track.

Answer. (1) 778.6 ft.; (2) 11° 39.5′; (3) 13.50, 13.03, 11.64, and 9.31 ft.

15. Use the same data as in Problem 14, except that the siding is to be on the *inside* of the main track.

Answer. (1) 2817.5 ft.; (2) 4° 14.6′; (3) 13.50, 13.02, 11.59, and 9.21 ft.

16. It is desired to locate a crossover between two parallel straight tracks, whose center lines are 13.0 ft. apart. A No. 10 turnout is to be used from one track and a No. 9 turnout from the other. The connecting curve is chosen from Table 8 to agree approximately with the degree of the sharper turnout, and is 9° 00′. The *T.C.* of the connecting curve is to come at the heel of the No. 10 frog, which

is 7.50 ft. from the actual point of frog. Find (1) the frog tangent between the *C.T.* of the connecting curve and the point of the No. 9 frog, and (2) the distance between actual points of frog, measured parallel to main tracks.

Answer. (1) 18.40 ft.; (2) 32.25 ft.

17. Find the distance between frog points on a ladder track for body tracks 13.0 ft. on centers, using No. 9 frogs throughout.

Answer. 117.36 ft.

18. In Fig. 84, $F_1 = 5° 43' 29''$, $F_2 = F_1 + a = 7° 09' 10''$, $K + P = 15.0$, $K_1 + P_1 = 8.0$, $D = 10° 00'$, $p = 13.0$.

Find AB, BC, CE, and the distance between frogs on body tracks.

Answer. $AB = 13.86$; $BC = 62.59$; $CE = 12.39$; distance between frog points = 104.40.

19. Two straight main tracks intersect at an angle of $74° 59' 30''$. The tracks are to be connected by a 6° curve. A No. 10 turnout and a frog tangent of 30 ft. are to be used from one main track, and a No. 9 turnout and a frog tangent of 40 ft. from the other track. Find the location of the two frog points with respect to the point of intersection of tracks.

Answer. No. 10 frog, 669.7 ft.; No. 9 frog, 668.6 ft.

20. When the distance A and B as computed in Problem 19 are measured off in the field, it is found that the No. 9 frog must be moved forward (toward the intersection) a distance of 4.35 ft. and the No. 10 frog moved backward 8.60 ft. in order that the frogs may abut existing rail joints. Compute the new frog tangents which must be used.

Answer. $K = 39.54$; $K_1 = 34.47$.

21. Given, in Fig. 87: $I = 35° 00'$; $T = 1044.4$ ft.; $R = R_1 = 716.34$; $D = D_1 = 8° 00'$; $N = N_1 = 9$; $K = K_1 = 30.0$.

Find: angles a and b; T_1; and x.

Answer. $a = 20° 12.5'$; $b = 55° 12.5'$; $T_1 = 99.21$ ft.; $x = 955.4$ ft.

22. Given, in Fig. 88: $I = 40° 24'$; $D = 3° 00'$; $D_1 = 6° 00'$; $N = 10$; $N_1 = 9$; $K = K_1 = 30$ ft.

Required: AH and AC, and angles x and y.

Answer. $AH = 475.5$; $AC = 347.6$; $x = 14° 16.0'$; $y = 42° 34.9'$.

23. Given, in Fig. 89: $I = 60° 00'$; $D = 4°$; $D_1 = 3°$; $D_2 = 8°$; $N = N_1 = 10$; $K = K_1 = 20$ ft.; $P = P_1 = 0.40$ ft.

Required: Distances AH and AC, and angle y.

Answer. $AH = 909.2$; $AC = 811.4$; $y = 109° 15.7'$.

24. Given, in Fig. 90: $N_1 = N_2 = 10$; $N_3 = 8$; $K_1 + P_1 = 20.0$; $K_2 + P_2 = 20.0$; $K_3 + P_3 = 7.5$; $D_1 = 8°$; $D_2 = 10°$.

Required: angles x and y and the distance along main track between frog points of turnouts.

Answer. $x = 74° 46.1'$; $y = 86° 37.8'$; distance between frogs = 1195.3.

25. Solve Problem 24, taking the main track as on a 6° curve. The wye is to be on the outside of the main line curve.

Answer. $x = 53° 31.5'$; $y = 63° 47.3'$; $z = 44° 05.1'$; distance between frog points = 734.7 ft.

CHAPTER 9

EARTHWORK

Introduction

230. Form of Cross-Section. (*a*) *Railroads.* The usual forms of earthwork cross-sections in cut, fill, and side-hill work are shown in Figs. 97, 98, and 99. Fig. 100 shows the form that a section in cut should take when it consists partly of earth and partly of rock. The roadbed is level transversely, both on tangents and on curves, since it has been demonstrated by experience that crowning the

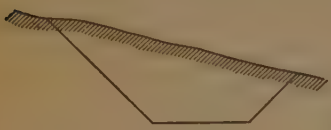

FIG. 97.

FIG. 98.

FIG. 99.

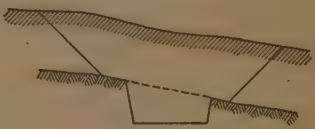

FIG. 100.

roadbed for drainage is a useless refinement, and that superelevation is best provided in the ballast.

Although ditches are constructed on each side of the roadbed in cuts, these are considered as supplemental cross-sections and are computed and recorded separately. The subject of ditches is discussed subsequently.

(*b*) *Highways.* The cross-sections for highways differ from those for railroads in the shape of the roadbed only. On tangents, both in cut and in fill, the roadbed is crowned to facilitate drainage; and on curves the roadbed becomes an inclined plane, the angle of inclination depending upon the amount of superelevation, which in turn is

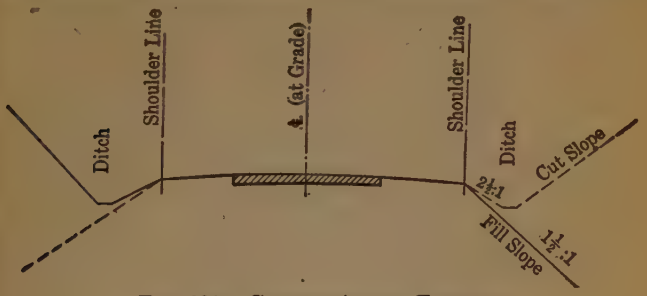

FIG. 101.—Cross-section on Tangents.

a function of the radius of curvature and of the maximum legal speed of vehicles. Also side ditches, both in cut and in fill, are considered part of the cross-section proper and are paid for on the same basis. Typical forms of highway cross-sections are shown in Figs. 101 and

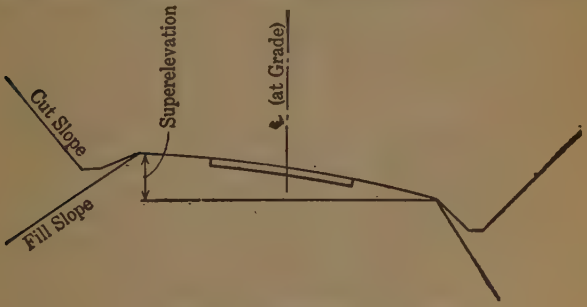

FIG. 102.—Cross-section on Curves.

102. If a curbed form of pavement is used, the cross-section becomes similar to that of a city street.

(*c*) *Streets.* The normal cross-section of a city street, as shown in Fig. 103, has a slope toward the middle from each property line or building line to the curb line, with the pavement itself depressed

at the curb line. On business streets the entire space from building line to curb is covered with a sidewalk; whereas in residential areas a planting space or parkway for trees or grass intervenes between the sidewalks and curbs.

In general, this section is obtained by excavation only. Sometimes on low ground or in business districts, the street is built up by

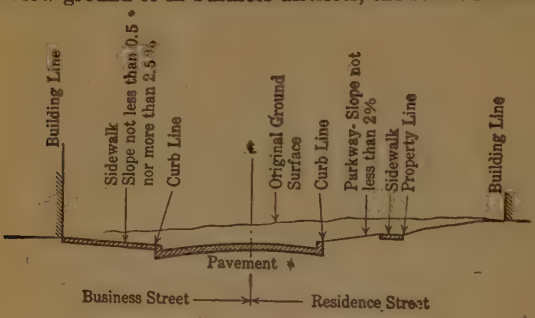

FIG. 103.—Street Cross-section.

filling to the same final shape. Sometimes a street is carried on a fill over low ground, and its section becomes similar or identical to that of a highway. If the pavement is curbed, the shoulders need to be only 3 or 4 feet wide, just enough to form a stable berm and possibly accommodate a guard fence.

Sometimes the entire street is excavated as a single project, and

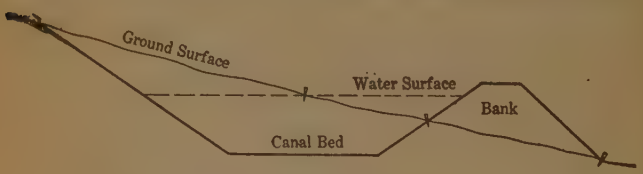

FIG. 104.—Canal Cross-section.

the earthwork is paid for at a uniform rate. More commonly, pavement excavation is figured as a trench just large enough to receive the pavement, while the excavation for sidewalks and the grading of parkways are computed as separate items.

(*d*) *Drainage Ditches.* The form of cross-section for drainage ditches is the same as that for railroad cuts and is shown in Fig. 97,

although generally the ground surface is more nearly level than is shown in the figure.

(*e*) *Canals.* If all of the canal section is in excavation, the form of section is the same as for railroad cuts. If the canal section is on a hillside, however, an artificial bank is necessary on the down-hill side as illustrated in Fig. 104. The bed of the canal should always be in excavation. Since the artificial bank is made of the material taken from the excavation, the latter area is the one used in determining the cost of construction.

(*f*) *Levees.* The standard form of cross-section for a levee is trapezoidal and differs from that for a railroad or a highway embankment in that the side slopes are flatter and the top widths are narrower. On levees along large rivers, such as the Mississippi, where the flood waters stand against the levees for weeks at a time, the land-side slope is reinforced by a banquette, or terrace of earth.

231. Width of Roadbed or Bottom. (*a*) *Railroads.* The width of roadbed on fills for single track varies from 16 to 24 ft., depending on the depth of ballast and the desired width of shoulder between the ballast and edge of roadbed. In cuts the width must be increased from 4 to 6 ft. on each side to provide room for ditches to drain the cut. For double tracks the width is increased by the distance between track centers and, for multiple tracks, by the distance between the center lines of the outside tracks.

(*b*) *Highways.* The width between shoulder lines on fills depends on the width of roadway or pavement and the desired amount of shoulder space. The graded width may therefore vary from 20 ft. on unimportant roads to 45 ft. for a modern 2-lane paved highway. Multiple-lane and divided highways require more width, depending on the number of lanes and the desired width of center strip. The best practice provides 10 to 12 ft. for each lane and at least 10 ft. for each shoulder to provide space for maneuvering in case the vehicle runs off the pavement, or for parking in emergencies. The center strip on divided highways may have any desired width. In cuts additional width is necessary for drainage ditches the same as on railroads unless a paved gutter is placed along the edges of the pavement or the pavement is widened enough to permit emergency parking and provided with curbs, in which case the width in cut may be the same as on fills.

(*c*) *Streets.* The width of pavement is often governed by the width of right of way which is generally established before the street

is improved or its character developed. Wherever possible 9 to 11 ft. should be provided for each running lane and 7 to 8 ft. for each parking lane. Thus a purely local residence street may have a pavement width of only 18 to 20 ft., whereas a 6-lane undivided thoroughfare would require 70 to 80 ft. In addition to the pavement the street must provide space for various utilities such as sidewalks, lights, trees, sewers, water and gas mains, etc. Hence the total width of right of way should be considerably wider than the width of pavement. A minimum of 60 ft. from property line to property line is desirable on any street. Whenever a 2-lane highway carrying a normal volume of traffic enters a municipality the street pavement should, if possible, be made wide enough to provide at least 3 running lanes in addition to the necessary parking space. With parking parallel to the curb this will require a minimum pavement width of about 43 feet.

(*d*) *Drainage Ditches.* The bottom width of drainage ditches is a function of the amount of water to be carried, and varies from 2 or 3 ft. to 60 ft. or more. For long ditches, the bottom width is increased as the area drained becomes greater.

(*e*) *Canals.* The bottom widths of canals also vary. For irrigation canals the bottom width varies with the discharge that is to be carried, and for navigation canals the width depends upon the widths of the boats and barges that are to use them.

232. Ditches. In order to provide drainage in railroad and highway cuts, side ditches are constructed on each side of the roadway. The ditches should be of such cross-section that they can be easily constructed and maintained and not easily obstructed. For these reasons broad shallow ditches are better than narrow deep ones. The usual width of ditches is from 3 to 6 ft., depending on the size of the cut and the amount and rate of rainfall.

Where there is considerable ground-water it may be necessary to construct tile sub-drains under one or both of the side ditches.

When there is likelihood of considerable surface water draining on to the roadbed, as for example in side-hill work, an open diversion ditch, termed a surface ditch, should be constructed outside the cut to carry this water to the end of the cut.

Side ditches are more expensive to excavate than the body of the cut, and therefore should preferably be included as a separate item in the contract.

233. Side Slopes. The slope of the sides of a cut or a fill is expressed as the ratio of the horizontal to the vertical distance and

depends on the material through or of which they are made. The following are commonly used:

(1) Cut

Solid rock	vertical or $\frac{1}{4}$: 1
Loose rock	$\frac{1}{4}$: 1 to 1 : 1
Hard earth	1 : 1 to $1\frac{1}{2}$: 1
Soft earth or sand	$1\frac{1}{2}$: 1 to 3 : 1

(2) Fill

Loose rock	1 : 1
Earth	1 : 1 to $1\frac{1}{2}$: 1
Sand	$1\frac{1}{2}$: 1 to 2 : 1

234. Borrow Pits. It frequently happens that the earth from the adjacent cuts is not sufficient to make the intervening fill. In this case the contractor is allowed to *borrow* earth from along the right of way. The pits thus dug are called *borrow pits*, and are staked out by the engineer before the work is started. In staking out borrow pits care should be taken to leave a berm of at least 5 ft. between the foot of the embankment and the edge of the borrow pit; a similar berm should also be left at the edge of the right of way. The contractor is required to leave all borrow pits in such a shape that water will not collect in them.

235. Shrinkage and Swell. Practically all materials increase in volume when first excavated. The amount of this increase for gravels and sands is from 5 to 12 per cent; for clays and loams, 10 to 30 per cent; and for solid rock, 30 to 100 per cent, depending on the size, shape, and uniformity of the rock fragments.

When the excavated material is placed in an embankment, it decreases in volume. The amount of this decrease varies with the kind of material, the method of making the embankment, and the weather conditions which prevail during the construction period, and may be greater or less than the increase in volume due to excavation. If the density of the embankment when it finally becomes fully compacted (ceases to settle) is greater than the density of the cut from which the material was taken, a unit volume from the cut will occupy less space in the fill. This permanent decrease in volume is called *shrinkage*.

The question of the amount of such shrinkage is one upon which there is considerable diversity of opinion, since there are few, if any, experimental data upon the subject. It is the authors' opinion

that clays, sands, and gravels will not decrease in volume, or shrink, when placed in embankment, and may actually swell or increase in volume. In either case the change of volume is certainly quite small and entirely negligible when considered with the other factors which affect the volume of cut and fill.

If the density of the settled embankment is less than the density of the cut from which the material was taken, there results an increase in volume which is termed *swell*. For soils, the amount of such swell is as indefinite as is the amount of shrinkage, and in fact is part of the same question. For solid rock, such swell is known to exist, and the amounts for given conditions are fairly well known.

It has been generally observed, however, that a cut in earth of a given volume, as determined by slope stake measurements, will not make an embankment of equal volume after it has thoroughly settled. This fact, in all probability, has been responsible for the belief that soils shrink. This *apparent shrinkage* is due not to an actual increase in the density of the soil, but (1) to loss of earth in transportation from cut to fill, (2) to a loss due to material being washed away by rains during the construction and settlement periods, (3) to more or less subsidence, compression, or displacement of the foundation due to the weight of the embankment, and (4) to certain uncompensated errors in field work and computations.

Because of this wastage or loss of material, it is necessary in balancing cuts and fills to multiply the fill volumes by a coefficient to determine the volumes of cut necessary to make the required fills. This coefficient is commonly termed the *shrinkage factor*. A more correct term would be *wastage factor*, although it includes any true shrinkage that occurs. If this factor is less than unity there is a net swell, if greater than unity there is wastage or apparent shrinkage. For solid rock, where the actual swell is considerable and the losses during construction are comparatively small, the shrinkage factor (or swell factor) should be from 0.5 to 0.8; and for soft and loose rock, from 0.7 to 1.0. For ordinary soils the factor is from 1.0 to 1.3. The shrinkage factor is often expressed as the percentage by which the fill volumes are to be increased, or decreased, to determine the equivalent cut volumes.

When cuts and fills are shallow, the loss of earth during construction is relatively high, as are also the errors in field work and computations. Consequently, the wastage or shrinkage factor must be larger for shallow cuts and fills than for deep ones. In ordinary

soils the factor should be about 1.3 for depths less than about 2 ft., and should decrease proportionally to about 1.1 for depths over 10 ft.

236. Settlement. A newly-built fill is never thoroughly compact and, therefore, will continue to settle for some time, possibly a year or two. The difference in height between the newly-built fill and the fill after it has fully settled is termed *settlement*. The amount of settlement depends on the nature of the filling material, the amount it is loosened during excavation, the method of forming the embankment, and the character of the ground or foundation upon which the fill is built. A fill of ordinary soil, well made with horses and scrapers or wagons, will settle about 5%. If the same material is simply dumped from a trestle, the settlement will be greater and in some cases may reach 15%. Rock, gravel, and sand fills will settle but little. These values apply when the embankment is made on firm ground. When the embankment is made on swampy or other unstable ground, the settlement will be much greater due to subsidence of the foundation. Obviously, the amount of subsidence is difficult, if not impossible, to estimate in advance.

Allowance for settlement is made in the field by marking on the slope stake the theoretical fill plus the allowance for settlement. For example, if the fill is 10 ft. and the settlement is assumed to be 5%, the slope stake is set at the proper point for a fill of 10 ft., *but the stake is marked* F 10.5. This means that the fill is finished 5% high, so that, when the soil has reached its final density, the top of the embankment will be at the desired elevation.

If a fill is short and high, the full allowance for settlement may make a hump in the grade line which may be very objectionable from the standpoint of operation. Of course this hump would ultimately disappear, but it may be preferable to avoid the hump by reducing the allowance for settlement and later to bring the track up to grade by the addition of ballast; or on paved highways to leave a gap in the pavement, filled in with stone or gravel, until the fill has settled, when the pavement can be completed.

In case a bridge is placed in a high fill, the full allowance for settlement can not be made at this point. Extra care should be taken to compact the fill immediately behind the abutments in order to reduce the amount of settlement, and the roadbed should be carefully maintained at these points until the embankment has fully settled.

Experiments have been made in *puddling* highway fills to secure

immediate compaction and to eliminate later settlement. This puddling may be accomplished either by thoroughly wetting down the fill as it is being built, or by saturating the entire embankment after it has been constructed in the ordinary way. During 1925, 1926, and 1927, the Illinois Division of Highways jetted a large number of newly made fills with almost uniformly satisfactory results. After the embankments had been built in the usual way, holes were bored in the roadbed at intervals of 6 to 10 ft. each way and extending to within 1 or 2 ft. of the natural ground surface. Water was then pumped into the holes until the entire fill was saturated, care being taken that the water did not break out through cracks or fissures and cause erosion. Settlement of the embankment always occurred during and immediately following the jetting, and practically none took place after the soil had dried out to normal moisture content. Where this method is used close up to bridge abutments, care must be taken to prevent excessive pressures against the abutments by the saturated soil during the puddling process.

Cross-Sectioning

237. Slope Stakes. Before construction work can be commenced, slope stakes must be set for the guidance of the contractor (1) at every full station, (2) at those intermediate points where the

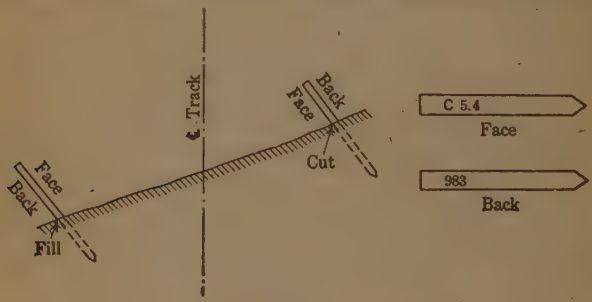

Fig. 105.

longitudinal slope of the ground changes, and (3) at other points required by special structures.

A *slope stake* is a stake that is set on each side of the center line at the point where the side slope of the cut or fill will intersect the ground surface. The points where the slope stakes are driven are

points of no cut or no fill. The vertical distance from the ground at the slope stake to the plane of the subgrade, however, is loosely referred to as the cut or the fill at the slope stake and is marked on one side of the stake. The station number is marked on the other side of the stake. The numbers are written lengthwise of the stake beginning at the top, and the cut or fill is given in feet and tenths, prefixing *C* for cut and *F* for fill. The stakes are driven with the tops slanting outward, and the sides upon which the cuts or fills are marked facing the roadbed, as shown in Fig. 105.

238. Cross-Sectioning. Cross-sectioning consists in setting the slope stakes, which mark the limits of excavation or embankment, and in taking sufficient elevations between the slope stakes to compute the area of the section to be excavated or filled in.

The data needed for cross-sectioning are the subgrade elevation at every station, the width of the roadbed in both cut and fill, the side slopes for different materials, and the elevations of all bench marks. The subgrade elevation for every station on the entire line can be obtained from the profile and should be written in the cross-section notebook opposite its station—as shown in the form of notes in Fig. 107—and carefully checked before any cross-sectioning is done. Also the description and elevation of every bench mark that was established during the location should be written in the back of the notebook.

239. Field Work. Starting with the nearest bench mark, differential levels are run to the part of the line that is to be cross-sectioned. The back sights, fore sights, heights of instrument, and elevations of turning points are recorded in the second column of the notes in such a way that the *H.I.* will be in the same line as the first station cross-sectioned from that set-up (see Fig. 107). The subgrade elevation in the third column is subtracted *algebraically* from the *H.I.* elevation in the second column and the difference is the *grade rod* and is recorded in the fourth column. A rod reading is then taken to the nearest tenth of a foot *on the ground* at the foot of the center stake. *All the rod readings are considered negative.* The rod reading and the grade rod are added *algebraically* and the sum represents the cut or fill at the point where the reading is taken, *minus* indicating fill and *plus* indicating cut. The cut or fill is marked on the back of the center stake and is recorded in the notes as the numerator of a fraction whose denominator is zero.

If the ground is level transversely of the line, the cut or fill at the slope stake will be the same as at the center, and the distance from

the center stake to the slope stake is found by multiplying the center cut or fill by the ratio of the side slopes and adding one-half the width of the roadbed. Such a section is called a *level section* (see Fig. 106*a*).

If the ground is not level transversely, the cut or fill will be different for various points, and the problem is to find the point on each side of the center line whose distance from the center is equal to the cut or fill at that point multiplied by the slope ratio and added to one-half the roadbed. This is necessarily a cut-and-try method, but proficiency is soon attained in approximating the correct position of the slope stake, and it is seldom that more than three

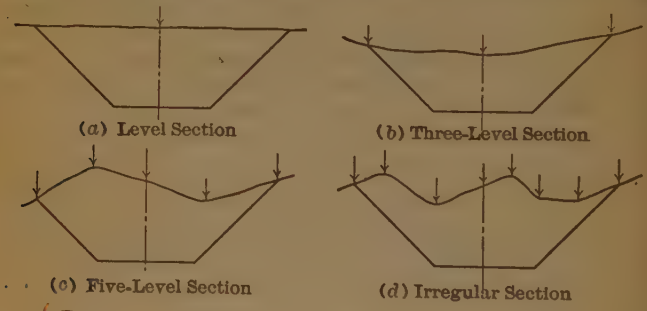

FIG. 106.

trials are required. The cut or fill is marked on the slope stake and is recorded in the notebook as the numerator of a fraction whose denominator is the distance out from the center. *Three-level*, *five-level*, and *irregular sections* come under this heading.

240. Illustrative Example. In Fig. 107 are shown the three typical cases which arise in setting slope stakes. Fig. 107*a* shows a cut. In this case the height of instrument is always above both the ground surface and the subgrade of the proposed roadbed, and, therefore, when the elevation of the subgrade is subtracted algebraically from the height of instrument, the difference, which is the grade rod, is always positive. In this particular instance the grade rod is +16.6 ft. The rodman now holds the leveling rod on the ground at the foot of the center stake, and the levelman obtains a rod reading of 7.3. As stated in the preceding section, rod readings are always considered negative. The algebraic sum of the rod reading, −7.3, and the grade rod, +16.6, is +9.3, which is the center cut.

In the notes, the plus sign is used to indicate cuts and the minus sign to indicate fills. The slope stakes, however, are marked "C" and "F," so as to avoid any misunderstanding on the part of the contractor. Having found the center cut to be 9.3 ft., the rodman now turns to the left, at right angles to the center line, and estimates what the cut at the slope stake will be. Of course, this estimate will be more or less in error. Suppose the rodman estimates the

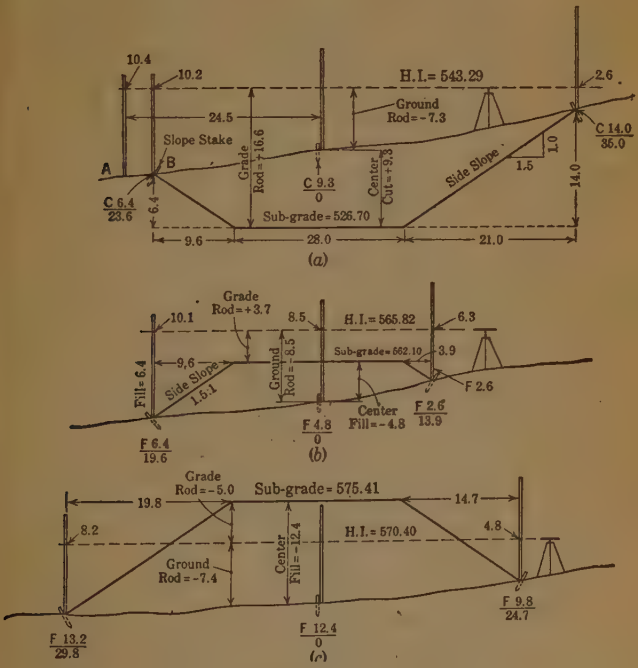

FIG. 107.

cut at the slope stake to be 7.0 ft. He multiplies this value by 1.5, which is the slope ratio, and the product is 10.5; to this he adds one-half the width of the roadbed, or 14.0 ft., and the sum is 24.5 ft. He then measures out this trial distance from the center and gives the levelman a reading at this point, *A*. Assume this rod reading to be 10.4. This value when subtracted (added algebraicly)

CROSS-SECTION NOTES.

Station	Turning Points	Grade	Grade Rod	L	C	R
55	740.51	727.50	13.0	+9.0	+9.8	+10.2
	−11.63			27.5	0	29.3
	728.88					
	+0.72					
56		727.00	13.5	+7.3	+7.3	+7.8
				25.0	0	25.7
57		726.50	14.0	+5.5	+6.0	+6.4
				22.3	0	23.6
58		726.00	14.5	+4.2	+4.4	+4.4
				20.3	0	20.6
59	729.60	725.50	4.1	+2.2	+2.7	+ 3.4
	−10.54			17.3	0	19.1
	719.06					
	+2.63					
10560		725.00	4.6	+1.0	+1.2	+ 1.4
				15.5	0	16.1
				Grade Point +65		
61		724.50	5.1	−1.2	−0.6	− 0.8
				10.8	0	10.2
62		724.00	5.6	−3.2	−2.4	− 2.1
				13.8	0	12.2
63		723.50	6.1	−4.8	−3.8	− 3.5
				16.2	0	14.3
64		723.00	6.6	−6.0	−5.2	− 5.0
				18.0	0	16.5
65	721.69	722.50	0.8	−6.8	−6.6	− 6.6
				19.2	0	18.9
66		722.00	0.3	−8.5	−7.8	− 7.0
				21.8	0	19.5

FIG. 108.

End Areas		Volumes		Mass Curve Volumes
Cut	Fill	Cut	Fill	
413				3653
		1303		
291				2350
		948		
221				1402
		687		
150				715
		442		
88				273
		230		
36				43
		43		
0.0	0.0			00
	15		10	10
			130	
	55			140
			278	
	95			418
			434	
	139			852
			602	
	186			1454
			772	
	231			2226

FIG. 108.

from the grade rod, 16.6, gives a cut of 6.2 ft.; therefore, the rodman's assumption of a 7.0-ft. cut was in error. He now uses the new cut value of 6.2 ft. at A as a guide. He multiplies this by 1.5 and adds 14.0, and obtains a sum of 23.3. That is, 23.3 ft. from the center stake is the correct distance for a cut of 6.2 ft.; but the point A at which the reading was taken is out 24.5 ft. from the center, hence A can not be the correct position of the slope stake, although it is within a foot of the correct position and beyond it. The rodman then moves the rod toward the center 1 ft. to the point B which is 23.5 ft. out from the center stake. At point B, the rod reading is 10.2, which gives a cut of 6.4 ft. The proper distance out for a cut of 6.4 ft. is $6.4 \times 1.5 + 14.0 = 23.6$ ft. This computed distance is only 0.1 ft. more than the measured distance of 23.5 ft. No more readings are necessary because the rod can be moved out 0.1 ft. without changing the rod reading of 10.2. The slope stake is marked C 6.4 and is driven 23.6 ft. from the center stake. The slope stake on the right is located similarly after one or two trial readings.

Fig. 107*b* illustrates a fill of which the finished roadbed is below the height of the instrument. In this case the grade rod will always be less numerically than the rod readings on the ground. The grade rod in this problem is $+3.7$, the rod reading at the center stake is -8.5, and their algebraic sum is -4.8, the minus sign indicating fill. The positions of the slope stakes are found by trial as previously explained.

In Fig. 107*c*, the finished roadbed is above the height of the instrument, and therefore the grade rod has a negative value. Since the rod readings are always considered negative, the fill in this instance is the arithmetical sum of the grade rod and the ground rod. Otherwise, this case is similar to the preceding ones.

241. Sections at Grade Points. Fig. 109 shows the junction between a cut and a fill. Stakes, marked "grade," are driven at

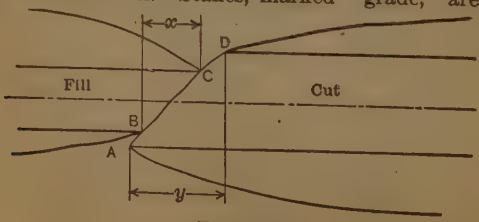

FIG. 109.

A, *B*, *C*, and *D*, which are grade points. Cross-sections are taken at *B* and *D*, and the plusses of *A*, *B*, *C*, and *D* are determined.

Area of Cross-Sections

242. Areas. Before the volume of earthwork can be computed, the area of each cross-section taken in the field must be found. This is done as follows:

In Fig. 110, it is desired to find the area of the figure *ABCEFGHK*. The cuts at the points, *A*, *B*, *C*, *E*, *F*, *G*, *H*, and *K* and their

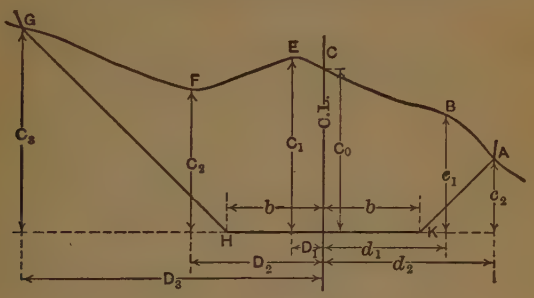

FIG. 110.

distances from the center line have been determined in the field and recorded in the field notebook as fractions, thus:

$$\frac{0}{-b+} \quad \frac{C_3}{-D_3+} \quad \frac{C_2}{-D_2+} \quad \frac{C_1}{-D_1+} \quad \frac{C_0}{0} \quad \frac{c_1}{+d_1-} \quad \frac{c_2}{+d_2-} \quad \frac{0}{+b}$$

It will be observed that each fraction consists of the coordinates of the respective points in the section. For example, in the fraction $\frac{0}{-b}$, 0 and $-b$ are the ordinate and the abscissa, respectively, of the point *H* referred to the center line as a vertical axis and the line of the roadbed *HK* as a horizontal axis. Likewise, C_3 and $-D_3$ are the coordinates of the point *G*, and C_2 and $-D_2$ are the coordinates of point *F*, and so on. Since the ordinates are *all above* the horizontal axis, *HK*, their signs are positive; and the abscissas to the left of the vertical axis, or center line, are negative and those to

the right are positive. The opposite sign is written arbitrarily after each denominator. The area is then found by the following rule:

RULE 1:—To find the area (1) multiply each numerator by the algebraic sum of its adjacent denominators, using the sign of the denominator which faces the numerator, and (2) divide the algebraic sum of the resulting products by 2.

Or,

$$\text{Area} = \frac{1}{2}[C_3(b - D_2) + C_2(D_3 - D_1) + C_1(D_2 + 0)$$

$$+C_0(D_1 + d_1) + c_1(0 + d_2) + c_2(b - d_1)] \quad (96)$$

(Let the student prove this equation geometrically.)

After the student becomes familiar with the rule, it will not be necessary for him actually to write down the plus and minus signs in the notes.

If the section is in fill, the numerators are ordinates measured downward from the horizontal axis through the roadbed, and hence they would *all* have minus signs. Since the effect of using minus signs in the numerators, however, would be simply to change the sign of the answer in Rule 1, without changing its numerical value, it is customary to consider the numerators as always positive.

Eq. 96 can be used for either cut or fill, and for level, three-level, five-level, and irregular sections.

For three-level sections, Eq. 96 reduces to a more simple form, as follows:

$\frac{0}{-b+}$	$\frac{C_1}{-D_1+}$	$\frac{C_0}{0}$	$\frac{c_1}{+d_1-}$	$\frac{0}{+b-}$

$$\text{Area} = \frac{1}{2}[b(C_1 + c_1) + C_0(D_1 + d_1)]$$

or expressed as a rule: The *area of a three-level section* is equal to the sum of (1) one-fourth the width of the roadbed multiplied by the sum of the slope-stake cuts or fills, and (2) the center cut or fill multiplied by one-half the distance between slope stakes.

243. Side-Hill Sections. When a section is partly in cut and partly in fill, as illustrated in Fig. 111, the field work and the computation of area are slightly modified. After the fill (or cut) at the center has been determined, the *grade point*, A, is found by

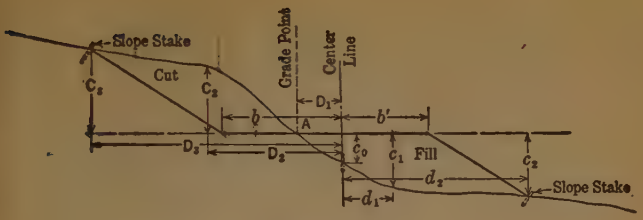

FIG. 111.

trial, and a stake marked "grade" is driven at that point. The notes for the section in Fig. 111 would be recorded as follows:

$\frac{0}{-b+}$	$\frac{C_3}{-D_3+}$	$\frac{C_2}{-D_2+}$	$\frac{0}{-D_1+}$	$\frac{c_0}{0}$	$\frac{c_1}{+d_1-}$	$\frac{c_2}{+d_2-}$	$\frac{0}{+b'-}$

<—————CUT—————×—————FILL—————>

It is to be noted that the denominators of the end fractions are not alike. This is caused by the fact that the center line of the track does not come in the center of the roadbed. The roadbed to the left of the center stake is wider than that to the right to allow for a ditch in the cut.

The areas of the cut and the fill are computed separately and are recorded separately in the notes. The fraction $\frac{0}{D_1}$ is used in computing both cut and fill.

244. Uncompleted Sections. It is sometimes necessary to find the area of an uncompleted section—for example, for monthly estimates—or of an irregular-shaped borrow pit. The field work for this does not require that the height of instrument be determined at each set-up of the level nor that the actual elevations of breaks in the cross-section be computed. The procedure at each section is to consider the line of sight through the telescope as the horizontal axis of coordinates; and, therefore, the rod readings taken at the breaks in the cross-section are the ordinates of these points. The vertical, or Y-axis, from which the horizontal distances, or abscissas, are measured may be taken through any point, although for convenience it is generally taken through the point at the

extreme left. Fig. 112 shows the field measurements required to compute the area of the uncompleted section, *ABCEFGHI*. The

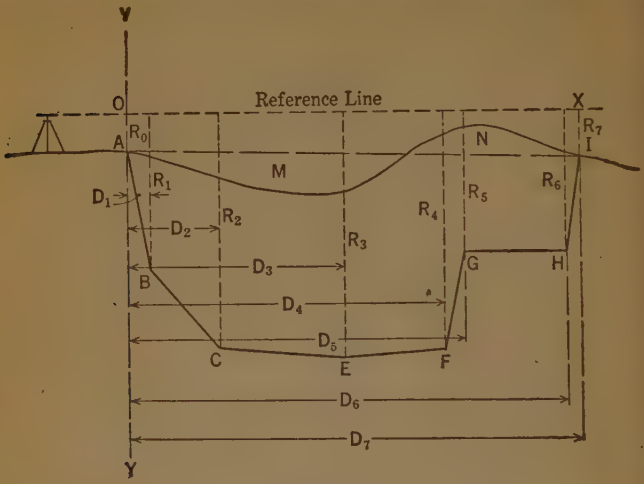

FIG. 112.

notes are written in the same manner as in the preceding problem, and are as follows:

$$\frac{0}{0}\ \frac{R_0}{0}\ \frac{R_1}{+D_1-}\ \frac{R_2}{+D_2-}\ \frac{R_3}{+D_3-}\ \frac{R_4}{+D_4-}\ \frac{R_5}{+D_5-}\ \frac{R_6}{+D_6-}\ \frac{R_7}{+D_7-}\ \frac{0}{+D_7}$$

When Rule 1 is applied to this series of fractions, the area obtained is that of the figure *OABCEFGHIX*. The area desired, however, is that of *ABCEFGHI*; hence, the area *OAIX* must be subtracted. This latter area is equal to $\frac{1}{2}D_7(R_0 + R_7)$. Instead of solving for the desired area in two steps, however, the area can be obtained directly from Rule 1, if the fraction $\frac{R_0}{0}$, the coordinates of point *A*, be substituted in the notes for the right-end fraction $\frac{0}{D_7}$. The series of fractions then becomes

$$\frac{0}{0}\ \frac{R_0}{0}\ \frac{R_1}{+D_1-}\ \frac{R_2}{+D_2-}\ \frac{R_3}{+D_3-}\ \frac{R_4}{+D_4-}\ \frac{R_5}{+D_5-}\ \frac{R_6}{+D_6-}\ \frac{R_7}{+D_7-}\ \frac{R_0}{0}$$

The area obtained in the above solution is the area of the cut below the line AI, and does not represent the actual amount of earth removed by the contractor, unless the area M below the line AI—and above the original ground surface—equals the area N above the line AI. Whether area N is equal to, less than, or greater than the area M can be easily determined from the original cross-

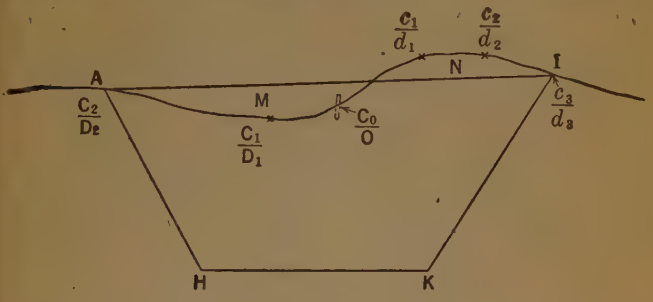

FIG. 113.

section notes. Fig. 113 shows the original field data for this section. The original cross-section notes should be recorded thus,

$$\frac{c_3}{0} \quad \frac{C_2}{-D_2+} \quad \frac{C_1}{-D_1+} \quad \frac{C_0}{0} \quad \frac{c_1}{+d_1-} \quad \frac{c_2}{+d_2-} \quad \frac{c_3}{+d_3-} \quad \frac{C_2}{0}$$

Observe the end fractions in this series.

When Rule 1 is applied to this series of fractions, the answer is the difference in the areas of M and N. If the sign of the answer is negative, M is greater than N, and vice versa. This difference is a correction which must be applied to the area of the uncompleted section, as determined above, to obtain the area of the true section.

245. Planimeter Method. Another method of obtaining the areas of cross-sections is to plat them to scale and use a planimeter for determining the area. This is a long and expensive method and is justified only when a graphical record of the sections is desired or when the individual sections may require detailed study. This latter condition frequently occurs in highway work. It is never necessary to plat the sections, regardless of how irregular they may be, if the area only is desired.

246. Consistent Accuracy. Since the cuts, or fills, and the horizontal distances are measured to the nearest tenth of a foot only, the

areas of the sections computed from them might very easily differ by several square feet from the areas of the sections as actually constructed. The areas of sections, therefore, should not be computed closer than the nearest square foot.

Volumes of Earthwork

Earthwork is paid for by the cubic yard, and in order to compute the cost it is necessary to find the volume of the material handled. The total volume is the sum of the volumes of the *prismoids* formed by adjacent cross-sections. The volume of the prismoid between two adjacent cross-sections may be found (1) by the end area method, or (2) by the prismoidal method.

247. End Area Method. In this method the prismoid is treated as a *prism*, whose cross-section is the mean of the two end areas of the prismoid.

$$v \text{ (cu. yds.)} = \frac{l}{27} \times \frac{A_1 + A_2}{2} \qquad (97)$$

where v is the volume, A_1 and A_2 the end areas in square feet, and l is the distance between cross-sections in feet.

For $l = 100$ ft.,

$$v = 1.85(A_1 + A_2) \qquad (97a)$$

This method is *exact* when the end areas are equal, but is only *approximate* when they are unequal. The result is usually too large and the maximum error occurs when one end area is zero and the other end area is large. When the latter section has a cut of 20 ft., the error is about 8%.

Owing to its simplicity, and to the fact that sections usually do not change rapidly, *this method is almost universally used.*

Table 19 gives the cubic yards per 100 ft. for center depths from 1 to 60 ft.

Table 20 gives the cubic yards per 100 ft. for end areas from 1 to 1000 sq. ft.

248. Prismoidal Method. When the changes from cut to fill are frequent and abrupt or the work is in rock, greater accuracy may be required; in this case the prismoidal method is used.

From Simpson's rule,

$$\text{Mean area of a prismoid} = \frac{A_1 + 4A_m + A_2}{6},$$

where A_1 and A_2 are the end areas and A_m the area of a section midway between the ends. Then,

$$v = \frac{l}{27} \times \frac{A_1 + 4A_m + A_2}{6} \qquad (98)$$

This is called the *prismoidal formula*. This form, however, is inconvenient to use since it requires the area of a mid-section. It is therefore more convenient to find the difference between the prismoidal and end area formulas and to apply this difference to the latter. Subtracting the end area formula from the prismoidal formula and reducing,

$$\text{Prismoidal Correction} = -\frac{l}{81}(A_1 - 2A_m + A_2)$$

Putting A_1, A_2, and A_m in terms of their slope stake dimensions—remembering that the dimensions of A_m are the average of those of A_1 and A_2—and reducing, we have

$$\text{Prism. Corr.} = \frac{l}{12 \times 27}(C_1 - C_2)(W_2 - W_1) \qquad (99)$$

in which C_1 and C_2 are the center cuts or fills at A_1 and A_2, and W_1 and W_2 are the corresponding distances between slope stakes. When

$$l = 100 \text{ ft., Prism. Corr.} = 0.31(C_1 - C_2)(W_2 - W_1) \qquad (99a)$$

This correction is added *algebraically* to Eq. 97. The result is exact for three-level sections and is sufficiently accurate for any other form of cross-section.

249. Earthwork on Curves. The volume on curves is generated by a section moving perpendicular to the center line of the curve. The section considered is the *mid-section* of the solid. The distance traveled by this section is the distance traveled by its center of gravity, and it is only when the center of gravity lies in a vertical line passing through the center line of the curve that the volume as computed by the prismoidal method is exact. If the center of gravity is inside the center line, the results obtained by the prismoidal method are too large; if outside, they are too small. The difference between the true volume and the volume determined by the prismoidal method is called the *curvature correction*.

If the volumes are computed by the end area method, it is inconsistent to apply the curvature correction; but if the prismoidal method is used, the curvature correction should be applied, especially in rock work.

The curvature correction is computed as follows:

Figure 114 represents the *mid-section* of the solid. Construct BC with the same slope as CF. Then the center of gravity of the part shown shaded is in a vertical line through C and therefore there is no correction to be applied to this portion. The center of gravity

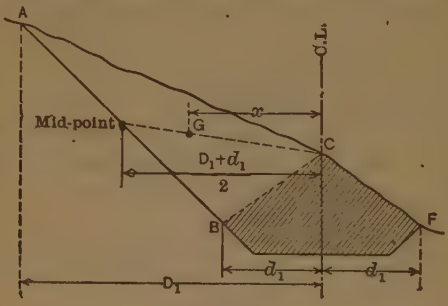

FIG. 114.

of the triangle ABC is at G, which is the distance x from the center, C.

The volume generated by the triangle ABC is

$$v = ABC \times (R + x) \times I$$

in which I is the angle at the center expressed in radians.

The volume as ordinarily computed, using the center line distance, is

$$v_1 = ABC \times R \times I$$

The curvature correction then is

$$C.C. = v - v_1 = ABC \times x \times I$$

But $x = \frac{2}{3} \times \frac{1}{2} \times (D_1 + d_1)$. Substituting this value of x and expressing I in degrees, we have

$$C.C. \text{ (in cu. ft.)} = 0.006ABC \times (D_1 + d_1) \times I^\circ \qquad (100)$$

For 100 ft., I becomes D, the degree of the curve.

Haul and Overhaul

250. Haul. Haul is the work done in transporting the material from the excavation to its place in the embankment. Its unit is the yard-station, or the work done in transporting one cubic yard 100 ft. The term *haul* is frequently used to indicate either the yardage or the distance, but this must not be confused with the true meaning.

Contract prices are usually based on a maximum length of haul—generally 500 ft.—called the *free-haul distance.* If the material is transported a longer distance the extra work involved is termed *overhaul* and the contractor is allowed extra compensation at a certain price per yard-station. The contractor is therefore paid for all excavation and for overhaul, but *he is not paid for making the fills nor for haul inside the free-haul distance.*

The amount of excavation is readily determined from the cross-section notes, hence this portion of the contractor's remuneration presents no particular difficulties except when the excavation involves several different kinds of materials to be paid for at different prices. The amount of overhaul, however, depends on the plan of distributing the excavated material, and the problem is to determine an economical distribution. This problem is not one of great difficulty, but unfortunately it has often been neglected, resulting in uneconomical work, misunderstandings with the contractor, and frequent law-suits.

The *limit of profitable haul* is that distance at which the cost of overhaul equals the cost of excavating a yard of earth. For example, if the contract price is 32 cents a cubic yard for excavation and 2 cents a yard-station for overhaul, and the free-haul distance is 500 ft., the limit of profitable haul is $32 \div 2 + 5 = 21$ stations. A yard of earth excavated, hauled 700 ft., and dumped into the fill costs the railroad company 32 cents plus 4 cents overhaul. If the yard excavated had been hauled only a few feet to one side of the cut and dumped, and *another* yard of earth had been taken from a borrow pit alongside of the fill and placed in the fill, the cost would have been 64 cents. It is evident, therefore, that it is much cheaper for the company to pay the additional amount for overhaul. In this instance the saving was 28 cents. If the haul had been longer, the saving would have been less. But the company saves money up to the point where the cost of overhaul equals the cost of making a yard of fill from a borrow pit.

251. Extra Right of Way. The preceding discussion is based on the assumption that the necessary borrow or waste can be made along the right of way within the free-haul distance of the particular operation. Sometime this can not be done, and extra right of way for borrow pit or spoil bank must be obtained. This adds to the cost of the work and thus may serve to lengthen the limit of profitable haul. The net value of the extra right of way, which is its first cost less its value after being used for borrow or waste, divided by the number of cubic yards it provides, gives the unit cost per cubic yard of the borrow or the waste which it supplies. This cubic yard price divided by the overhaul price is the additional distance which may be added to the limit of profitable haul. If the earthwork can be done on the normal right of way within this new limit of haul, the new right of way will not be required.

Mass Diagram

It is very difficult to determine haul and overhaul or the economical distribution of the earth directly from the cross-section notes. The problem, however, can be readily solved graphically by means of the *mass diagram*, which might more appropriately be termed the *distribution diagram.*

The basis of the mass diagram is the *mass curve*, which may be defined as a graph whose abscissas are distances and whose ordinates are volumes of earthwork. The manner of constructing the mass curve and the methods of using it in solving the earthwork distribution are given in following sections.

In computing the ordinates to the mass curve, called *mass ordinates*, allowance must be made for shrinkage or swell. This is done by changing the fill volumes to equivalent cut volumes, i.e., the volumes of cut required to make the fills.

Under conditions where both cut and fill occur at the same station, the earthwork is first balanced laterally, and the residual amounts of cut and fill used in making the mass curve. For example, if between Stas. 50 and 51 there are 2400 cu. yd. of cut and 1500 cu. yd. of fill (corrected for shrinkage), only the net cut of 900 cu. yd. would be used in the mass curve.

For simplicity in explanation, and to illustrate special procedures, the mass diagram will be considered under four cases as follows:

1. Where the cuts and fills in general do not greatly exceed the limit of profitable haul.

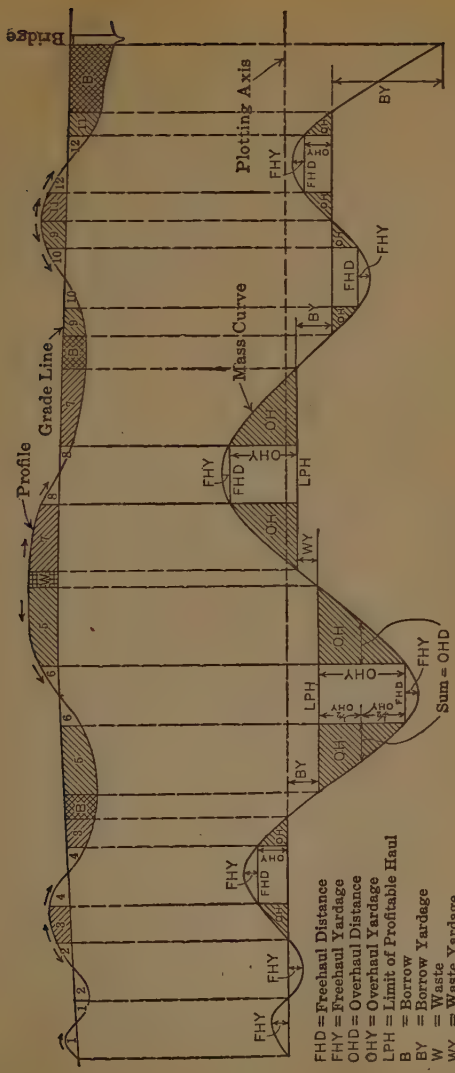
Bridge
Plotting Axis
Grade Line
Profile
Mass Curve
FHY
FHD
OHD
OHY
LPH
BY
WY
FHD = Freehaul Distance
FHY = Freehaul Yardage
OHD = Overhaul Distance
OHY = Overhaul Yardage
LPH = Limit of Profitable Haul
B = Borrow
BY = Borrow Yardage
W = Waste
WY = Waste Yardage
Numbers and arrows show earth movements
Sum = OHD

FIG. 115.

2. Where a short fill lies between two long cuts, or a short cut lies between two long fills.

3. Where a long fill lies between two cuts, or a long cut lies between two fills.

4. Where the cuts and fills are so short that earth may be moved from one cut across a fill and through another cut to a fill beyond.

252. Case 1. This might be considered the general case. The mass curve is platted continuously from some point past which earth will not be moved, such as the beginning of the line or a stream crossing, to another point past which earth will also not be moved. This is shown in Fig. 115.

The first step in constructing the mass curve is to prepare a table of mass ordinates. In the first column is listed the stations from the cross-section notes, beginning at the chosen point. In the second column, opposite each station, is recorded the amount of cut between it and the preceding station. The third column gives the fills, corrected for shrinkage or swell. Assuming that cut and fill do not occur at the same station, the fourth and last column gives the mass ordinates found by successive algebraic additions of the values in the cut and fill columns, considering cut as positive and fill as negative. If cut and fill do occur at the same stations, then two additional columns are inserted before the mass ordinate column, in which are recorded the respective net cut or fill at the respective stations, and these values are used in computing the mass ordinates.

The second step is to plat the mass curve. A suitable horizontal base line is chosen on the profile sheet. From this base line the mass ordinates are platted at their respective stations to some suitable scale of cubic yards per inch. These points are then connected by a smooth curve which is the mass curve.

Since the cuts have been considered positive and the fills negative, it is evident that the mass curve must rise over cuts and descend over fills. Consequently, if any horizontal line, called a *distribution line*, is drawn cutting the mass curve at two points, the earthwork between these two points is balanced, since the curve rises and falls the same amount. It is also evident that the maximum ordinate from this distribution line to the curve represents the total cut (or fill) between the two points. Furthermore, since the mass curve is a curve between distances in stations and earth quantities in cubic yards, it follows that the area between the distribution line and the curve represents haul in yard-stations. These characteristics form the basis of the use

of the mass curve in determining economical distribution, balance points, borrow, waste, and overhaul.

The first step in using the mass curve is to determine the freehaul limits and quantities. At each of the sags or summits of the mass curve a horizontal line equal in length to the freehaul distance is drawn with its ends in the mass curve. If several adjacent cuts and fills are short, several of these lines may connect with some sections less than the freehaul distance in length. The points where these distribution lines cut the curve are projected vertically onto the profile, where they show the points between which earth is distributed without overhaul.

The second step is to make use of the limit of profitable haul in a similar manner. In doing so several distribution lines shorter than the limit of profitable haul may join together. On the other hand, there may be a gap between ends of adjacent distribution lines. In this case the vertical distance between the ends represents borrow or waste, depending on whether the curve is falling or rising. The horizontal distance between the ends gives the distance in which borrow or waste is required. The ordinate between any distribution line as just constructed and the freehaul distribution line corresponding to it gives the number of cubic yards on which overhaul will be charged. This process is continued until the entire mass curve is accounted for in freehaul yardage, overhaul yardage, borrow, or waste. It should be remembered that no part of the mass curve may be used twice.

If cut and fill do not occur at the same station, the summits and sags of the mass curve will be directly below the grade points on the profile. If, however, both cut and fill do occur at the same stations, then the summits and sags will not be directly below the profile grade points. This does not affect the use of the mass curve except that the distributions can not be shown on the profile as mentioned above.

The next step is to determine the amount of overhaul but, in order to make use of the larger diagram in Fig. 116, this is explained under Case 2.

Sometimes, especially under "lump sum" contracts, the freehaul distance is omitted and *balance points* determined within a fixed distance which corresponds to the limit of profitable haul. These balance points are shown on the profile, and borrow and waste are determined as explained above.

253. Case 2. Figure 116 represents a fill between two cuts where it is readily seen that there is more than enough earth in the two cuts within the limits of profitable haul to make the fill, and the question

is how much of the fill should be made from each cut. Since the cut to the right is the deeper, it is evident that the fill can be made with a minimum of haul, if the greater volume of earth is taken from the right-hand cut. The problem is to locate the points A, E, and J, which are the limits of the distribution from the two cuts, for economical movement of the earth.

In this case the mass curve is constructed somewhat differently from the method in Case 1. Beginning at the point C' on a suitable base line, the cumulative cuts are platted to the left past A'. Beginning also at C', the cumulative fills are platted upward to the right past E'. In a similar manner a curve is platted from G' with fills to the left and cuts to the right. These two curves correspond to what

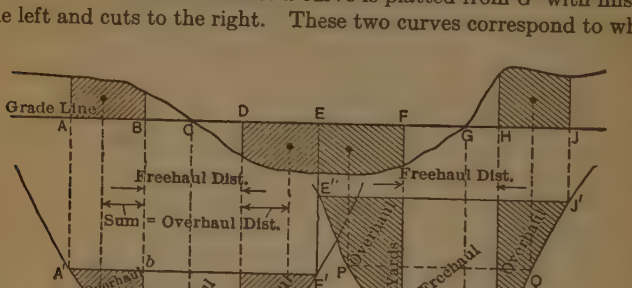

FIG. 116.

would have been a summit at C' and a sag at G', if the curve had been platted as in Case 1, and are used in the same way. The advantage in platting the curves in this manner, rather than as a continuous curve, is that in many cases a larger scale can be used in the platting without interfering with the profile.

The limits of freehaul are found by platting $B'D'$ and $F'H'$, each equal in length to the freehaul distance. These points are projected onto the profile and establish the points B, D, F, and H. The ordinate to $B'D'$ is the freehaul yardage past C, and the ordinate to $F'H'$ is the freehaul yardage past G.

The next step is to find the positions of A, E, and J so that the haul will be a minimum. For all practical purposes, under most conditions, this occurs when $AE = EJ$. To accomplish this, two distribution

lines, $A'E'$ and $E''J'$, of equal length are located by trial in such a manner that E' and E'' are in the same vertical line. These points are projected onto the profile to locate A, E, and J. These points should be located in the field and distinctly marked for the guidance of the contractor. The ordinates between $B'D'$ and $A'E'$ and also between $F'H'$ and $E''J'$ give the respective volumes on which there is an overhaul charge. The total volume moved is given by the sum of the ordinates to $A'E'$ and $E''J'$, respectively.

As stated under Case 1, the area between the distribution lines and the mass curve represents haul in yard-stations. Consequently the overhaul is given by the shaded portions between the curves and the lines $A'E'$ and $E''J'$. The amount of overhaul may be found by determining the area of the shaded portions with a planimeter and multiplying this by the product of the horizontal and vertical scales of the mass curve. This method is frequently used in highway work. It is rarely used in railroad work, the preference being for the method described in the following paragraph.

If the ordinate $B'b$ is bisected, dividing the overhaul yardage into two equal parts, and the horizontal line MN is drawn, the points M and N will for all practical purposes be directly under the centers of mass of the cuts and fill, respectively. Therefore the length of MN minus the freehaul distance is the *overhaul distance* corresponding to $B'b$. The overhaul can now be found by multiplying the cubic yards represented by $B'b$ by this overhaul distance. The overhaul past G is determined in a similar manner.

When two large fills are separated by a short cut, the cut may be distributed into the fills in a similar manner.

254. Case 3. In Fig. 117, C and G are so far apart and the cuts are so long that it is evident that the limit of profitable haul will be reached. Construct the mass curve and locate the freehaul limits as before. Lay off $A'E'$ and $K'J'$, making each equal to the limit of profitable haul. The centers of mass, M, N, P, and Q, are found as in Case 2. The points A', E', K', and J', projected onto the profile, give the points A, E, K, and J, which should be marked in the field.

The fill between E and K must be borrowed, and the yardage required is given by the difference in the ordinates to E' and K''. If the distance between E and K is long, there is no need of platting the mass curve to obtain the amount of borrow. In this case, the first curve is stopped at a convenient station beyond E' and the second curve at a convenient station ahead of K'. The amount of fill which is all borrow between these two stations is then computed directly

from the cross-section notes. The total amount of borrow is found by adding to this quantity the amounts given by the vertical ordinate from E' and K', respectively, to the horizontal line through the corresponding stations at which the curves are ended. The amount of overhaul is determined as in the preceding cases.

The complement of this Case is the determination of the waste in a long cut.

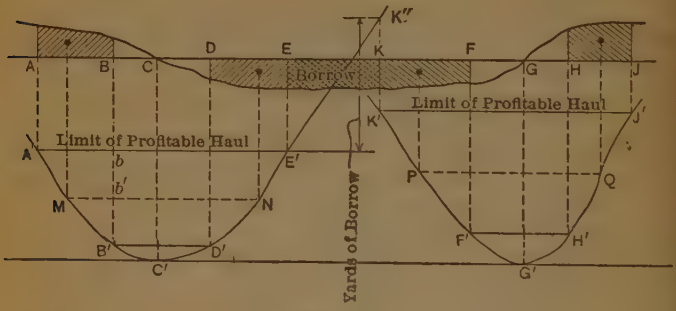

FIG. 117.

255. Case 4. It frequently happens that the cuts and the fills are short, and that earth is moved from one cut, across a short fill, through another cut, and deposited in the fill beyond. For these cases the construction of the mass curve is slightly modified, as is seen in Fig. 118, where one continuous curve is used instead of two separate curves as in the preceding cases. Because of the stream at A, all of the cut AC is to be moved to the right. The point A is projected to the base of the mass curve at A'. The curve $A'C'$ is a graphical summation of the total excavation as one advances from A to C. The ordinate at C' is the total yardage in the cut AC. The mass curve is continued from C' by subtracting the successive fill volumes through the fill CD. The end of the fill, D on the profile, is represented by point D' on the mass curve. The difference between the ordinates at C' and D' is the total yardage in the fill CD. The cut from D to E is represented on the mass curve by the curve $D'E'$ which is an ascending one. From E' the mass curve descends to J', which is the projection of the end of the fill at J. Thus a continuous curve is constructed. The curve rises for the cuts and falls for the fills. The crests and the troughs on the mass curve come under the grade points on the profile.

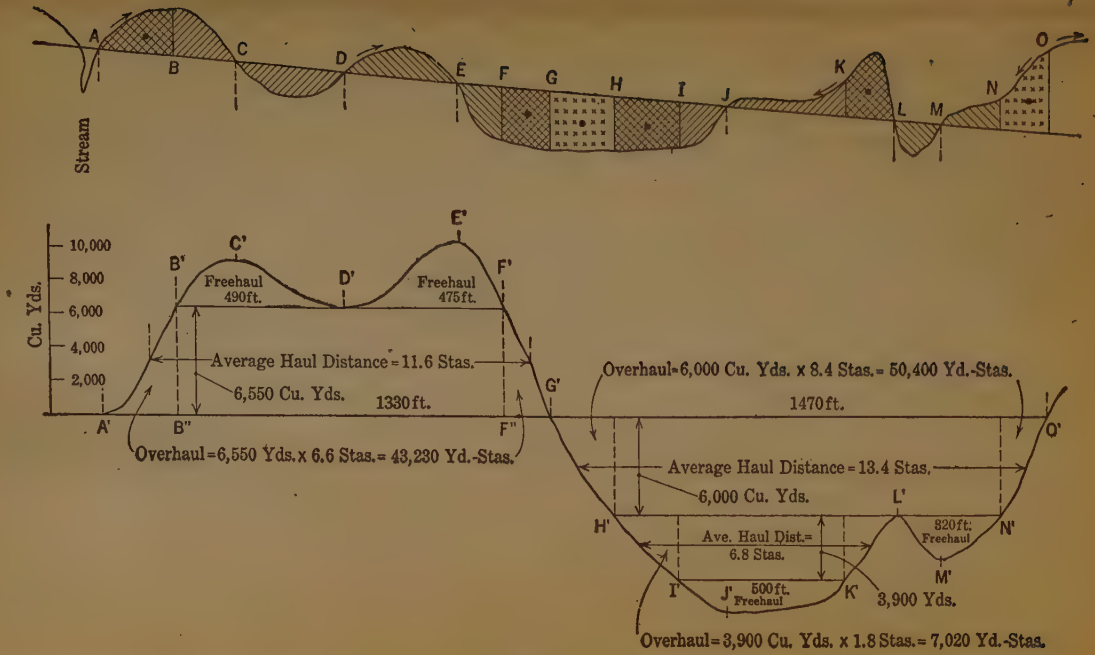
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
Stream
10,000
8,000
6,000
4,000
2,000
Cu. Yds.
A'
B'
C'
D'
E'
F'
G'
B"
F"
Freehaul 490ft.
Freehaul 475ft.
Average Haul Distance = 11.6 Stas.
6,550 Cu. Yds.
1330ft.
Overhaul = 6,550 Yds. x 6.6 Stas. = 43,230 Yd.-Stas.
Overhaul = 6,000 Cu. Yds. x 8.4 Stas. = 50,400 Yd.-Stas.
1470ft.
O'
Average Haul Distance = 13.4 Stas.
6,000 Cu. Yds.
H'
I'
J'
K'
L'
M'
N'
Ave. Haul Dist. = 6.8 Stas.
500ft. Freehaul
820ft. Freehaul
3,900 Yds.
Overhaul = 3,900 Cu. Yds. x 1.8 Stas. = 7,020 Yd.-Stas.

FIG. 118.

Since the excavation from the cuts AC and DE is to be placed in the fills CD and EJ, all the excavated material is moved to the right. A horizontal line through D' on the mass curve shows that it will take the material between B and C to make the fill CD, and that the maximum distance hauled is 490 ft.; hence, the fill CD is made free of charge. The horizontal line $D'F'$ shows, also, that the cut DE will make the fill to the right up to the point F; and, since the distance from D to F is only 475 ft., there is no charge for making the fill EF.

It is obvious that the material excavated between A and B should be hauled to the fill EJ. The question is how far will it go and what will be the overhaul charge. The first of these questions is answered by the horizontal line $A'G''$. The cut from AB is represented on the mass curve by the ordinate $B'B''$. Since $F'F''$ is equal to $B'B''$, the cut AB will make the fill FG. The ordinate $B'B'' = F'F'' = 6550$ yd. is the yardage upon which there is a charge. The average distance hauled—from center of mass of AB to center of mass of FG—is 11.6 stations, as shown on the figure. Since there is no extra charge for the first 500 ft., this overhaul distance is 6.6 stations. The overhaul is, therefore, 6550 cu. yd. times 6.6 sta., or 43,230 yd.-sta.

The horizontal 500-ft. line, $I'K'$, on the mass curve determines the points I and K on the profile, and the fill IJ is made from the cut JK without extra charge. The horizontal line $H'L'$ through the crest of the curve at L' shows that the material excavated between K and L will make the fill between H and I. The yardage $KL = HI$ is 3900 yd. as shown on the diagram. The average distance hauled is 6.8 stations; therefore, the overhaul on KL is 3900 yd. times 1.8 sta., or 7020 yd.-sta.

The horizontal line $L'N'$ equals 320 ft. and shows that the cut from M to N is required to make the fill LM, and that the fill is made free of charge. To determine how much of the cut to the right is needed to make the fill GH, and to complete the fill EJ, the horizontal line $G'O'$ is drawn. The point O' projected to the profile shows how much of the cut is required. The volume required is the ordinate between $G'O'$ and $H'N'$, and is 6000 cu. yd. The distance between centers of mass in this instance is 1340 ft. Subtracting the free-haul distance of 500 ft. leaves 840 ft., or 8.4 sta., upon which there is an overhaul charge. The overhaul is, therefore, 6000 cu. yd. times 8.4 sta., or 50,400 yd.-sta.

256. Uses of the Mass Diagram. The principal use of the mass

diagram is to balance cuts and fills within the limits of profitable haul. As shown in the preceding examples, if any horizontal line is drawn intersecting the curve in two points, the cuts and fills between these two points will balance. In highway grading it is especially desirable that the cuts and the fills balance, and the mass diagram provides the most convenient method of accomplishing this. First, a tentative grade line is established on the profile so that the cuts and the fills will balance as nearly as can be determined by inspection. The cut and the fill quantities are then computed and the mass curve constructed. From the diagram it can be quickly ascertained how nearly the cuts and the fills actually do balance, and what changes in the grade line are necessary to make them balance. The grade line is then changed and a new mass curve drawn. This process is repeated until the cuts and the fills balance satisfactorily.

Where a long high fill occurs, it may not be economical to make it from the adjoining cuts—assuming that there is enough material in the cuts to make the fill—because of the long haul distance. In this case the central portion of the fill will be made from material taken from borrow pits alongside the fill. Since the fill is high, runways will be required to enable the hauling equipment to reach the top of the fill; and, if the distance from the point of excavation in the borrow pit to the point of deposition in the fill is more than the free-haul distance, there will be an overhaul charge.

The question of the amount of overhaul has been a source of much controvery between railroads and contractors in the past. So much so that many railroads have dispensed with the overhaul clause and let contracts on a flat unit-price basis. The mass curve is used to determine the most economical distribution of earthwork and to indicate to the contractor where the material from the cuts is to go and where there will be borrow and waste. The contractor then adds enough to his bid for excavation to cover the cost of extra haul. It is also common practice in highway construction to show on the plans the balance points and the amounts of total cut, fill, borrow and waste, and then to ask for a lump-sum bid on the whole job, or sometimes a unit price per cubic yard, including all hauls.

257. Street Excavation. City pavements are normally in cut, consequently cuts and fills can not ordinarily be balanced. The excavated material is usually wasted on adjoining property, but if of unsuitable quality, or if not wanted by the property owners, it may be necessary to haul it considerable distances. It is custom-

ary, therefore, to consider the conditions applying to each separate project and to establish a free-haul distance so great that overhaul is eliminated.

It is customary to compute the excavation for a trench just wide and deep enough to receive the pavement. In modern practice the parkways are usually graded as part of the project, but the quantities are ordinarily computed separately. If the parkways are not graded and the cuts to the top of curb are more than 3 or 4 ft., side slopes may be included with the pavement excavation.

Since the subgrade must be finished accurately and prepared to receive the pavement, and since the cuts are comparatively shallow and the free-haul distances long, the unit price of pavement excavation and of parkway excavation is generally considerably higher than for railroad excavation.

The following method of computing street excavation, called the *average cut* method, is adaptable to most conditions and is convenient, rapid and sufficiently accurate.

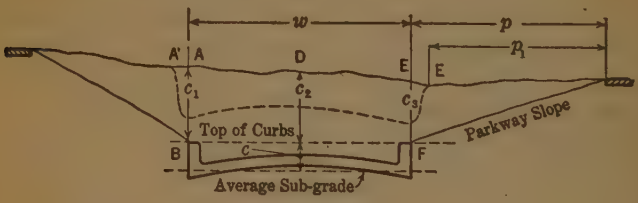

FIG. 119.

In Fig. 119, the area of the cross-section is the sum of the two trapezoids *ABCD* and *CDEF*, or

$$a = wC = \frac{w(c_1 + 2c_2 + c_3)}{4} \tag{101}$$

in which a is the area of any section, w is the width between outside curb lines, and C is the average cut for the section and is given by

$$C = \frac{c_1 + 2c_2 + c_3}{4} \tag{102}$$

in which c_1 and c_3 are the cuts at the curb lines and c_2 is the cut on the center line. If an occasional value of c_1, c_2, or c_3 is negative, indicating fill, it is included with its negative sign, as this approximates finding a grade point. If an entire cross-section becomes

fill, the grade points should be found and the fill computed separately. This is done by finding its *average fill* in the same way as for the cuts.

In general the cuts c_1, c_2, and c_3 should extend to the subgrade, but in practice it is more convenient to consider them as extending only to the plane of the tops of the curbs, or to the top of pavement on the center line, as fixed by the grade line on the profile, and then to correct them to an average subgrade line by the *section constant*, which for a crowned subgrade is

$$K = h_c - h_p + t + \frac{h_s}{3} \tag{103}$$

where h_c is the height of the curb, h_p the amount of pavement crown, t the thickness of the pavement on the center line, and h_s the amount of subgrade crown.

The volume between any two sections by the end area method is

$$v = \left(\frac{C_1 + C_2}{2} + K\right)wl \tag{104}$$

The term in parenthesis is the average of the two average cuts as found from Eq. 102, and the term wl is the surface area of the excavation. Therefore, we can write

$$V = AC_a \tag{105}$$

where V is the total volume, A is the total area of the excavation, and C_a is the grand average of the cuts, or the *average cut* for the project. Summing up the successive sections and reducing, we find

$$C_a = \frac{C_1 + 2C_2 + 2C_3 \cdots + 2C_{n-1} + C_n}{2(n-1)} + K \tag{106}$$

Theoretically these equations apply only to a pavement of uniform width and to one where the cross-sections are equally spaced, but experience has shown that they are entirely satisfactory for pavements of variable width and with cross-sections at unequal distances apart, provided the total number of sections is the same as if they were equally spaced. Experience has also shown that, except under very unusual conditions, the method can be used to include turnouts to intersecting streets and alleys, including the

curves at the corners by making A include the total surface area, since these additional areas will rarely change the value of C_a.

258. Parkways. In Fig. 119, the end area of the parkway excavation to the right is equal to the triangle whose altitude is p and whose base is the cut c_3, taken to the top of the curb. Then for a constant value of p,

$$V_p = \tfrac{1}{27} pLC_p \qquad (107)$$

in which L is the total length of the excavation and C_p is the mean of all the cuts c_3.

It is usually sufficiently accurate to make L equal to the total length of the project minus the sum of the widths of pavement on turnouts to intersecting streets and alleys. This neglects the curved corners, but the amount is generally quite small.

If the parkway is the same width on both sides of the pavement, then

$$V_p \doteq pLx \qquad (108)$$

where x is the grand average of all the cuts, c_1 and c_3, at the curbs.

Frequently, the street has already been partially graded as an earth road, as shown by the dotted line in Fig. 119. In this case, extra readings should be taken at A' and E' and p_1 determined, and these values used in the equations.

APPENDIX

THEORY OF SPIRAL

259. Derivation of δ and Δ.

From definition, $d = ks$; for the *S.C.*, $D = kS$ (30)

The radius of curvature, $r = \dfrac{dl}{d\delta}$

Substituting for r its value, $\dfrac{5730}{d}$, and then for d its value from Eq. 30, and solving,

$$d\delta = \frac{kl\ dl}{573000}$$

Integrating, $\delta = \dfrac{kl^2}{1146000} = \dfrac{ks^2}{114.6}$

in which δ is in circular measure. Changing δ to degrees,

$$\delta = \tfrac{1}{2}ks^2 \qquad (31)$$

For the *S.C.* $\Delta = \tfrac{1}{2}kS^2$

260. Derivation of the Co-ordinates x and y.

The differential equations for the co-ordinates of a spiral are:

$$dx = ds \cos \delta$$

$$dy = ds \sin \delta$$

Expanding the sin and cos into series,

$$dx = ds\left(1 - \frac{\delta^2}{\lfloor 2} + \frac{\delta_4}{\lfloor 4} - \text{etc.}\right)$$

$$dy = ds\left(\delta - \frac{\delta^3}{\lfloor 3} + \frac{\delta^5}{\lfloor 5} - \text{etc.}\right)$$

Substituting $\frac{ks^2}{114.6}$ for δ

$$dx = ds\left(1 - \frac{k^2s^4}{2(114.6)^2} + \text{etc.}\right)$$

$$dy = ds\left(\frac{ks^2}{114.6} - \frac{k^3s^6}{6(114.6)^3} + \text{etc.}\right)$$

Integrating, $x = s - \frac{k^2s^5}{10(114.6)^2} + \text{etc.}$

$$y = \frac{ks^3}{3(114.6)} - \frac{k^3s^7}{42(114.6)^3} + \text{etc.}$$

in which x and y are in stations. Reducing to feet and dropping small terms,

$$x = 100s - 0.000762k^2s^5 \qquad (35)$$

$$y = 0.291ks^3 - 0.0000158k^3s^7 \qquad (34)$$

261. Derivation of a, A, b, and B.

$$\tan a = \frac{y}{x} = \frac{\frac{ks^3}{3 \times 114.6} - \frac{k^3s^7}{42 \times (114.6)^3} + \text{etc.}}{s - \frac{k^2s^5}{10 \times (114.6)^2} + \text{etc.}}$$

$$= \frac{ks^2}{3 \times 114.6} + \frac{k^3s^6}{(114.6)^3 \times 105} + \text{etc.}$$

Substituting δ for $\frac{ks^2}{114.6}$

$$\tan a = \frac{\delta}{3} + \frac{1}{105}\delta^3 + \text{etc.}$$

$$\tan \frac{\delta}{3} = \frac{\delta}{3} + \frac{1}{81}\delta^3 + \text{etc. (tangent series).}$$

Whence, $\tan a = \tan \frac{1}{3}\delta - \frac{24}{8505}\delta^3$

Reducing to degrees,

$$\tan a = \tan \frac{1}{3}\delta - 0.000000015\delta^3$$

Investigating the second term, it is found that δ must be about 25° before there is an error of *one minute* in a by considering it as $\frac{1}{3}\delta$; and since δ rarely reaches 25° and any error of direction is corrected at the *S.C.*, it is sufficient to write

$$\left.\begin{aligned} a &= \frac{1}{3}\delta = 10ks^2 \text{ (in minutes)} \\ A &= \frac{1}{3}\Delta = 10kS^2 \text{ (in minutes)} \end{aligned}\right\} \quad (32)$$

and,

$$b = \frac{2}{3}\delta = 2a, \text{ and } B = \frac{2}{3}\Delta = 2A \quad (33)$$

262. Derivation of o and t.

From Fig. 120.

$$o = Y - R \text{ vers } \Delta$$

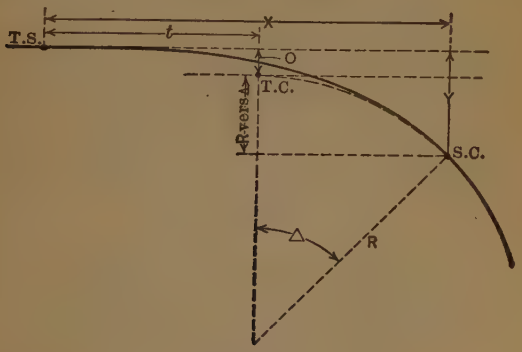

FIG. 120.

Substituting for Y its value, $0.291kS^3$, for R its value, $\frac{5730}{kS}$, and expanding the vers series,

$$o = 0.291kS^3 - 0.0000158k^3S^7 - \frac{5730}{kS}\left(\frac{k^2S^4}{2 \times (114.6)^2} - \frac{k^4S^8}{24(114.6)^4}\right)$$

$$o = 0.291kS^3 - 0.00000158k^3S^7 - 5730\left(\frac{kS^3}{2(114.6)^2} - \frac{k^3S^7}{24(114.6)^4}\right)$$

Reducing,

$$o = 0.0727kS^3 - 0.0000002k^3S^7$$

Evidently the second term is very small, hence,

$$o = 0.0727kS^3 \tag{36}$$

From figure,

$$t = X - R\sin\Delta$$

Substituting for X its value from Eq. 35, and for R its value, $\frac{5730}{kS}$, and expanding the sin series,

$$t = 100s - 0.000762\,k^2S^5 - \frac{5730}{kS}\left(\Delta - \frac{\Delta^3}{3} + \text{etc.}\right)$$

$$= 100s - 0.000762\,k^2S^5 - \frac{5730}{kS}\left(\frac{kS^3}{114.6} - \frac{k^3S^6}{6(114.6)^3} + \text{etc.}\right)$$

Reducing,

$$t = 50S - 0.000127k^2S^5 \tag{37}$$

263. S and o Mutually Bisect.

Substituting $\frac{1}{2}S$ in Eq. 35, the abscissa of the middle of the spiral is $50S - 0.000024k^2S^5$. The difference between this and Eq. 37 is only $0.0001k^2S^5$, showing that *the spiral is practically bisected by the ordinate to the T.C.*

Substituting $\frac{1}{2}S$ in Eq. 34, the ordinate for the middle of the spiral is $0.0364kS^3$.

$$\tfrac{1}{2}o = 0.0363kS^3$$

showing that *o is bisected by the spiral.* If the second term of Eq. 34 had been used, the difference would have been still less.

264. Osculating Circle. If at any point on a spiral a circle is drawn whose radius coincides with and is the same length as the radius of curvature of the spiral at that point, such a circle is termed an osculating circle.

In the forward direction, the osculating circle lies outside the spiral since the curvature of the circle is constant while the spiral, by definition, is becoming sharper at the rate k. In the reverse direction (toward the *T.S.*) the circle lies inside the spiral since, by definition, the spiral is becoming flatter at the rate k. The oscu-

lating circle, therefore, both intersects the spiral and is tangent to it at the chosen point, and the spiral changes its curvature with respect to the circle at a constant rate k.

Since an osculating circle can be made at any point, and since the tangent at the $T.S.$ is a circle with an infinite radius, it follows that *the spiral departs from any osculating circle at the same rate as from the initial tangent.*

265. Superelevation.

From mechanics,

$$F = \frac{Wv^2}{gR}$$

where F is the centrifugal force, W is the weight, v is the velocity in feet per second, R is the radius of curvature, and g is the acceleration of gravity (equal to 32.16).

In Fig. 121, AB represents the horizontal distance between rail heads, which for standard gage is about 4.9 ft.; CB, the superelevation; EH, the weight W; and GE, the centrifugal force. Then from similar triangles,

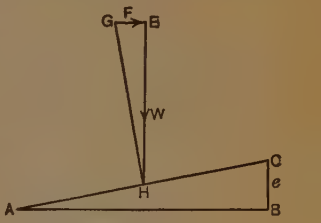

FIG. 121.

$$F = W\frac{e}{AB}$$

Substituting the above value of F in this equation, we have

$$\frac{Wv^2}{gR} = \frac{We}{AB}$$

whence,

$$e \text{ (in feet)} = \frac{ABv^2}{gR} = \frac{4.9v^2}{32.16R}$$

Reducing v to miles per hour, substituting $\frac{5730}{D}$ for R, and reducing

$$e \text{ (in inches)} = 0.00069DV^2 \qquad (42)$$

On highways e is desired in feet per foot of width hence the term 4.9 in the above expression becomes 1.0 and Eq. 42 reduces to

$$e \text{ (in feet per foot of width)} = \frac{0.067V^2}{R} \qquad (59)$$

TRIGONOMETRIC FORMULAS

Right Triangles

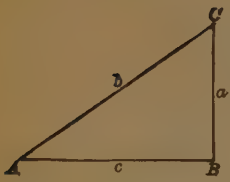

Fig. 122.

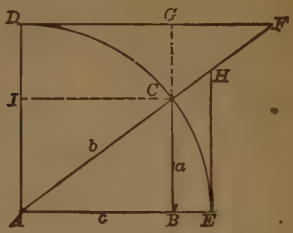

Fig. 123.

In Fig. 123, let DCE be the arc of a quadrant, ABC a right triangle, the angle BAC subtended by the arc $CE = A$, and consider the radius AC = unity. Then

$BC = \sin A.$ $\qquad$ $AF = \operatorname{cosec} A.$

$AB = \cos A.$ $\qquad$ $BE = \operatorname{versin} A.$

$HE = \tan A.$ $\qquad$ $DI = \operatorname{coversin} A.$

$DF = \cot A.$ $\qquad$ $CH = \operatorname{exsec} A.$

$AH = \sec A.$ $\qquad$ $CF = \operatorname{coexsec} A.$

Using the small letters a, b, c, to represent the sides of a right triangle in Fig. 122 or 123, we may write

$$\sin A = \frac{a}{b}; \ \operatorname{cosec} A = \frac{b}{a}; \ \therefore \sin A = \frac{1}{\operatorname{cosec} A}.$$

$$\cos A = \frac{c}{b}; \quad \sec A = \frac{b}{c}; \ \therefore \cos A = \frac{1}{\sec A}.$$

$$\tan A = \frac{a}{c}; \quad \cot A = \frac{c}{a}; \ \therefore \tan A = \frac{1}{\cot A}.$$

Formulas for Logarithmic Computation

Given one side, a, and the three angles A, B, and C.

The "sine proportion" states that each side of a triangle is proportional to the sine of the angle opposite it.

Then, $$b = \frac{a \sin B}{\sin A} \quad \text{and} \quad C = \frac{a \sin C}{\sin A}.$$

Area $$= \frac{a^2 \sin B \sin C}{2 \sin (B + C)}$$

Given two sides a and b and the included angle C.

First method:—Determining the angles first.

$$\tfrac{1}{2}(A + B) = \tfrac{1}{2}(180 - C)$$

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \tfrac{1}{2}(A + B)$$

$$A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)$$

$$B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B).$$

The third side, c, can now be determined by the sine proportion.

Second method:—Determining the third side first.

$$\tan x = \frac{2 \sin \tfrac{1}{2}C}{a - b} \sqrt{ab}.$$

$$c = \frac{a - b}{\cos x}.$$

The angles A and B can now be found by the sine proportion

$$\text{Area} = \tfrac{1}{2}ab \sin C$$

Third Method:—Using two right triangles.

In Fig. 124 drop a perpendicular p from the extremity of the shorter side a onto the longer side b forming two right triangles.

In triangle 1,

$$m = a \cos C \quad \text{and} \quad p = a \sin C$$

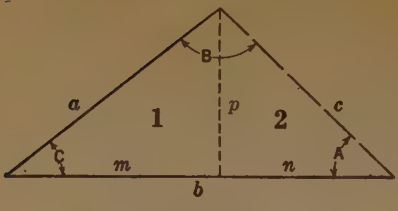

FIG. 124.

In triangle 2,

$$n = b - m$$

$$\tan A = \frac{p}{n}$$

and

$$c = \frac{p}{\sin A}$$

Given the three sides a, b, and c.

Let

$$s = \tfrac{1}{2}(a + b + c).$$

Then

$$*\tan \tfrac{1}{2}A = \frac{\sqrt{\frac{1}{s}(s-a)(s-b)(s-c)}}{s-a}$$

$$*\tan \tfrac{1}{2}B = \frac{\sqrt{\frac{1}{s}(s-a)(s-b)(s-c)}}{s-b}$$

$$*\tan \tfrac{1}{2}C = \frac{\sqrt{\frac{1}{s}(s-a)(s-b)(s-c)}}{s-c}$$

$$\text{Area} = \sqrt{s(s-a(s-b)(s-c)}$$

* It will be noted that the numerator is the same in each case and that the denominators are quantities whose logarithms are known and hence two additional logarithmic operations over that required for one angle gives all three angles and a complete check on the entire computation since $A + B + C = 180°$.

TABLES

TABLE 10.—RADII

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
0°– 0′	∞		0°–30′	11460.00	4.0591846
1	343800.00	5.5363059	31	11090.32	4.0449440
2	171900.00	5.2352759	32	10743.75	4.0311559
3	114600.00	5.0591846	33	10418.18	4.0177919
4	85950.00	4.9342459	34	10111.76	4.0048268
5	68760.00	4.8373359	35	9822.86	3.9922379
6	57300.00	4.7581546	36	9550.00	3.9800034
7	49114.29	4.6912079	37	9291.89	3.9681040
8	42975.00	4.6332159	38	9047.37	3.9565224
9	38200.00	4.5820634	39	8815.38	3.9452410
10	34380.00	4.5363059	40	8595.00	3.9342459
11	31254.55	4.4949132	41	8385.37	3.9235222
12	28650.00	4.4571246	42	8185.71	3.9130563
13	26446.15	4.4223625	43	7995.35	3.9028375
14	24557.14	4.3901780	44	7813.64	3.8928534
15	22920.00	4.3602146	45	7640.00	3.8830934
16	21487.50	4.3321859	46	7473.91	3.8735479
17	20223.53	4.3058570	47	7314.89	3.8642078
18	19100.00	4.2810334	48	7162.50	3.8550646
19	18094.74	4.2575524	49	7016.33	3.8461100
20	17190.00	4.2352759	50	6876.00	3.8373359
21	16371.43	4.2140866	51	6741.18	3.8287359
22	15627.27	4.1938831	52	6611.54	3.8203026
23	14947.83	4.1745782	53	6486.79	3.8120298
24	14325.00	4.1560946	54	6366.67	3.8039124
25	13752.00	4.1383659	55	6250.91	3.7959433
26	13223.08	4.1213326	56	6139.29	3.7881182
27	12733.33	4.1049420	57	6031.58	3.7804311
28	12278.57	4.0891478	58	5927.59	3.7728782
29	11855.17	4.0739078	59	5827.12	3.7654540
30	11460.00	4.0591846	60	5730.00	3.7581546

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
1°– 0′	5730.00	3.7581546	1°–30′	3820.00	3.5820634
1	5636.07	3.7509764	31	3778.02	3.5772643
2	5545.16	3.7439141	32	3736.96	3.5725185
3	5457.14	3.7369651	33	3696.77	3.5678225
4	5371.87	3.7301255	34	3657.46	3.5631775
5	5289.23	3.7233925	35	3618.95	3.5585826
6	5209.09	3.7167619	36	3581.25	3.5540347
7	5131.34	3.7102308	37	3544.33	3.5495342
8	5055.88	3.7037968	38	3508.16	3.5450794
9	4982.61	3.6974569	39	3472.73	3.5406711
10	4911.43	3.6912079	40	3438.00	3.5363059
11	4842.25	3.6850472	41	3403.96	3.5319845
12	4775.00	3.6789734	42	3370.59	3.5277059
13	4709.59	3.6729831	43	3337.86	3.5234681
14	4645.95	3.6670746	44	3305.77	3.5192726
15	4584.00	3.6612446	45	3274.29	3.5151172
16	4523.68	3.6554919	46	3243.40	3.5110005
17	4464.94	3.6498156	47	3213.08	3.5069215
18	4407.69	3.6442110	48	3183.33	3.5028817
19	4351.90	3.6386789	49	3154.13	3.4988796
20	4297.50	3.6332159	50	3125.46	3.4949139
21	4244.44	3.6278204	51	3097.30	3.4909833
22	4192.68	3.6224917	52	3069.64	3.4870874
23	4142.17	3.6172280	53	3042.48	3.4832277
24	4092.86	3.6120269	54	3015.79	3.4794011
25	4044.70	3.6068863	55	2989.56	3.4756073
26	3997.68	3.6018080	56	2963.79	3.4718474
27	3951.72	3.5967862	57	2938.46	3.4681198
28	3906.82	3.5918234	58	2913.56	3.4644239
29	3862.92	3.5869157	59	2889.08	3.4607596
30	3820.00	3.5820634	60	2865.00	3.4571246

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

Whence $R = \frac{5730.00}{D}$

D	R	Log R	D	R	Log R
2°– 0′	2865.00	3.4571246	2°–30′	2292.00	3.3602146
1	2841.32	3.4535202	31	2276.82	3.3573287
2	2818.03	3.4499456	32	2261.84	3.3544619
3	2795.12	3.4464005	33	2247.06	3.3516147
4	2772.58	3.4428841	34	2232.47	3.3487857
5	2750.40	3.4393959	35	2218.06	3.3459733
6	2728.57	3.4359351	36	2203.85	3.3431821
7	2707.09	3.4325027	37	2189.81	3.3404065
8	2685.94	3.4290963	38	2175.95	3.3376489
9	2665.12	3.4257168	39	2162.26	3.3349079
10	2644.62	3.4223633	40	2148.75	3.3321859
11	2624.43	3.4190450	41	2135.43	3.3294853
12	2604.55	3.4157327	42	2122.22	3.3267904
13	2584.96	3.4124538	43	2109.20	3.3241178
14	2565.71	3.4092076	44	2096.34	3.3214617
15	2546.67	3.4059727	45	2083.64	3.3188227
16	2527.94	3.4027668	46	2071.08	3.3161969
17	2509.49	3.3995855	47	2058.68	3.3135889
18	2491.30	3.3964260	48	2046.43	3.3109969
19	2473.38	3.3932909	49	2034.32	3.3084193
20	2455.71	3.3901771	50	2022.35	3.3058564
21	2438.30	3.3870871	51	2010.53	3.3033106
22	2421.13	3.3840181	52	1998.84	3.3007780
23	2404.20	3.3809706	53	1987.28	3.2982590
24	2387.50	3.3779434	54	1975.86	3.2957562
25	2371.03	3.3749371	55	1964.57	3.2932675
26	2354.79	3.3719522	56	1953.41	3.2907934
27	2338.78	3.3689894	57	1942.37	3.2883320
28	2322.97	3.3660436	58	1931.46	3.2858857
29	2307.38	3.3631191	59	1920.67	3.2834527
30	2292.00	3.3602146	60	1910.00	3.2810334

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
3° –0′	1910.00	3.2810334	3°–30′	1637.14	3.2140858
1	1899.48	3.2786347	31	1629.38	3.2120224
2	1889.01	3.2762343	32	1621.70	3.2099705
3	1878.69	3.2738551	33	1614.08	3.2079250
4	1868.48	3.2714885	34	1606.54	3.2058915
5	1858.38	3.2691345	35	1599.07	3.2038675
6	1848.39	3.2667937	36	1591.67	3.2018530
7	1838.50	3.2644636	37	1584.33	3.1998456
8	1828.72	3.2621472	38	1577.06	3.1978482
9	1819.05	3.2598447	39	1569.86	3.1958609
10	1809.47	3.2575514	40	1562.73	3.1938839
11	1800.00	3.2552725	41	1555.66	3.1919147
12	1790.62	3.2530034	42	1548.65	3.1899533
13	1781.35	3.2507493	43	1541.70	3.1879999
14	1772.16	3.2485029	44	1534.82	3.1860575
15	1763.07	3.2462695	45	1528.00	3.1841234
16	1754.08	3.2440494	46	1521.24	3.1821977
17	1745.18	3.2418402	47	1514.54	3.1802808
18	1736.36	3.2396398	48	1507.89	3.1783696
19	1727.64	3.2374532	49	1501.31	3.1764704
20	1719.00	3.2352759	50	1494.78	3.1745773
21	1710.45	3.2331104	51	1488.31	3.1726934
22	1701.98	3.2309545	52	1481.90	3.1708189
23	1693.60	3.2288108	53	1475.54	3.1689510
24	1685.29	3.2266746	54	1469.23	3.1670898
25	1677.07	3.2245512	55	1462.98	3.1652384
26	1668.93	3.2224381	56	1456.78	3.1633939
27	1660.87	3.2203356	57	1450.63	3.1615567
28	1652.88	3.2182413	58	1444.54	3.1597295
29	1644.98	3.2161606	59	1438.49	3.1579069
30	1637.14	3.2140858	60	1432.50	3.1560946

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

Whence $R = \frac{5730.00}{D}$

D	R	Log R	D	R	Log R
4°– 0′	1432.50	3.1560946	4°–30′	1273.33	3.1049409
1	1426.56	3.1542900	31	1268.63	3.1033350
2	1420.66	3.1524902	32	1263.97	3.1017368
3	1414.81	3.1506982	33	1259.34	3.1001430
4	1409.02	3.1489172	34	1254.74	3.0985537
5	1403.27	3.1471412	35	1250.18	3.0969726
6	1397.56	3.1453705	36	1245.65	3.0953961
7	1391.90	3.1436080	37	1241.16	3.0938278
8	1386.29	3.1418541	38	1236.69	3.0922609
9	1380.72	3.1401056	39	1232.26	3.0907023
10	1375.20	3.1383659	40	1227.86	3.0891488
11	1369.72	3.1366318	41	1223.49	3.0876005
12	1364.29	3.1349067	42	1219.15	3.0860572
13	1358.89	3.1331843	43	1214.84	3.0845191
14	1353.54	3.1314711	44	1210.57	3.0829899
15	1348.24	3.1297672	45	1206.32	3.0814625
16	1342.97	3.1280663	46	1202.10	3.0799406
17	1337.74	3.1263717	47	1197.91	3.0784242
18	1332.56	3.1246868	48	1193.75	3.0769134
19	1327.41	3.1230051	49	1189.62	3.0754083
20	1322.31	3.1213333	50	1185.52	3.0739089
21	1317.24	3.1196649	51	1181.44	3.0724117
22	1312.21	3.1180033	52	1177.40	3.0709240
23	1307.22	3.1163487	53	1173.38	3.0694387
24	1302.27	3.1147010	54	1169.39	3.0679594
25	1297.36	3.1130605	55	1165.42	3.0664825
26	1292.48	3.1114239	56	1161.49	3.0650155
27	1287.64	3.1097945	57	1157.58	3.0635510
28	1282.84	3.1081725	58	1153.69	3.0620891
29	1278.07	3.1065546	59	1149.83	3.0606336
30	1273.33	3.1049409	60	1146.00	3.0591846

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
5°– 0′	1146.00	3.0591846	5°–30′	1041.81	3.0177886
1	1142.19	3.0577384	31	1038.67	3.0164776
2	1138.41	3.0562987	32	1035.54	3.0151669
3	1134.65	3.0548620	33	1032.43	3.0138606
4	1130.92	3.0534319	34	1029.34	3.0125589
5	1127.21	3.0520049	35	1026.27	3.0112616
6	1123.53	3.0505847	36	1023.21	3.0099648
7	1119.87	3.0491676	37	1020.18	3.0086768
8	1116.23	3.0477537	38	1017.16	3.0073893
9	1112.62	3.0463469	39	1014.16	3.0061065
10	1109.03	3.0449433	40	1011.18	3.0048285
11	1105.47	3.0435469	41	1008.21	3.0035501
12	1101.92	3.0421501	42	1005.26	3.0022784
13	1098.40	3.0407605	43	1002.33	3.0010107
14	1094.90	3.0393745	44	999.42	2.9997480
15	1091.43	3.0379958	45	996.52	2.9984860
16	1087.97	3.0366169	46	993.64	2.9972291
17	1084.54	3.0352456	47	990.78	2.9959772
18	1081.13	3.0338779	48	987.93	2.9947262
19	1077.74	3.0325140	49	985.10	2.9934803
20	1074.38	3.0311579	50	982.29	2.9922397
21	1071.03	3.0298017	51	979.49	2.9910000
22	1067.70	3.0284492	52	976.70	2.9897612
23	1064.40	3.0271049	53	973.94	2.9885322
24	1061.11	3.0257604	54	971.19	2.9873042
25	1057.85	3.0244241	55	968.45	2.9860772
26	1054.60	3.0230878	56	965.73	2.9848557
27	1051.38	3.0217597	57	963.03	2.9836398
28	1048.17	3.0204318	58	960.34	2.9824250
29	1044.98	3.0191080	59	957.66	2.9812113
30	1041.81	3.0177886	60	955.00	2.9800034

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
6°– 0′	955.00	2.9800034	6°–30′	881.54	2.9452420
1	952.35	2.9787966	31	879.28	2.9441272
2	949.72	2.9775956	32	877.04	2.9430194
3	947.11	2.9764004	33	874.81	2.9419137
4	944.51	2.9752066	34	872.59	2.9408102
5	941.92	2.9740140	35	870.38	2.9397089
6	939.34	2.9728228	36	868.18	2.9386098
7	936.78	2.9716376	37	865.99	2.9375129
8	934.24	2.9704585	38	863.81	2.9364182
9	931.71	2.9692808	39	861.65	2.9353309
10	929.19	2.9681045	40	859.50	2.9342459
11	926.68	2.9669298	41	857.36	2.9331632
12	924.19	2.9657613	42	855.22	2.9320778
13	921.72	2.9645990	43	853.10	2.9309999
14	919.25	2.9634336	44	850.99	2.9299245
15	916.80	2.9622746	45	848.89	2.9288514
16	914.36	2.9611172	46	846.80	2.9277808
17	911.94	2.9599663	47	844.72	2.9267128
18	909.52	2.9588123	48	842.65	2.9256472
19	907.13	2.9576695	49	840.59	2.9245842
20	904.74	2.9565238	50	838.54	2.9235238
21	902.36	2.9553798	51	836.50	2.9224659
22	900.00	2.9542425	52	834.47	2.9214107
23	897.65	2.9531070	53	832.45	2.9203582
24	895.31	2.9519734	54	830.43	2.9193030
25	892.99	2.9508466	55	828.43	2.9182558
26	890.67	2.9497168	56	826.44	2.9172113
27	888.37	2.9485939	57	824.46	2.9161696
28	886.08	2.9474729	58	822.49	2.9151306
29	883.80	2.9463540	59	820.53	2.9140945
30	881.54	2.9452420	60	818.57	2.9130558

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
7°– 0′	818.57	2.9130558	7°–30′	764.00	2.8830934
1	816.63	2.9120253	31	762.31	2.8821316
2	814.69	2.9109924	32	760.62	2.8811677
3	812.77	2.9099677	33	758.94	2.8802074
4	810.85	2.9089405	34	757.27	2.8792508
5	808.94	2.9079163	35	755.60	2.8782919
6	807.04	2.9068951	36	753.95	2.8773425
7	805.15	2.9058768	37	752.30	2.8763911
8	803.27	2.9048615	38	750.66	2.8754433
9	801.38	2.9038385	39	749.02	2.8744934
10	799.53	2.9028348	40	747.39	2.8735473
11	797.68	2.9018287	41	745.77	2.8726049
12	795.83	2.9008203	42	744.16	2.8716663
13	794.00	2.8998205	43	742.55	2.8707257
14	792.17	2.8988184	44	740.95	2.8697889
15	790.34	2.8978140	45	739.35	2.8688501
16	788.53	2.8968182	46	737.77	2.8679210
17	786.73	2.8958257	47	736.19	2.8669899
18	784.93	2.8948309	48	734.62	2.8660627
19	783.14	2.8938394	49	733.05	2.8651336
20	781.36	2.8928512	50	731.49	2.8642084
21	779.59	2.8918663	51	729.94	2.8632872
22	777.82	2.8908791	52	728.39	2.8623640
23	776.07	2.8899009	53	726.85	2.8614448
24	774.32	2.8889205	54	725.32	2.8605297
25	772.58	2.8879435	55	723.79	2.8596126
26	770.85	2.8869699	56	722.27	2.8586996
27	769.13	2.8859998	57	720.75	2.8577847
28	767.41	2.8850218	58	719.25	2.8568799
29	765.70	2.8840586	59	717.75	2.8559732
30	764.00	2.8830934	60	716.25	2.8550646

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
8°– 0′	716.25	2.8550646	8°–30′	674.12	2.8287372
1	714.76	2.8541602	31	672.80	2.8278860
2	713.28	2.8532600	32	671.48	2.8270331
3	711.80	2.8523580	33	670.17	2.8261850
4	710.33	2.8514602	34	668.87	2.8253417
5	708.87	2.8505666	35	667.57	2.8244968
6	707.41	2.8496712	36	666.28	2.8236568
7	705.95	2.8487739	37	664.99	2.8228151
8	704.51	2.8478872	38	663.71	2.8219784
9	703.07	2.8469986	39	662.43	2.8211400
10	701.63	2.8461082	40	661.15	2.8203000
11	700.20	2.8452221	41	659.88	2.8194650
12	698.78	2.8443405	42	658.62	2.8186349
13	697.36	2.8434570	43	657.36	2.8178033
14	695.95	2.8425780	44	656.11	2.8169767
15	694.55	2.8417035	45	654.86	2.8161485
16	693.15	2.8408272	46	653.61	2.8153187
17	691.75	2.8399492	47	652.37	2.8144940
18	690.36	2.8390756	48	651.14	2.8136744
19	688.98	2.8382066	49	649.91	2.8128532
20	687.60	2.8373359	50	648.68	2.8120305
21	686.23	2.8364697	51	647.46	2.8112129
22	684.86	2.8356018	52	646.24	2.8103938
23	683.50	2.8347385	53	645.03	2.8095799
24	682.14	2.8338735	54	643.82	2.8087645
25	680.79	2.8330132	55	642.62	2.8079542
26	679.45	2.8321575	56	641.42	2.8071425
27	678.11	2.8313001	57	640.22	2.8063292
28	676.77	2.8304411	58	639.03	2.8055212
29	675.44	2.8295868	59	637.85	2.8047186
30	674.12	2.8287372	60	636.67	2.8039144

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
9°– 0′	636.67	2.8039144	9°–30′	603.16	2.7804325
1	635.49	2.8031087	31	602.10	2.7796686
2	634.32	2.8023084	32	601.05	2.7789106
3	633.15	2.8015066	33	600.00	2.7781513
4	631.99	2.8007102	34	598.95	2.7773906
5	630.83	2.7999123	35	597.91	2.7766358
6	629.67	2.7991130	36	596.87	2.7758798
7	628.52	2.7983191	37	595.84	2.7751297
8	627.37	2.7975237	38	594.81	2.7743783
9	626.23	2.7967339	39	593.78	2.7736256
10	625.09	2.7959426	40	592.76	2.7728789
11	623.96	2.7951567	41	591.74	2.7721309
12	622.83	2.7943695	42	590.72	2.7713817
13	621.70	2.7935809	43	589.71	2.7706385
14	620.58	2.7927978	44	588.70	2.7698940
15	619.46	2.7920133	45	587.69	2.7691483
16	618.35	2.7912344	46	586.69	2.7684087
17	617.24	2.7904541	47	585.69	2.7676678
18	616.13	2.7896724	48	584.69	2.7669257
19	615.03	2.7888963	49	583.70	2.7661897
20	613.93	2.7881189	50	582.71	2.7654525
21	612.83	2.7873400	51	581.73	2.7647215
22	611.74	2.7865669	52	580.74	2.7639817
23	610.66	2.7857995	53	579.76	2.7632482
24	609.57	2.7850236	54	578.79	2.7625210
25	608.50	2.7842606	55	577.82	2.7617926
26	607.42	2.7834891	56	576.85	2.7610629
27	606.35	2.7827234	57	575.88	2.7603320
28	605.28	2.7819563	58	574.92	2.7596074
29	604.22	2.7811951	59	573.96	2.7588816
30	603.16	2.7804325	60	573.00	2.7581546

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

Whence $R = \frac{5730.00}{D}$

D	R	Log R	D	R	Log R
10°– 0′	573.00	2.7581546	10°–30′	545.71	2.7369619
1	572.05	2.7574340	31	544.85	2.7362770
2	571.10	2.7567122	32	543.99	2.7355909
3	570.15	2.7559891	33	543.13	2.7349038
4	569.21	2.7552725	34	542.27	2.7342156
5	568.26	2.7545471	35	541.42	2.7335343
6	567.33	2.7538357	36	540.57	2.7328519
7	566.39	2.7531156	37	539.72	2.7321685
8	565.46	2.7524019	38	538.87	2.7314840
9	564.53	2.7516870	39	538.03	2.7308065
10	563.61	2.7509787	40	537.19	2.7301279
11	562.68	2.7502615	41	536.35	2.7294483
12	561.76	2.7495508	42	535.51	2.7287676
13	560.85	2.7488467	43	534.68	2.7280939
14	559.93	2.7481337	44	533.85	2.7274192
15	559.02	2.7474273	45	533.02	2.7267435
16	558.11	2.7467198	46	532.20	2.7260749
17	557.21	2.7460189	47	531.38	2.7254052
18	556.31	2.7453169	48	530.56	2.7247345
19	555.41	2.7446137	49	529.74	2.7240628
20	554.52	2.7439172	50	528.92	2.7233900
21	553.62	2.7432118	51	528.11	2.7227244
22	552.73	2.7425130	52	527.30	2.7220578
23	551.85	2.7418210	53	526.49	2.7213901
24	550.96	2.7411201	54	525.69	2.7207297
25	550.08	2.7404259	55	524.89	2.7200683
26	549.20	2.7397305	56	524.09	2.7194059
27	548.33	2.7390420	57	523.29	2.7187424
28	547.45	2.7383445	58	522.49	2.7180780
29	546.58	2.7376537	59	521.70	2.7174208
30	545.71	2.7369619	60	520.91	2.7167627

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
11°– 0′	520.91	2.7167627	11°–30′	498.26	2.6974560
1	520.12	2.7161036	31	497.54	2.6968280
2	519.34	2.7154518	32	496.82	2.6961992
3	518.55	2.7147906	33	496.10	2.6955692
4	517.77	2.7141369	34	495.39	2.6949472
5	516.99	2.7134821	35	494.68	2.6943244
6	516.22	2.7128348	36	493.97	2.6937006
7	515.44	2.7121781	37	493.26	2.6930759
8	514.67	2.7115289	38	492.55	2.6924503
9	513.90	2.7108786	39	491.85	2.6918327
10	513.13	2.7102274	40	491.14	2.6912053
11	512.37	2.7095837	41	490.44	2.6905859
12	511.61	2.7089390	42	489.74	2.6899656
13	510.85	2.7082934	43	489.05	2.6893533
14	510.09	2.7076468	44	488.35	2.6887312
15	509.33	2.7069993	45	487.66	2.6881171
16	508.58	2.7063593	46	486.97	2.6875022
17	507.83	2.7057184	47	486.28	2.6868864
18	507.08	2.7050765	48	485.59	2.6862697
19	506.33	2.7044337	49	484.91	2.6856611
20	505.59	2.7037985	50	484.23	2.6850517
21	504.85	2.7031624	51	483.54	2.6844324
22	504.11	2.7025253	52	482.87	2.6838302
23	503.37	2.7018873	53	482.19	2.6832182
24	502.63	2.7012484	54	481.51	2.6826053
25	501.90	2.7006172	55	480.84	2.6820006
26	501.17	2.6999851	56	480.17	2.6813950
27	500.44	2.6993520	57	479.50	2.6807886
28	499.71	2.6987180	58	478.83	2.6801814
29	498.98	2.6980831	59	478.16	2.6795732
30	498.26	2.6974560	60	477.50	2.6789734

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
12°– 0′	477.50	2.6789734	13°– 0′	440.77	2.6442120
2	476.18	2.6777712	2	439.64	2.6430972
4	474.86	2.6765656	4	438.52	2.6419894
6	473.55	2.6753658	6	437.40	2.6408788
8	472.25	2.6741720	8	436.29	2.6397753
10	470.96	2.6729840	10	435.19	2.6386789
12	469.67	2.6717928	12	434.09	2.6375798
14	468.39	2.6706076	14	433.00	2.6364879
16	467.12	2.6694285	16	431.91	2.6353933
18	465.85	2.6682461	18	430.83	2.6343059
20	464.59	2.6670699	20	429.75	2.6332159
22	463.34	2.6658998	22	428.68	2.6321332
24	462.10	2.6647360	24	427.61	2.6310479
26	460.86	2.6635690	26	426.55	2.6299699
28	459.63	2.6624084	28	425.50	2.6288996
30	458.40	2.6612446	30	424.45	2.6278265
32	457.18	2.6600872	32	423.40	2.6267509
34	455.97	2.6589363	34	422.36	2.6256828
36	454.76	2.6577823	36	421.32	2.6246121
38	453.56	2.6566347	38	420.29	2.6235491
40	452.37	2.6554938	40	419.27	2.6224938
42	451.18	2.6543498	42	418.25	2.6214359
44	450.00	2.6532125	44	417.23	2.6203755
46	448.83	2.6520819	46	416.22	2.6193229
48	447.66	2.6509483	48	415.22	2.6182783
50	446.50	2.6498215	50	414.22	2.6172311
52	445.34	2.6486917	52	413.22	2.6161813
54	444.19	2.6475688	54	412.23	2.6151396
56	443.04	2.6464429	56	411.24	2.6140954
58	441.90	2.6453240	58	410.26	2.6130592
60	440.77	2.6442120	60	409.29	2.6120311

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
14°– 0′	409.29	2.6120311	15°– 0′	382.00	2.5820634
2	408.31	2.6109900	2	381.15	2.5810959
4	407.35	2.6099677	4	380.31	2.5801377
6	406.38	2.6089323	6	379.47	2.5791774
8	405.42	2.6079052	8	378.63	2.5782150
10	404.47	2.6068863	10	377.80	2.5772620
12	403.52	2.6058651	12	376.97	2.5763068
14	402.58	2.6048522	14	376.15	2.5753611
16	401.64	2.6038370	16	375.33	2.5744133
18	400.70	2.6028193	18	374.51	2.5734634
20	399.77	2.6018102	20	373.70	2.5725231
22	398.84	2.6007987	22	372.89	2.5715807
24	397.92	2.5997958	24	372.08	2.5706363
26	397.00	2.5987905	26	371.27	2.5696899
28	396.08	2.5977829	28	370.47	2.5687530
30	395.17	2.5967840	30	369.68	2.5678260
32	394.27	2.5957937	32	368.88	2.5668851
34	393.37	2.5948012	34	368.09	2.5659540
36	392.47	2.5938065	36	367.31	2.5650328
38	391.57	2.5928094	38	366.52	2.5640977
40	390.68	2.5918212	40	365.74	2.5631725
42	389.79	2.5908307	42	364.97	2.5622572
44	388.91	2.5898491	44	364.19	2.5613280
46	388.04	2.5888765	46	363.42	2.5604088
48	387.16	2.5878905	48	362.66	2.5594997
50	386.29	2.5869135	50	361.89	2.5585766
52	385.43	2.5859455	52	361.13	2.5576636
54	384.56	2.5849641	54	360.38	2.5567607
56	383.70	2.5839918	56	359.62	2.5558438
58	382.85	2.5830287	58	358.87	2.5549372
60	382.00	2.5820634	60	358.12	2.5540286

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
16°– 0′	358.12	2.5540286	17°– 0′	337.06	2.5277072
2	357.38	2.5531302	2	336.40	2.5268560
4	356.64	2.5522301	4	335.74	2.5260031
6	355.90	2.5513280	6	335.09	2.5251615
8	355.17	2.5504363	8	334.44	2.5243182
10	354.43	2.5495305	10	333.79	2.5234733
12	353.70	2.5486351	12	333.14	2.5226268
14	352.98	2.5477501	14	332.50	2.5217916
16	352.25	2.5468510	16	331.85	2.5209418
18	351.53	2.5459624	18	331.21	2.5201034
20	350.82	2.5450843	20	330.58	2.5192766
22	350.10	2.5441921	22	329.94	2.5184350
24	349.39	2.5433105	24	329.31	2.5176049
26	348.68	2.5424270	26	328.68	2.5167733
28	347.98	2.5415543	28	328.05	2.5159400
30	347.27	2.5406673	30	327.43	2.5151185
32	346.57	2.5397910	32	326.81	2.5142953
34	345.88	2.5389255	34	326.19	2.5134706
36	345.18	2.5380456	36	325.57	2.5126444
38	344.49	2.5371766	38	324.95	2.5118165
40	343.80	2.5363059	40	324.34	2.5110005
42	343.11	2.5354334	42	323.73	2.5101829
44	342.43	2.5345718	44	323.12	2.5093638
46	341.75	2.5337085	46	322.51	2.5085432
48	341.07	2.5328435	48	321.91	2.5077345
50	340.40	2.5319896	50	321.31	2.5069294
52	339.72	2.5311211	52	320.71	2.5061125
54	339.05	2.5302637	54	320.11	2.5052992
56	338.39	2.5294175	56	319.52	2.5044980
58	337.72	2.5285568	58	318.92	2.5036818
60	337.06	2.5277072	60	318.33	2.5028776

TABLE 10.—RADII—*Continued*

Based on 5730.00 as the radius of a 1° curve

$$\text{Whence } R = \frac{5730.00}{D}$$

D	R	Log R	D	R	Log R
18°– 0′	318.33	2.5028776	19°– 0′	301.58	2.4794025
2	317.74	2.5020719	2	301.05	2.4786386
4	317.16	2.5012784	4	300.52	2.4778734
6	316.57	2.5004698	6	300.00	2.4771213
8	315.99	2.4996733	8	299.48	2.4763678
10	315.41	2.4988755	10	298.96	2.4756131
12	314.84	2.4980899	12	298.44	2.4748570
14	314.26	2.4972891	14	297.92	2.4740997
16	313.69	2.4965007	16	297.40	2.4733410
18	313.11	2.4956969	18	296.89	2.4725956
20	312.55	2.4949195	20	296.38	2.4718489
22	311.98	2.4941268	22	295.87	2.4711009
24	311.41	2.4933326	24	295.36	2.4703517
26	310.85	2.4925509	26	294.85	2.4696011
28	310.29	2.4917678	28	294.35	2.4688640
30	309.73	2.4909833	30	293.85	2.4681257
32	309.17	2.4901973	32	293.34	2.4673713
34	308.62	2.4894241	34	292.84	2.4666304
36	308.06	2.4886353	36	292.35	2.4659031
38	307.51	2.4878592	38	291.85	2.4651597
40	306.96	2.4870818	40	291.36	2.4644299
42	306.42	2.4863171	42	290.86	2.4636840
44	305.87	2.4855369	44	290.37	2.4629517
46	305.33	2.4847695	46	289.88	2.4622183
48	304.79	2.4840007	48	289.39	2.4614835
50	304.25	2.4832306	50	288.91	2.4607626
52	303.71	2.4824591	52	288.42	2.4600254
54	303.17	2.4816862	54	287.94	2.4593020
56	302.64	2.4809263	56	287.46	2.4585774
58	302.11	2.4801615	58	286.98	2.4578516
60	301.58	2.4794025	60	286.50	2.4571246

TABLE 11

Even Radius Curves

Radius, Ft.	Station Length, Ft.	Deflection for One Station	Chord for One Station, Ft.	Deflection for One Foot	Length for One Minute, Ft.
50	25	14° 19.43′	24.74	34.377′	0.01454
60	25	11 56.19	24.82	28.647	.01745
70	25	10 13.88	24.87	24.555	.02036
80	25	8 57.14	24.90	21.485	.02327
90	25	7 57.46	24.92	19.098	.02618
100	25	7 09.72	24.94	17.188	.02908
110	25	6 30.65	24.95	15.626	.03199
120	25	5 58.10	24.94	14.323	.03490
130	25	5 30.55	24.97	13.222	.03781
140	25	5 06.94	24.97	12.277	.04072
150	25	4 46.68	24.97	11.459	.04363
160	25	4 28.57	24.98	10.742	.04654
170	25	4 12.77	24.98	10.111	.04945
180	25	3 58.73	24.98	9.549	.05236
190	25	3 46.17	24.98	9.046	.05526
200	25	3 34.86	24.98	8.594	.05817
225	25	3 10.98	24.99	7.639	.06545
250	25	2 51.89	24.99	6.875	.07272
275	25	2 36.26	24.99	6.250	.07999
300	25	2 23.24	24.99	.5.729	.08726
325	25	2 12.22	24.99	5.288	.09453
350	25	2 02.78	24.99	4.911	.10181
375	25	1 54.59	24.99	4.583	.10908
400	25	1 47.43	24.99	4.297	.11635
425	25	1 41.11	24.99	4.044	.12362
450	25	1 35.49	24.99	3.819	.13090
475	25	1 30.47	24.99	3.618	.13817
500	50	2 51.89	49 98	3.437	.14544
525	50	2 43.70	49.98	3.274	.15271
550	50	2 36.26	49.98	3.125	.15998
575	50	2 29.47	49.99	2.989	.16726
600	50	2 23.24	49.99	2.864	.17453
625	50	2 17.51	49.99	2.750	.18180
650	50	2 12.22	49.99	2.644	.18907
675	50	2 07.32	49.99	2.546	.19635
700	50	2 02.78	49.99	2.455	.20362
725	50	1 58.54	49.99	2.370	.21089
750	100	3 49.19	99.93	2.291	.21816
775	100	3 41.79	99.93	2.178	.22543
800	100	3 34.86	99.94	2.148	.23271
825	100	3 28.35	99.94	2.083	.23998
850	100	3 22.22	99.94	2.022	.24725
875	100	3 16.44	99.94	1.964	.25452
900	100	3 10.98	99.95	1.909	.26180
925	100	3 05.82	99.95	1.858	.26907
950	100	3 00.93	99.95	1.809	.27634
975	100	2 56.29	99.95	1.762	.28361
1000	100	2 51.89	99.96	1.718	.29089
1025	100	2 47.69	99.96	1.676	.29816
1050	100	2 43.70	99.96	1.637	.30543
1075	100	2 39.89	99.96	1.598	.31270

TABLE 11.—*Continued*

EVEN RADIUS CURVES—*Continued*

Radius, Ft.	Station Length, Ft.	Deflection for One Station	Chord for One Station, Ft.	Deflection for One Foot	Length for One Minute, Ft.
1100	100	2° 36.26′	99.96	1.562′	0.31997
1125	100	2 32.79	99.97	1.527	.32725
1150	100	2 29.47	99.97	1.494	.33452
1175	100	2 26.29	99.97	1.462	.34179
1200	100	2 23.24	99.97	1.432	.34906
1225	100	2 20.32	99.97	1.403	.35634
1250	100	2 17.51	99.97	1.375	.36361
1275	100	2 14.81	99.97	1.348	.37088
1300	100	2 12.22	99.97	1.322	.37815
1350	100	2 07.32	99.97	1.273	.39270
1400	100	2 02.78	99.98	1.227	.40724
1450	100	1 58.54	99.98	1.185	.42179
1500	100	1 54.59	99.98	1.145	.43633
1550	100	1 50.89	99.98	1.108	.45087
1600	100	1 47.43	99.98	1.074	.46542
1650	100	1 44.17	99.98	1.041	.47996
1700	100	1 41.11	99.98	1.011	.49451
1750	100	1 38.22	99.99	.982	.50905
1800	100	1 35.49	99.99	.954	.52360
1850	100	1 32.91	99.99	.929	.53814
1900	100	1 30.47	99.99	.904	.55269
1950	100	1 28.15	99.99	.881	.56723
2000	100	1 25.94	99.99	.859	.58178
2050	100	1 23.85	99.99	.838	.59632
2100	100	1 21.85	99.99	.818	.61086
2150	100	1 19.95	99.99	.799	.62541
2200	100	1 18.13	99.99	.781	.63995
2250	100	1 16.35	99.99	.763	.65450
2300	100	1 14.73	99.99	.747	.66904
2350	100	1 13.14	99.99	.731	.68359
2400	100	1 11.62	99.99	.716	.69813
2450	100	1 10.16	99.99	.701	.71268
2500	100	1 08.75	99.99	.687	.72722
2550	100	1 07.40	99.99	.674	.74176
2600	100	1 06.11	99.99	.661	.75631
2650	100	1 04.86	99.99	.648	.77085
2700	100	1 03.66	99.99	.636	.78540
2750	100	1 02.50	99.99	.625	.79994
2800	100	1 01.39	99.99	.613	.81449
2850	100	1 00.31	99.99	.603	.82903
2900	100	0 59.27	99.99	.592	.84358
2950	100	0 58.26	99.99	.582	.85812
3000	100	0° 57.29	99.99	.572	.87267
3500	100	0 49.11	99.99	.491	1.08111
4000	100	0 42.97	100.00	.429	1.16356
5000	100	0 34.37	100.00	.343	1.45445
6000	100	0 28.64	100.00	.286	1.74534
7000	100	0 24.55	100.00	.245	2.03623
8000	100	0 21.48	100.00	.214	2.32712
9000	100	0 19.09	100.00	.190	2.61801
10000	100	0 17.18	100.00	.171	2.90890

TABLE 12

SPIRAL DATA

$k = \frac{1}{4}°$

D	L	Δ	o	t	X	Y
0° 30′	200.00	0° 30.0′	0.15	100.00	200.00	0.58
40	266.67	0 53.3	0.34	133.33	266.66	1.38
50	333.33	1 23.3	0.67	166.67	333.29	2.69
1° 00′	400.00	2° 00.0′	1.17	199.99	399.94	4.66
10	466.67	2 43.3	1.85	233.31	466.55	7.39
20	533.33	3 33.3	2.76	266.64	533.15	11.04
30	600.00	4 30.0	3.93	299.94	599.64	15.71

$k = \frac{1}{3}°$

0° 30′	133.33	0° 20.0′	0.06	66.67	133.33	0.26
40	177.78	0 35.6	0.15	88.89	177.77	0.61
50	222.22	0 55.6	0.29	111.11	222.21	1.16
1° 00′	266.67	1° 20.0′	0.52	133.33	266.65	2.06
10	311.11	1 48.9	0.82	155.55	311.07	3.29
20	355.56	2 22.2	1.23	177.77	355.50	4.91
30	400.00	3 00.0	1.75	199.98	399.88	6.98
40	444.44	3 42.2	2.32	222.19	444.26	9.28
50	488.89	4 28.9	3.16	244.39	488.59	12.63
2° 00′	533.33	5° 20.0′	4.14	266.59	532.85	16.54
10	577.78	6 15.6	5.25	288.77	577.06	20.99
20	622.22	7 15.6	6.56	311.04	621.20	26.26
30	666.67	8 20.0	8.07	333.09	665.23	31.29

$k = \frac{1}{2}°$

0° 30′	100.00	0° 15.0′	0.04	50.00	100.00	0.14
40	133.33	0 26.7	0.09	66.67	133.33	0.34
50	166.67	0 41.7	0.17	83.33	166.66	0.67
1° 00′	200.00	1° 00.0′	0.29	100.00	199.98	1.16
10	233.33	1 21.7	0.46	116.67	233.30	1.84
20	266.67	1 46.7	0.69	133.33	266.63	2.76
30	300.00	2 15.0	0.98	149.99	299.95	3.93
40	333.33	2 46.7	1.34	166.66	333.25	5.37
50	366.67	3 21.7	1.79	183.31	366.55	7.18
2° 00′	400.00	4° 00.0′	2.33	199.97	399.82	9.31
10	433.33	4 41.7	2.96	216.62	433.03	11.83
20	466.67	5 26.7	3.69	233.26	466.25	14.74
30	500.00	6 15.0	4.54	249.90	499.40	18.17
40	533.33	7 06.7	5.52	266.53	532.45	22.08
50	566.67	8 01.7	6.60	283.14	565.53	26.42
3° 00′	600.00	9° 00.0′	7.86	299.75	598.50	31.43

TABLE 12—*Continued*—SPIRAL DATA—*Continued*

$k = \frac{1}{4}°$

D	L	Δ	o	t	X	Y
1° 00′	133.33	0° 40.0′	0.13	66.67	133.33	0.52
10	155.56	0 54.4	0.20	77.78	155.56	0.82
20	177.78	1 11.1	0.31	88.89	177.78	1.23
30	200.00	1 30.0	0.44	100.00	200.00	1.75
40	222.22	1 51.1	0.60	111.11	222.21	2.39
50	244.44	2 14.4	0.80	122.22	244.42	3.20
2° 00′	266.67	2° 40.0′	1.04	133.32	266.63	4.14
10	288.89	3 07.8	1.32	144.43	288.82	5.26
20	311.11	3 37.8	1.64	155.53	310.99	6.55
30	333.33	4 10.0	2.02	166.63	333.15	8.08
40	355.56	4 44.4	2.45	177.74	355.32	9.81
50	377.78	5 21.1	2.94	188.84	377.48	11.75
3° 00′	400.00	6° 00.0′	3.49	199.93	399.58	13.96
10	422.22	6 41.1	4.10	211.01	421.62	16.28
20	444.44	7 24.4	4.79	222.10	443.72	19.85
30	466.67	8 10.0	5.54	233.17	465.71	21.76
40	488.89	8 57.8	6.40	244.24	487.69	25.18
50	511.11	9 47.8	7.28	255.30	509.61	28.53
4° 00′	533.33	10° 40.0′	8.26	266.36	531.47	32.28

$k = 1°$

1° 00′	100.00	0° 30.0′	0.07	50.00	100.00	0.29
10	116.67	0 40.8	0.11	58.33	116.67	0.46
20	133.33	0 53.3	0.17	66.67	133.33	0.69
30	150.00	1 07.5	0.24	75.00	150.00	0.98
40	166.67	1 23.3	0.34	83.33	166.66	1.34
50	183.33	1 40.8	0.45	91.67	183.32	1.80
2° 00′	200.00	2° 00.0′	0.58	100.00	199.98	2.33
10	216.67	2 20.8	0.74	108.33	216.64	2.96
20	233.33	2 43.3	0.92	116.66	233.28	3.69
30	250.00	3 07.5	1.14	124.99	249.92	4.55
40	266.67	3 33.3	1.38	133.31	266.55	5.52
50	283.33	4 00.8	1.65	141.65	283.19	6.61
3° 00′	300.00	4° 30.0′	1.96	149.97	299.82	7.85
10	316.67	5 00.8	2.31	158.29	316.43	9.22
20	333.33	5 33.3	2.69	166.62	333.03	10.75
30	350.00	6 07.5	3.12	174.93	349.58	12.46
40	366.67	6 43.3	3.58	183.25	366.19	14.33
50	383.33	7 20.8	4.09	191.57	382.73	16.37
4° 00′	400.00	8 00.0	4.65	199.87	399.22	18.59
10	416.67	8 40.8	5.26	208.17	415.72	21.06
20	433.33	9 23.3	5.91	216.48	432.19	23.64
30	450.00	10 07.5	6.61	224.77	448.62	26.45
40	466.67	10 53.3	7.37	233.04	464.93	29.49
50	483.33	11 40.8	8.19	241.33	481.35	32.76
5° 00′	500.00	12° 30.0′	9.06	249.60	497.60	36.25

TABLE 12—*Continued*—SPIRAL DATA—*Continued*

$k = 1\frac{1}{4}°$

D	L	Δ	m	t	X	Y
1° 00′	80.00	0° 24.0′	0.05	40.00	80.00	0.19
10	93.33	0 32.7	0.07	46.67	93.33	0.30
20	106.67	0 42.7	0.11	53.33	106.67	0.44
30	120.00	0 54.0	0.16	60.00	120.00	0.61
40	133.33	1 06.7	0.21	66.67	133.33	0.86
50	146.67	1 20.7	0.29	73.33	146.67	1.15
2 00′	160.00	1° 36.0	0.37	80.00	160.00	1.49
10	173.33	1 52.6	0.47	86.67	173.32	1.89
20	186.67	2 10.7	0.59	93.33	186.65	2.36
30	200.00	2 30.0	0.73	100.00	199.97	2.90
40	213.33	2 50.7	0.88	106.66	213.28	3.52
50	226.67	3 12.7	1.06	113.32	226.60	4.23
3° 00′	240.00	3° 36.0′	1.26	119.98	239.89	5.02
10	253.33	4 00.6	1.48	126.65	253.20	5.90
20	266.67	4 26.7	1.72	133.30	266.51	6.88
30	280.00	4 54.0	1.99	139.97	279.81	7.97
40	293.33	5 22.6	2.29	146.63	293.09	9.16
50	306.67	5 52.6	2.74	153.28	306.37	10.47
4° 00′	320.00	6° 24.0	2.98	159.93	319.58	11.88
10	333.33	6 56.7	3.36	166.59	332.85	13.43
20	346.67	7 30.7	3.79	173.23	346.07	15.10
30	360.00	8 06.0	4.24	179.88	359.28	16.91
40	373.33	8 42.6	4.73	186.53	372.49	18.42
50	386.67	9 20.6	5.25	193.16	385.65	20.94
5° 00′	400.00	10° 00.0′	5.82	199.80	398.80	23.18
20	426.67	11 22.8	7.39	212.89	424.99	28.11
40	453.33	12 50.6	8.46	226.29	451.05	33.68
6° 00′	480.00	14° 24.0′	10.03	239.49	476.94	39.95

$k = 1\frac{1}{2}°$

1° 30′	100.00	0° 45.0′	0.11	50.00	100.00	0.44
40	111.11	0 55.6	0.15	55.55	111.11	0.60
50	122.22	1 07.2	0.20	61.11	122.22	0.80
2° 00′	133.33	1° 20.0′	0.26	66.67	133.33	1.01
10	144.44	1 33.9	0.33	72.22	144.44	1.31
20	155.56	1 49.0	0.42	77.78	155.55	1.64
30	166.67	2 05.0	0.49	83.33	166.66	2.02
40	177.78	2 22.3	0.61	88.89	177.77	2.45
50	188.89	2 40.6	0.74	94.44	188.87	2.94
3° 00′	200.00	3° 00.0′	0.87	99.99	199.96	3.49
10	211.11	3 20.6	1.04	105.54	211.05	4.11
20	222.22	3 42.2	1.20	111.10	222.14	4.79
30	233.33	4 05.0	1.38	116.65	233.22	5.54
40	244.44	4 28.9	1.59	122.20	244.30	6.37
50	255.56	4 53.9	1.82	127.75	255.38	7.29

TABLE 12—*Continued*—SPIRAL DATA—*Continued*

D	L	Δ	o	t	X	Y
4° 00′	266.67	5° 20.0′	2.07	133.29	266.43	8.27
10	277.78	5 47.2	2.34	138.84	277.58	9.35
20	288.89	6 15.5	2.63	144.38	288.53	10.51
30	300.00	6 45.0	2.94	149.93	299.58	11.76
40	311.11	7 15.5	3.30	155.47	310.63	13.11
50	322.22	7 47.1	3.65	161.01	321.62	14.56
5° 00′	333.33	8° 20.0′	4.04	166.55	332.61	16.11
20	355.56	9 28.8	4.90	177.62	354.60	19.55
40	377.78	10 42.2	5.88	188.67	376.46	23.43
6° 00′	400.00	12° 00.0′	6.98	199.70	398.20	27.77
20	422.22	13 22.2	8.21	210.74	419.98	32.61
40	444.44	14 48.9	9.57	221.72	441.44	37.96
7° 00′	466.67	16° 20.0′	11.80	232.70	462.89	43.88
		$k = 2°$				
2° 00′	100.00	1° 00.0	0.14	50.00	100.00	0.58
10	108.33	1 10.4	0.18	54.17	108.33	0.74
20	116.67	1 21.7	0.23	58.33	116.67	0.92
30	125.00	1 33.7	0.28	62.50	124.99	1.14
40	133.33	1 46.7	0.34	66.67	133.32	1.38
50	141.67	2 00.4	0.41	70.83	141.65	1.66
3° 00′	150.00	2° 15.0′	0.49	75.00	149.98	1.97
10	158.33	2 30.4	0.58	79.17	158.30	2.31
20	166.67	2 46.7	0.67	83.33	166.63	2.70
30	175.00	3 03.8	0.78	87.49	174.95	3.12
40	183.33	3 21.7	0.88	91.66	183.27	3.58
50	192.67	3 40.4	1.04	96.32	192.59	4.16
4° 00′	200.00	4° 00.0′	1.16	99.98	199.90	4.66
10	208.33	4 20.4	1.31	104.15	208.21	5.26
20	216.67	4 41.7	1.48	108.31	216.53	5.91
30	225.00	5 03.8	1.66	112.47	224.83	6.63
40	233.33	5 26.7	1.85	116.64	233.13	7.38
50	241.67	5 50.4	2.05	120.79	241.43	8.22
5° 00′	250.00	6° 15.0′	2.27	124.95	249.70	9.07
20	266.67	7 06.7	2.76	133.26	266.25	11.02
40	283.33	8 01.7	3.31	141.58	282.79	13.20
6° 00′	300.00	9° 00.0′	3.93	149.88	299.28	15.66
20	316.67	10 01.7	4.62	158.17	315.71	18.41
40	333.33	11 06.7	5.38	166.46	332.07	21.43
7° 00′	350.00	12° 15.0′	6.23	174.73	348.38	24.80
20	366.67	13 26.7	7.17	182.99	364.63	28.48
40	383.33	14 41.7	8.18	191.25	380.81	32.47
8° 00′	400.00	16° 00.0′	9.27	199.48	396.88	37.04

TABLE 12—*Continued*—SPIRAL DATA—*Continued*

$k = 3°$

D	L	Δ	o	t	X	Y
3° 00′	100.00	1° 30.0′	0.23	50.00	100.00	0.87
10	105.56	1 40.3	0.26	52.78	105.56	1.05
20	111.11	1 51.1	0.30	55.55	111.11	1.20
30	116.67	2 02.5	0.35	58.33	116.67	1.39
40	122.22	2 14.4	0.40	61.11	122.22	1.59
50	127.78	2 26.9	0.45	63.89	127.78	1.82
4° 00′	133.33	2° 40.0	0.52	66.67	133.33	2.07
10	138.89	2 53.6	0.58	69.44	138.88	2.34
20	144.44	3 07.8	0.66	72.22	144.42	2.63
30	150.00	3 22.5	0.74	75.00	149.97	2.95
40	155.56	3 37.8	0.82	77.77	155.52	3.29
50	161.11	3 53.6	0.91	80.54	161.05	3.65
5° 00′	166.67	4° 10.0′	1.01	83.32	166.58	4.04
20	177.78	4 44.4	1.23	88.87	177.70	4.91
40	188.89	5 21.1	1.47	94.41	188.71	5.88
6° 00′	200.00	6° 00.0′	1.74	99.96	199.76	6.97
20	211.11	6 41.1	2.06	105.50	210.81	8.20
40	222.22	7 24.4	2.39	111.05	221.86	9.55
7° 00′	233.33	8° 10.0′	2.77	116.59	232.85	11.06
20	244.44	8 57.8	3.18	122.02	243.84	12.70
40	255.56	9 47.8	3.64	127.66	254.84	14.52
8° 00′	266.67	10° 40.0′	4.14	133.18	265.77	16.48
20	277.78	11 34.4	4.68	138.70	276.75	18.63
40	288.89	12 31.1	5.26	144.21	287.51	20.91
9° 00′	300.00	13° 30.0′	5.89	149.72	298.32	23.39
20	311.11	14 31.1	6.57	155.22	309.13	26.03
40	322.22	15 34.4	7.29	160.71	319.82	28.90
10° 00′	333.33	16° 40.0′	8.04	166.21	330.57	31.94

$k = 4°$

4° 00′	100.00	2° 00.0′	0.29	50.00	100.00	1.16
10	104.22	2 10.2	0.33	52.11	104.22	1.32
20	108.33	2 20.8	0.37	54.17	108.33	1.48
30	112.50	2 31.9	0.41	56.25	112.49	1.66
40	116.67	2 43.3	0.46	58.33	116.65	1.85
50	120.83	2 55.2	0.51	60.42	120.80	2.05
5° 00′	125.00	3° 07.5′	0.57	62.50	124.95	2.27
20	133.33	3 33.3	0.69	66.66	133.26	2.76
40	141.67	4 00.8	0.83	70.82	141.58	3.31
6° 00′	150.00	4° 30.0′	0.98	74.98	149.89	3.93
20	158.33	5 00.8	1.15	79.15	158.20	4.62
40	166.67	5 33.3	1.35	83.30	166.51	5.39
7° 00′	175.00	6° 07.5′	1.56	87.47	174 81	6.22
20	183.33	6 43.3	1.79	91.63	183.09	7.16
40	191.67	7 20.8	2.04	95.78	191.37	8.18

TABLE 12—*Continued*—SPIRAL DATA—*Continued*

D	L	Δ	o	t	X	Y
8° 00′	200.00	8° 00.0′	2.33	99.93	199.58	9.29
20	208.33	8° 40.8	2.63	104.09	207.85	10.49
40	216.67	9 23.3	2.96	108.23	216.07	11.76
9° 00′	225.00	10° 07.5′	3.31	112.38	224.28	13.20
20	233.33	10 53.3	3.69	116.53	232.49	14.71
40	241.67	11 40.8	4.11	120.66	240.65	16.35
10° 00′	250.00	12° 30.0′	4.54	124.80	248.80	18.08
30	262.50	13 46.9	5.26	131.00	261.00	20.88
11° 00′	275.00	15 07.5	6.05	137.18	273.08	23.98
30	287.50	16 31.9	6.91	143.35	285.10	27.36
12° 00′	300.00	18° 00.0′	7.85	149.51	297.06	31.01
30	312.50	19 31.9	8.87	155.64	308.83	34.96
13° 00′	325.00	21° 07.5′	9.98	161.76	320.56	39.22
30	337.50	22 46.9	11.18	167.86	332.16	43.78
14° 00′	350.00	24° 30.0′	12.47	143.93	343.58	48.67
			$k = 6°$			
6° 00′	100.00	3° 00.0′	0.44	50.00	99.99	1.75
20	105.56	3 20.5	0.51	52.78	105.54	2.06
40	111.11	3 42.2	0.60	55.55	111.07	2.39
7° 00′	116.67	4° 05.0′	0.69	58.32	116.61	2.77
20	122.22	4 28.9	0.80	61.10	122.14	3.19
40	127.78	4 53.9	0.91	63.87	127.68	3.64
8° 00′	133.33	5 20.0′	1.01	66.65	133.21	4.14
20	138.89	5 47.2	1.17	69.42	138.75	4.68
40	144.44	6 15.5	1.31	72.19	144.27	5.25
9° 00′	150.00	6° 45.0′	1.47	74.97	149.80	5.88
20	155.56	7 15.5	1.64	77.74	155.32	6.57
40	161.11	7 47.2	1.82	80.51	160.81	7.28
10° 00′	166.67	8° 20.0	2.02	83.77	166.31	8.07
30	175.00	9 11.3	2.34	87.43	174.58	9.33
11° 00′	183.33	10° 05.0′	2.69	91.58	182.79	10.46
30	191.67	11 01.2	3.07	95.71	190.95	12.24
12° 00′	200.00	12° 00.0′	3.49	99.85	199.10	13.89
30	208.33	13 01.2	3.94	103.99	207.25	15.67
13° 00′	216.67	14° 05.0′	4.44	108.11	215.35	17.58
30	225.00	15 11.2	4.96	112.24	223.44	19.70
14° 00′	233.33	16° 20.0′	5.53	116.35	231.41	21.92
15 00	250.00	18 45.0	6.79	124.55	247.30	26.87
16 00	266.67	21 20.0	8.23	132.71	262.95	32.48
17 00	283.33	24 05.0	9.91	140.84	278.35	38.72
18 00	300.00	27 00.0	11.68	148.89	293.33	45.68

TABLE 12—*Continued*—SPIRAL DATA—*Continued*

$k = 8°$

D	L	Δ	o	t	X	Y
8° 00′	100.00	4° 00.0′	0.58	49.99	99.97	2.33
20	104.22	4 20.4	0.66	52.10	104.18	2.63
40	108.33	4 41.7	0.74	54.16	108.27	2.96
9° 00′	112.50	5° 03.8′	0.83	56.24	112.42	3.31
20	116.67	5 26.7	0.92	58.31	116.57	3.70
40	120.83	5 50.4	1.02	60.40	120.71	4.10
10° 00′	125.00	6° 15.0′	1.14	62.48	124.85	4.55
30	131.22	6 53.4	1.31	65.58	131.03	5.25
11° 00′	137.50	7° 33.7′	1.51	68.71	137.26	6.04
30	143.67	8 15.9	1.73	71.84	143.37	6.89
12° 00′	150.00	9° 00.0′	1.96	74.94	149.64	7.83
30	156.22	9° 45.9′	2.22	78.03	155.74	8.83
13° 00′	162.50	10° 33.7′	2.50	81.16	161.96	9.94
30	168.67	11 23.4	2.79	84.22	168.02	11.12
14° 00′	175.00	12° 15.0′	3.12	87.37	174.22	12.40
30	181.22	13 08.4	3.46	90.45	180.26	13.75
15° 00′	187.50	14° 03.7′	3.83	93.56	186.36	15.22
16 00	200.00	16 00.0	4.64	99.74	198.44	18.42
17 00	212.50	18 03.7	5.57	106.10	210.40	22.03
18 00	225.00	20 15.0	6.60	112.03	222.18	26.06
19 00	237.50	22 33.7	7.75	118.14	233.74	30.43
20 00	250.00	25 00.0	9.02	124.21	245.26	35.41

$k = 10°$

10° 00′	100.00	5° 00.0′	0.73	49.99	99.92	2.91
30	105.00	5 30.8	0.84	52.48	104.88	3.36
11° 00′	110.00	6° 03.0′	0.97	54.98	109.86	3.87
30	115.00	6 36.8	1.11	57.47	114.83	4.42
12° 00′	120.00	7° 12.0′	1.26	59.97	119.80	5.02
30	125.00	7 48.8	1.42	62.46	124.76	5.67
13° 00′	130.00	8° 27.0′	1 60	64.95	129.70	6.37
30	135.00	9 06.8	1.79	67.44	134.64	7.13
14° 00′	140.00	9° 48.0′	1.99	69.93	139.58	7.95
30	145.00	10 30.8	2.22	72.42	144.52	8.83
15° 00′	150.00	11° 15.0′	2.45	74.90	149.40	9.77
16 00	160.00	12 48.0	2.97	79.87	159.22	11.84
17 00	170.00	14 27.0	3.56	84.82	168.92	14.18
18 00	180.00	16 12.0	4.23	89.76	178.56	16.81
19 00	190.00	18 03.0	4.97	94.69	188.14	19.68
20 00	200.00	20 00.0	5.79	99.60	197.60	22.89
21 00	210.00	22 03.0	6.70	104.48	206.88	26.36
22 00	220.00	24 12.0	7.69	109.34	216.04	30.21
23 00	230.00	26 27.0	8.78	114.18	225.08	34.36
24 00	240.00	28 48.0	9.96	118.99	233.94	38.82
25 00	250.00	31 15.0	11.24	123.76	242.56	43.59

TABLE 13.—FUNCTIONS OF A ONE-DEGREE CURVE.

The Long Chords, Mid-Ordinates, Externals, and Tangent Distances of this table are for a curve of 5730 feet radius. To find the corresponding functions of any other curve divide the tabular values by the degree of curve.

For metric curves having 20-metre chords, multiply the degree by 5 and enter the table with the result as a value of D, the tabular values being taken as metres instead of feet

Thus for a 1° 30′ metric curve having $I = 45°$ the tangent distance is $T = \frac{2373.4}{1.5 \times 5} = 316.45$ metres. Again, suppose $I = 38°$ and the long chord = 373.1 m. known and D required. The tabular $L.\ C.$ is 3731 m.; therefore $D = \frac{3731.0}{373.1 \times 5} = 2°\ 0'$.

	0°				1°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	0.00	0.000	0.000	0.00	100.00	0.218	0.218	50.00	0
2	3.33	0 000	0.000	1.67	103.33	0.233	0.233	51.67	2
4	6.67	0 001	0 001	3.33	106.66	0.248	0.248	53.33	4
6	10.00	0.002	0.002	5.00	110.00	0.264	0.264	55.00	6
8	13.33	0.004	0.004	6.67	113.33	0.280	0.280	56.67	8
10	16.67	0 006	0.006	8.33	116.66	0.297	0.297	58.33	10
12	20.00	0.009	0.009	10.00	120 00	0.314	0.314	60.00	12
14	23.33	0.012	0.012	11.67	123.33	0.332	0.332	61.67	14
16	26.67	0.015	0.015	13.33	126.66	0.350	0.350	63.33	16
18	30.00	0.019	0.019	15.00	130.00	0.368	0.368	65.00	18
20	33.33	0.024	0.024	16.67	133.33	0.388	0.388	66.67	20
22	36.67	0.029	0.029	18.33	136.66	0.407	0.407	68.33	22
24	40.00	0.035	0.035	20.00	140.00	0.427	0.427	70.00	24
26	43.33	0.041	0.041	21.67	143.33	0.448	0.448	71.67	26
28	46 67	0.048	0.048	23.33	146.66	0.469	0.469	73.33	28
30	50.00	0.054	0.054	25.00	150.00	0.491	0.491	75.00	30
32	53 33	0 062	0.062	26.67	153.33	0.513	0.513	76.67	32
34	56 67	0 070	0 070	28.33	156.66	0.536	0 536	78.33	34
36	60 00	0.079	0.079	30.00	160.00	0.559	0 559	80.00	36
38	63.33	0.088	0.088	31.67	163.33	0.582	0.582	81.67	38
40	66.67	0.097	0.097	33.33	166.66	0.606	0.606	83.33	40
42	70.00	0.107	0.107	35.00	170.00	0.630	0.630	85 00	42
44	73.33	0.117	0 117	36.67	173.33	0.655	0 655	86.67	44
46	76.67	0 128	0.128	38.33	176.66	0.681	0.681	88 33	46
48	80.00	0.140	0.140	40.00	180.00	0.706	0.706	90.00	48
50	83.33	0.151	0.151	41.67	183.33	0 733	0.733	91.67	50
52	86 67	0.164	0.164	43.33	186.66	0.760	0.760	93.33	52
54	90.00	0 176	0.176	45.00	190.00	0.788	0 788	95.00	54
56	93 33	0.190	0.190	46.67	193.33	0.815	0.815	96.67	56
58	96.67	0.204	0.204	48 33	196.66	0.844	0.844	98.33	58
60	100.00	0.218	0.218	50 00	199.98	0.873	0 873	100.00	60

′	2°				3°				′
	L. C.	M.	E.	T.	L. C	M.	E.	T.	
0	199.98	0.873	0.873	100.00	299.96	1.964	1.964	150.07	0
2	203.31	0.902	0.902	101.67	303.29	2.008	2.009	151.74	2
4	206.64	0 932	0.932	103.34	306.62	2.053	2.054	153.41	4
6	209.97	0.962	0.962	105.01	309 95	2.098	2.099	155 08	6
8	213.31	0.993	0.993	106.68	313.29	2.143	2.144	156.75	8
10	216.64	1.024	1.024	108.35	316.62	2.188	2.189	158.42	10
12	219.97	1.056	1.056	110.02	319.95	2.235	2.236	160.09	12
14	223.30	1.088	1.088	111.69	323.28	2.282	2.283	161.76	14
16	226.64	1.121	1.121	113.36	326.62	2.329	2.330	163.43	16
18	229.97	1.154	1.154	115.02	329.95	2.376	2.377	165.09	18
20	233.30	1.188	1.188	116.69	333.28	2.424	2.425	166.76	20
22	236.63	1.222	1.222	118.36	336.61	2.473	2.474	168.43	22
24	239.97	1.256	1.256	120.03	339.95	2.523	2.523	170.10	24
26	243.30	1.292	1.292	121.70	343.28	2.573	2.573	171.77	26
28	246.63	1.328	1.328	123.37	346.61	2.622	2.623	173.44	28
30	249.96	1.364	1.364	125.03	349.94	2.672	2.673	175.10	30
32	253.29	1.399	1.399	126.70	353.27	2.724	2.725	176.72	32
34	256.62	1.437	1.437	128.37	356.60	2.776	2.777	178.39	34
36	259.96	1.475	1.475	130.04	359.94	2.828	2.829	180.06	36
38	263.29	1.513	1.513	131.71	363.27	2.880	2.881	181.73	38
40	266.62	1.552	1.552	133.38	366 60	2.933	2.934	183.40	40
42	269.96	1.592	1.592	135.05	369.94	2.987	2.988	185.07	42
44	273.29	1.632	1.632	136.72	373.27	3.042	3 043	186.74	44
46	276.62	1.672	1.672	138.38	376.60	3.096	3 097	188.40	46
48	279.96	1.712	1.712	140.05	379.94	3.151	3.152	190.07	48
50	283.29	1.752	1.752	141.72	383.27	3.206	3.207	191.74	50
52	286.62	1.794	1.794	143.39	386 60	3.263	3.264	193.41	52
54	289.96	1.836	1.836	145.06	389.94	3.320	3.321	195.08	54
56	293 29	1.878	1.878	146.73	393.27	3.377	3 378	196.75	56
58	296.62	1.921	1.921	148.40	396.60	3.434	3.435	198.42	58
60	299.96	1.964	1.964	150.07	399.94	3.491	3.492	200 09	60

′	4°				5°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	399.94	3.491	3.492	200.09	499.88	5.454	5.459	250.17	0
2	403.27	3.550	3.551	201.76	503.21	5.527	5.533	251.84	2
4	406.60	3.609	3.610	203.43	506.54	5.601	5.607	253.51	4
6	409.93	3.668	3.670	205.10	509.87	5.675	5 681	255.18	6
8	413.26	3.727	3.730	206.77	513.20	5.749	5.755	256.85	8
10	416.59	3.787	3.790	208.44	516 53	5.823	5.829	258.52	10
12	419 92	3.848	3.851	210.11	519.86	5.899	5.905	260.20	12
14	423.26	3.910	3.913	211.77	523.19	5.975	5 981	261.86	14
16	426 59	3.972	3.975	213.45	526.52	6.052	6.058	263.54	16
18	429.92	4.034	4.037	215.11	529.85	6.129	6.135	265.20	18
20	433.25	4.096	4.099	216.78	533.18	6.206	6.212	266.87	20
22	436.58	4.160	4.163	218.45	536.51	6.284	6.290	268 54	22
24	439.91	4.224	4.227	220.12	539.84	6.362	6.369	270.21	24
26	443.24	4.288	4.291	221.79	543.17	6.441	6.448	271.88	26
28	446.58	4.353	4.356	223.46	546.50	6.520	6.527	273.54	28
30	449 91	4.418	4.421	225.13	549.83	6.599	6.606	275.21	30
32	453.24	4.484	4.487	226.80	553 17	6.680	6.687	276.88	32
34	456.57	4.550	4.554	228.47	556.50	6.761	6.768	278.55	34
36	459.90	4.617	4.621	230.14	559.83	6.842	6.849	280.23	36
38	463.23	4.684	4.688	231.81	563.16	6.923	6.931	281.90	38
40	466.56	4.751	4.755	233.48	566.49	7.005	7.013	283.57	40
42	469.89	4.820	4.824	235.15	569.82	7.088	7.096	285.24	42
44	473 23	4.889	4.893	236.82	573.15	7 171	7.180	286.91	44
46	476 56	4.958	4 962	238.48	576.48	7.255	7.264	288.59	46
48	479.89	5.027	5.031	240.15	579.81	7.339	7.348	290.26	48
50	483 22	5.096	5 100	241.82	583.14	7 423	7.432	291.93	50
52	486.55	5.167	5.171	243.49	586.47	7.508	7.517	293.60	52
54	489 88	5.238	5.243	245.16	589.80	7.593	7.603	295.27	54
56	493.21	5.310	5.315	246.83	593.13	7.678	7 689	296 95	56
58	496 54	5.382	5.387	248.50	596.46	7.764	7.775	298.62	58
60	499.88	5 454	5.459	250.17	599.80	7.850	7 861	300.30	60

′	6°				7°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	599.80	7.850	7.861	300.30	699.60	10.69	10.71	350.44	0
2	603.13	7.940	7.951	301.97	702.93	10.79	10.81	352.11	2
4	606 46	8.030	8.041	303.64	706.26	10.90	10.92	353.79	4
6	609.78	8.120	8.131	305 31	709.58	11.00	11.02	355.46	6
8	613.11	8.210	8.221	306 98	712.91	11.11	11.13	357.13	8
10	616.44	8.300	8.311	308.65	716.24	11.21	11.23	358.81	10
12	619.76	8.390	8.401	310.32	719.56	11.31	11.33	360.48	12
14	623.09	8.480	8 491	311.99	722.89	11.42	11.44	362.15	14
16	626.42	8.570	8.581	313.66	726.21	11.52	11.54	363 83	16
18	629.74	8.660	8.671	315.33	729.53	11.63	11.65	365.50	18
20	633.07	8.750	8.761	317.00	732.86	11.73	11.75	367.17	20
22	636.40	8.844	8.856	318.67	736.19	11.84	11.86	368.85	22
24	639.72	8.939	8.951	320.34	739.51	11.95	11.97	370.52	24
26	643.05	9.033	9.046	322.01	742.84	12.06	12.08	372.19	26
28	646.38	9.128	9.141	323.68	746.17	12.17	12.19	373.86	28
30	649.70	9.222	9.236	325.35	749.49	12.27	12.30	375.54	30
32	653.03	9.317	9.331	327.02	752.82	12.38	12.41	377.22	32
34	656.36	9.411	9.426	328.69	756.15	12.49	12 52	378.89	34
36	659.69	9.506	9.521	330.37	759.47	12.60	12.63	380 57	36
38	663.02	9.600	9.616	332.04	762.80	12.71	12.74	382.24	38
40	666.34	9.695	9.712	333.71	766.13	12.82	12.85	383.92	40
42	669.67	9.794	9.812	335.38	769.45	12.93	12.96	385.60	42
44	673.00	9.894	9.912	337.05	772.78	13.04	13.08	387.27	44
46	676.32	9.993	10.01	338.73	776.11	13.15	13.19	388.95	46
48	679.65	10.09	10.11	340.40	779.43	13.26	13.31	390.62	48
50	682 98	10.19	10.21	342.07	782.76	13.37	13.42	392.30	50
52	686.30	10.29	10.31	343.74	786.09	13.48	13.53	393.98	52
54	689 63	10.39	10.41	345.41	789.41	13.59	13.65	395.65	54
56	692.96	10.49	10.51	347.08	792.74	13.70	13.76	397.33	56
58	696.28	10.59	10.61	348.76	796.07	13.81	13.88	399.01	58
60	699 60	10.69	10.71	350.44	799.40	13 96	13.99	400.70	60

′	8°				9°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	799.40	13.96	13.99	400.70	899.10	17.66	17.71	450.95	0
2	802.72	14.07	14.10	402.37	902.42	17.79	17.84	452.63	2
4	806 04	14.19	14.22	404.05	905.74	17.92	17.98	454.31	4
6	809.37	14 31	14.34	405.72	909.07	18.06	18.11	455.98	6
8	812.69	14 43	14.46	407.39	912.39	18.19	18 25	457.66	8
10	816.01	14.55	14.58	409.06	915.71	18.32	18.38	459.34	10
12	819.34	14.66	14.70	410.74	919.04	18.46	18 52	461.02	12
14	822.66	14.78	14.82	412.41	922.36	18.59	18.65	462.70	14
16	825.98	14.90	14.94	414.08	925.68	18.72	18.79	464.37	16
18	829.31	15.02	15.06	415.75	929.01	18.86	18.92	466.05	18
20	832.63	15.14	15.18	417.43	932.33	18.99	19.06	467.73	20
22	835 95	15.26	15.30	419.10	935.65	19.12	19.19	469.41	22
24	839.28	15.38	15.43	420.77	938.98	19.26	19.33	471.08	24
26	842.60	15.51	15.55	422.45	942.30	19.40	19.47	472.76	26
28	845.92	15.63	15.68	424.12	945.62	19.54	19.61	474.43	28
30	849.25	15.75	15.80	425.79	948.95	19.68	19.75	476.10	30
32	852.57	15.88	15.93	427.47	952.27	19.82	19.89	477.78	32
34	855.89	16.00	16.05	429.15	955.59	19.96	20.03	479.46	34
36	859.22	16.12	16.18	430.82	958.92	20.10	20.17	481.14	36
38	862.54	16.25	16.30	432.50	962.24	20.24	20.31	482.83	38
40	865.86	16.38	16.43	434.18	965.56	20.38	20.45	484.51	40
42	869.19	16.50	16.55	435.86	968.89	20.52	20.59	486.19	42
44	872.51	16.63	16.68	437.54	972.21	20.66	20.74	487.87	44
46	875.83	16.76	16.81	439.21	975.53	20.80	20.88	489.56	46
48	879.16	16.89	16.94	440.89	978.86	20.94	21.03	491.24	48
50	882.48	17.02	17.07	442.57	982.18	21.09	21.17	492.92	50
52	885.80	17.14	17.19	444.25	985.50	21.23	21.31	494 60	52
54	889.13	17.27	17.32	445.93	988.83	21.37	21.46	496.28	54
56	892.45	17.40	17.45	447.60	992.15	21.51	21.60	497.96	56
58	895.77	17.53	17.58	449.28	995.47	21.65	21.75	499.65	58
60	899.10	17.66	17.71	450.95	998.80	21.80	21.89	501.32	60

′	10°				11°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	998.8	21.80	21.89	501.32	1098.4	26.38	26.50	551.74	0
2	1002.1	21.94	22.03	503.00	1101.7	26.54	26.66	553.42	2
4	1005.4	22.09	22.18	504.68	1105.0	26.70	26.83	555.10	4
6	1008.8	22.24	22.33	506.36	1108.3	26.86	26.99	556.78	6
8	1012.1	22.39	22.48	508.04	1111.7	27.02	27.16	558.46	8
10	1015.4	22.54	22.63	509.72	1115.0	27.18	27.32	560.14	10
12	1018.7	22.68	22.78	511.40	1118.3	27.34	27.48	561.82	12
14	1022 0	22.83	22.93	513.08	1121.6	27.51	27.65	563.50	14
16	1025.4	22.98	23.08	514.76	1124.9	27.67	27.81	565.18	16
18	1028.7	23.13	23.23	516.44	1128.2	27.83	27.98	566.86	18
20	1032.0	23.28	23.38	518.12	1131.6	28.00	28.14	568.54	20
22	1035.3	23.43	23.53	519.80	1134.9	28.17	28.30	570.22	22
24	1038.6	23.58	23.68	521.48	1138.2	28.34	28.47	571.90	24
26	1042.0	23.73	23.84	523.16	1141.5	28.50	28 64	573.58	26
28	1045 3	23.88	23.99	524.85	1144.8	28.67	28.81	575.27	28
30	1048.6	24.04	24.14	526.53	1148.1	28.84	28.98	576.95	30
32	1051.9	24.19	24.30	528.21	1151.5	29.00	29.14	578.63	32
34	1055.2	24.34	24.45	529.89	1154.8	29.17	29.31	580.32	34
36	1058.6	24.49	24.60	531.57	1158.1	29.34	29.48	582.00	36
38	1061.9	24.64	24.76	533.25	1161.4	29.50	29.65	583.69	38
40	1065.2	24.80	24.91	534.93	1164.7	29.67	29.82	585.37	40
42	1068.5	24.95	25.06	536.61	1168.0	29.84	29.99	587.05	42
44	1071.8	25.11	25.22	538.29	1171.4	30.01	30.17	588.74	44
46	1075.2	25.27	25.38	539.97	1174.7	30.18	30.34	590.42	46
48	1078.5	25.43	25.54	541.65	1178.0	30.35	30.52	592.11	48
50	1081.8	25.59	25.70	543.33	1181.3	30.51	30.69	593.79	50
52	1085.1	25.74	25.86	545.01	1184.6	30.70	30.86	595.47	52
54	1088.4	25.90	26.02	546.69	1187.9	30.87	31.04	597.16	54
56	1091.8	26.06	26.18	548.37	1191.3	31.01	31.21	598.84	56
58	1095.1	26.22	26.34	550.06	1194.6	31.21	31.39	600.53	58
60	1098.4	26.38	26.50	551.74	1197.9	31.39	31.56	602.22	60

′	12°				13°				′
	L. C.	M.	E.	T.	L. C.	M.	E	T.	
0	1197.9	31.39	31.56	602.22	1297.3	36.83	37.07	652.87	0
2	1201.2	31.57	31.73	603.91	1300.6	37.02	37.26	654.56	2
4	1204.5	31.74	31.91	605.60	1303.9	37.21	37.46	656.25	4
6	1207.8	31.92	32.09	607.28	1307.2	37.40	37.65	657.93	6
8	1211 1	32.09	32.27	608.97	1310.5	37.59	37.85	659.62	8
10	1214.5	32.27	32.45	610.66	1313.8	37 79	38.04	661.31	10
12	1217.8	32.45	32.63	612.35	1317.2	37.98	38.23	663.00	12
14	1221.1	32.62	32.81	614.04	1320.5	38.17	38.43	664.69	14
16	1224.4	32.80	32.99	615.72	1323.8	38.36	38.62	666.37	16
18	1227.7	32.97	33.17	617.41	1327.1	38.55	38.82	668.06	18
20	1231.0	33.15	33.35	619.10	1330.4	38.75	39.01	669.75	20
22	1234.3	33.33	33.53	620.79	1333.7	38.95	39.20	671.44	22
24	1237.7	33.51	33.72	622.48	1337.0	39.15	39.40	673.13	24
26	1241.0	33.69	33.90	624.16	1340.3	39.35	39.60	674.81	26
28	1244.3	33.87	34.09	625.85	1343.6	39.54	39.80	676.51	28
30	1247.6	34.06	34.27	627.55	1346.9	39.74	40.00	678.20	30
32	1250.9	34.24	34.45	629.24	1350.3	39.94	40.19	679.89	32
34	1254.2	34.42	34.64	630.93	1353.6	40.13	40.39	681.58	34
36	1257.5	34.60	34.82	632.61	1356.9	40.33	40.59	683.26	36
38	1260.8	34.78	35.01	634.30	1360.2	40.52	40.79	684.95	38
40	1264.2	34 97	35.19	635.99	1363.5	40.71	40.99	686.64	40
42	1267.5	35.16	35.37	637.68	1366.8	40.91	41.19	688 33	42
44	1270.8	35.34	35.56	639.37	1370.1	41.11	41.40	690.02	44
46	1274.1	35.53	35.75	641.05	1373.4	41.31	41.60	691.70	46
48	1277.4	35.71	35.94	642.74	1376.7	41.51	41.81	693.39	48
50	1280.7	35.90	36.13	644.43	1380.0	41.71	42.01	695.08	50
52	1284.0	36.09	36.31	646.12	1383.4	41.91	42 21	696.77	52
54	1287.4	36.27	36.50	647.81	1386.7	42.11	42.42	698.46	54
56	1290.7	36.46	36.69	649.49	1390.0	42.31	42.62	700.14	56
58	1294.0	36.64	36.88	651.18	1393.3	42.51	42.83	701.83	58
60	1297.3	36.83	37.07	652.87	1396.6	42.71	43.03	703.53	60

′	14°				15°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	1396.6	42.71	43.08	703.53	1495.9	49.02	49.44	754.35	0
2	1399.9	42 92	43.23	705.23	1499.2	49.24	49.66	756.05	2
4	1403.2	43.12	43.44	706.92	1502.5	49.46	49.89	757.74	4
6	1406.5	43.33	43.65	708.62	1505.8	49.68	50.11	759.44	6
8	1409.8	43.53	43.86	710.31	1509.1	49 90	50.34	761.13	8
10	1413.1	43.74	44.07	712.01	1512.4	50.12	50.56	762.83	10
12	1416 5	43.94	44.28	713.71	1515.7	50.34	50.78	764.53	12
14	1419.8	44.15	44.49	715.40	1519.0	50.56	51.01	766.22	14
16	1423.1	44.35	44.70	717.10	1522.3	50.78	51.23	767.92	16
18	1426.4	44.56	44.91	718.79	1525.6	51.00	51.46	769.61	18
20	1429.7	44.77	45.12	720.49	1528.9	51.22	51.68	771.31	20
22	1433 0	44 98	45.33	722.20	1532.2	51.44	51.90	773.01	22
24	1436.3	45.19	45.54	723.89	1535.5	51.67	52.13	774.70	24
26	1439.6	45.40	45.76	725.59	1538.8	51.89	52.36	776.40	26
28	1442.9	45.61	45.97	727.28	1542.1	52.12	52.59	778.09	28
30	1446 2	45 82	46.18	728.97	1545.4	52.34	52.82	779.79	30
32	1449.6	46.03	46.40	730.66	1548.7	52.57	53.05	781.49	32
34	1452.9	46 24	46.61	732.35	1552.0	52.79	53 28	783.19	34
36	1456.2	46.45	46.82	734.05	1555.3	53.02	53.51	784.89	36
38	1459.5	46.66	47.04	735.74	1558.6	53.24	53.74	786.59	38
40	1462.8	46.87	47.25	737.43	1561.9	53.47	53.97	788.29	40
42	1466.1	47.08	47.46	739.12	1565.2	53.69	54.20	789.99	42
44	1469.4	47.30	47.68	740.81	1568.5	53.92	54.44	791.69	44
46	1472.7	47.51	47.90	742.51	1571.8	54.15	54.67	793.39	46
48	1476.0	47.73	48.12	744.20	1575.1	54.38	54.91	795.09	48
50	1479.3	47.94	48.34	745.89	1578.4	54.61	55.14	796.79	50
52	1482.7	48.16	48.56	747.58	1581.7	54.84	55.37	798.49	52
54	1486.0	48.37	48.78	749.27	1585.0	55.07	55 61	800.19	54
56	1489.3	48.59	49.00	750.97	1588.3	55.30	55.84	801.89	56
58	1492.6	48.80	49.22	752.66	1591.6	55 53	56.08	803.59	58
60	1495.9	49.02	49.44	754.35	1594.9	55.76	56.31	805.29	60

′	16°				17°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	1594.9	55.76	56.31	805.29	1693.9	62.94	63.64	856.35	0
2	1598.2	55.99	56.54	806.99	1697.2	63.18	63.89	858.05	2
4	1601.5	56.23	56.78	808.64	1700.5	63.43	64.15	859.76	4
6	1604.8	56.46	57.02	810.39	1703.8	63.68	64.40	861.46	6
8	1608.1	56.70	57.26	812.09	1707.1	63 93	64.66	863.16	8
10	1611.4	56.93	57.50	813.79	1710.4	64.18	64.91	864.87	10
12	1614.7	57.17	57.74	815.49	1713.7	64.42	65.16	866.57	12
14	1618.0	57.40	57.98	817.19	1716.9	64.67	65.42	868.27	14
16	1621.3	57.64	58.22	818.89	1720.2	64.92	65.67	869.98	16
18	1624.6	57.87	58.46	820.59	1723.5	65.17	65.93	871.68	18
20	1627.9	58.11	58.70	822.29	1726.8	65.42	66.18	873.38	20
22	1631.2	58 34	58.94	823.99	1730.1	65.67	66.43	875.09	22
24	1634.5	58.58	59 19	825.69	1733.4	65.93	66.69	876.79	24
26	1637.8	58.82	59.43	827.39	1736.7	66.18	66.95	878.49	26
28	1641.1	59.06	59.68	829.09	1740.0	66.44	67.21	880.20	28
30	1644.4	59.30	59.92	830.79	1743.3	66.69	67.47	881.90	30
32	1647.7	59.54	60.16	832.49	1746.6	66.94	67.72	883.61	32
34	1651.0	59.78	60.41	834.20	1749.9	67.20	67.98	885.32	34
36	1654.3	60.02	60 65	835.90	1753.2	67.45	68.24	887.02	36
38	1657.6	60.26	60.90	837.61	1756.5	67.71	68.50	888.73	38
40	1660 9	60.50	61.14	839.31	1759.8	67.96	68.76	890.44	40
42	1664.2	60.74	61.39	841.01	1763.1	68.21	69.03	892.15	42
44	1667.5	60 99	61.64	842.72	1766.3	68.47	69.29	893.86	44
46	1670.8	61.23	61.89	844.42	1769.6	68.73	69.56	895.56	46
48	1674.1	61.48	62.14	846.13	1772.9	68.99	69.82	897.27	48
50	1677.4	61.72	62 39	847.83	1776 2	69.25	70.09	898.98	50
52	1680.7	61.96	62.64	849.53	1779.5	69.50	70.36	900.69	52
54	1684.0	62.21	62.89	851 24	1782.8	69.76	70.62	902.40	54
56	1687.3	62.45	63.14	852.94	1786.1	70.02	70.89	904.10	56
58	1690.6	62.70	63 39	854.65	1789.4	70.28	71.17	905.81	58
60	1693.9	62.94	63.64	856.35	1792.7	70.54	71.42	907.52	60

′	18°				19°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	1793.7	70.54	71.42	907.52	1891.5	78.58	79.65	958.86	0
2	1796.0	70.80	71.69	909.23	1894.8	78.86	79.94	960.57	2
4	1799.3	71.06	71.96	910 94	1898.1	79.13	80.22	962.30	4
6	1802.6	71.33	72.23	912.65	1901.3	79.41	80.51	964.00	6
8	1805.9	71.59	72.50	914.36	1904.6	79.68	80.79	965.72	8
10	1809.2	71.85	72.77	916.07	1907.9	79.96	81.08	967.43	10
12	1812.5	72.12	73.04	917.78	1911.2	80.24	81.37	969.15	12
14	1815.7	72.38	73.31	919.49	1914.5	80.51	81.65	970.86	14
16	1819.0	72.64	73.58	921.20	1917.8	80.79	81.94	972.58	16
18	1822.3	72 91	73.85	922.91	1921.0	81.07	82.22	974.29	18
20	1825.6	73.17	74.12	924.63	1924.3	81.35	82.51	976.01	20
22	1828.9	73.43	74.39	926.34	1927.6	81.63	82.80	977.72	22
24	1832 2	73.70	74.67	928.05	1930.9	81.91	83.09	979.44	24
26	1835.5	73.97	74.94	929.76	1934.2	82.20	83 38	981.15	26
28	1838.8	74.24	75.22	931.47	1937.5	82.48	83.67	982.86	28
30	1842.1	74.51	75.49	933.18	1940.7	82.76	83.97	984.58	30
32	1845.4	74.77	75.77	934.89	1944.0	83.05	84.26	986.30	32
34	1848.7	75.04	76.04	936.60	1947.3	83.33	84.55	988.02	34
36	1852.0	75.31	76.32	938.32	1950.6	83.61	84.84	989.74	36
38	1855.3	75.58	76.59	940.03	1953.9	83.90	85.13	991.46	38
40	1858.6	75.85	76.87	941.74	1957.2	84.18	85.43	993.18	40
42	1861.9	76.12	77.14	943.45	1960 4	84.47	85.73	994.90	42
44	1865.1	76.39	77.42	945.16	1963.7	84.75	86.02	996.62	44
46	1868.4	76.67	77.70	946.88	1967.0	85.04	86 32	998.34	46
48	1871.7	76.94	77.98	948.59	1970.3	85.32	86.61	1000.0	48
50	1875.0	77.21	78.26	950.30	1973.6	85.61	86.91	1001.8	50
52	1878.3	77.49	78.53	952.01	1976.9	85.90	87.21	1003.5	52
54	1881.6	77.76	78.81	953.72	1980.1	86.19	87.50	1005.2	54
56	1884.9	78.03	79.09	955.44	1983.4	86.47	87.80	1006.9	56
58	1888.2	78.31	79.37	957.15	1986.7	86.76	88.09	1008.6	58
60	1891.5	78.58	79.65	958 86	1990.0	87.05	88.39	1010.4	60

′	20°				21°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	1990.0	87.05	88.39	1010.4	2088.5	95.95	97 58	1062.0	0
2	1993.3	87.34	88.69	1012.1	2091.8	96.26	97.90	1063.7	2
4	1996.6	87.63	88 99	1013.8	2095.0	96.56	98.21	1065.4	4
6	1999.8	87.92	89.29	1015.5	2098.3	96.87	98.53	1067.2	6
8	2003.1	88.21	89.59	1017.2	2101.6	97.17	98.84	1068.9	8
10	2006.4	88.50	89 89	1019.0	2104.9	97.48	99.16	1070.6	10
12	2009.7	88.79	90.19	1020.7	2108.1	97.79	99.48	1072.4	12
14	2013.0	89.08	90.49	1022 4	2111.4	98.09	99.79	1074.1	14
16	2016 3	89.37	90.79	1024.1	2114.7	98.40	100.1	1075.8	16
18	2019.5	89.66	91.09	1025.8	2118.0	98.70	100.4	1077.5	18
20	2022.8	89.96	91.40	1027.6	2121.2	99.00	100.7	1079.3	20
22	2026.1	90.25	91.71	1029.3	2124.5	99.30	101.1	1081.0	22
24	2029.4	90.55	92.01	1031.0	2127.8	99.60	101.4	1082.7	24
26	2032.7	90.85	92.32	1032.7	2131.0	99.90	101.7	1084.4	26
28	2036.0	91.15	92.62	1034.4	2134.3	100.2	102.0	1086.2	28
30	2039.2	91.45	92.93	1036.1	2137.6	100.5	102.3	1087.9	30
32	2042.5	91.74	93.24	1037.9	2140.9	100.8	102.7	1089.6	32
34	2045.8	92.04	93.54	1039.6	2144.1	101.1	103.0	1091.3	34
36	2049.1	92.34	93.85	1041.3	2147.4	101.4	103.3	1093.1	36
38	2052.4	92.64	94.15	1043.0	2150.7	101.7	103.6	1094.8	38
40	2055.7	92.94	94.46	1044.8	2154.0	102.1	104.0	1096.5	40
42	2058.9	93.24	94.78	1046.5	2157.2	102.4	104.3	1098.3	42
44	2062.2	93.54	95.09	1048.2	2160.5	102.7	104.6	1100.0	44
46	2065.5	93.84	95.40	1049.9	2163.8	103.0	104.9	1101.7	46
48	2068.8	94.14	95.71	1051.7	2167.1	103.3	105.3	1103.4	48
50	2072.1	94.44	96.03	1053.4	2170.3	103.6	105.6	1105.2	50
52	2075.4	94.74	96.34	1055.1	2173.6	103.9	105.9	1106.9	52
54	2078.6	95.04	96.65	1056.8	2176.9	104.2	106.3	1108.6	54
56	2081.9	95.34	96.96	1058.6	2180.1	104.5	106.6	1110.3	56
58	2085.2	95.64	97.27	1060.3	2183.4	104.8	106.9	1112.1	58
60	2088.5	95.95	97.58	1062.0	2186.7	105.2	107.2	1113.8	60

′	22°				23°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	2186.7	105.2	107.2	1113.8	2284.8	115.0	117.4	1165.8	0
2	2190.0	105.6	107.6	1115.5	2288.1	115.3	117.7	1167.5	2
4	2193.2	105.9	107.9	1117.3	2291.3	115.7	118.1	1169.2	4
6	2196.5	106.2	108.2	1119.0	2294.6	116.0	118.4	1171.0	6
8	2199.8	106.5	108.6	1120.7	2297.8	116.4	118.8	1172.7	8
10	2203.0	106.8	108.9	1122.4	2301.1	116.7	119.1	1174.4	10
12	2206 3	107.1	109.2	1124.2	2304.4	117.0	119 5	1176.2	12
14	2209.6	107.4	109.6	1125 9	2307.6	117.4	119.8	1177.9	14
16	2212 9	107.7	109.9	1127.6	2310.9	117.7	120.2	1179.7	16
18	2216.1	108.0	110.2	1129.4	2314.1	118.1	120.5	1181.4	18
20	2219.4	108.4	110.6	1131.1	2317.4	118.4	120.9	1183.1	20
22	2222.7	108.7	110.9	1132 8	2320.7	118 7	121 2	1184.9	22
24	2225.9	109.0	111.2	1134.6	2323.9	119.1	121 6	1186.6	24
26	2229.2	109.4	111.6	1136.3	2327.2	119.4	121.9	1188.4	26
28	2232.5	109.7	111.9	1138.0	2330.4	119.8	122.3	1190.1	28
30	2235.7	110.0	112.3	1139.7	2333.7	120.1	122.6	1191.8	30
32	2239.0	110.4	112.6	1141.5	2337.0	120 4	123.0	1193.6	32
34	2242 3	110.7	112.9	1143 2	2340.2	120.8	123.3	1195.3	34
36	2245.6	111.0	113.3	1144.9	2343.5	121.1	123.7	1197.1	36
38	2248.8	111.4	113.6	1146.7	2346.7	121.5	124.1	1198.8	38
40	2252.1	111.7	113.9	1148.4	2350 0	121.8	124.4	1200.5	40
42	2255.4	112 0	114.3	1150.1	2353.3	122.1	124.8	1202.3	42
44	2258.6	112.3	114.6	1151.9	2356.5	122 5	125.1	1204.0	44
46	2261.9	112.7	115.0	1153.6	2359.8	122.8	125.5	1205.8	46
48	2265.2	113.0	115.3	1155.4	2363.0	123.2	125.8	1207.5	48
50	2268.4	113.3	115.7	1157.1	2366.3	123.5	126.2	1209.2	50
52	2271.7	113.7	116.0	1158.8	2369.6	123.8	126.6	1211.0	52
54	2275.0	114.0	116.3	1160.6	2372.8	124.2	126.9	1212.7	54
56	2278.3	114.3	116.7	1162.3	2376.1	124.5	127.3	1214.5	56
58	2281.5	114.7	117.0	1164.0	2379.3	124.9	127.6	1216.2	58
60	2284.8	115.0	117.4	1165.8	2382.6	125.2	128.0	1218.0	60

′	24°				25°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	2382.6	125.2	128.0	1218.0	2480.4	135.8	139.1	1270.3	0
2	2385.9	125.5	128.4	1219.7	2483.6	136.2	139.5	1272.0	2
4	2389.1	125.9	128.7	1221.4	2486.9	136.5	139.9	1273.8	4
6	2392.4	126.2	129.1	1223.2	2490.1	136.9	140.3	1275.5	6
8	2395.6	126.6	129.5	1224.9	2493.4	137.2	140.6	1277.3	8
10	2398.9	126.9	129.8	1226.7	2496.6	137.6	141.0	1279.0	10
12	2402.2	127.3	130.2	1228.4	2499.9	138.0	141.4	1280.8	12
14	2405.4	127.6	130.6	1230.2	2503.1	138.3	141.8	1282.5	14
16	2408.7	128.0	130.9	1231.9	2506.4	138.7	142.2	1284.3	16
18	2411.9	128.3	131.3	1233.6	2509.6	139.0	142.5	1286.1	18
20	2415.2	128.7	131.7	1235.4	2512.9	139.4	142.9	1287.8	20
22	2418.5	129.0	132.0	1237.1	2516.1	139.8	143.3	1289.6	22
24	2421.7	129.4	132.4	1238.9	2519.4	140.1	143.7	1291.3	24
26	2425.0	129.7	132.8	1240.6	2522.6	140.5	144.1	1293.1	26
28	2428.2	130.1	133.1	1242.4	2525.9	140.8	144.5	1294.8	28
30	2431.5	130.4	133.5	1244.1	2529.1	141.2	144.9	1296.6	30
32	2434.8	130 8	133.9	1245.8	2532.4	141.6	145.3	1298.3	32
34	2438.0	131.1	134.2	1247.6	2535.6	142.0	145.6	1300.1	34
36	2441.3	131.5	134.6	1249.3	2538.9	142.3	146.0	1301.8	36
38	2444.5	131.8	135.0	1251.1	2542.1	142.7	146.4	1303.6	38
40	2447.8	132.2	135.4	1252.8	2545.4	143.1	146.8	1305.3	40
42	2451.1	132.6	135.7	1254.6	2548.6	143 5	147.2	1307.1	42
44	2454.3	132.9	136.1	1256.3	2551.9	143.8	147.6	1308.8	44
46	2457.6	133.3	136.5	1258.1	2555.1	144.2	148.0	1310.6	46
48	2460.8	133.6	136.9	1259.8	2558.4	144.5	148.4	1312.4	48
50	2464.1	134.0	137.2	1261.5	2561.6	144.9	148.8	1314.1	50
52	2467.4	134.4	137.6	1263.3	2564.9	145.3	149.2	1315.9	52
54	2470.6	134.7	138.0	1265.0	2568.1	145.7	149.5	1317.6	54
56	2473.9	135.1	138.4	1266.8	2571.4	146.0	149.9	1319.4	56
58	2477.1	135.4	138.7	1268.5	2574.6	146.4	150.3	1321.1	58
60	2480.4	135.8	139.1	1270.3	2577.9	146.8	150.7	1322.9	60

′	26°				27°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	2577.9	146.8	150.7	1322.9	2675.3	158.3	162.8	1375.6	0
2	2581.1	147.1	151.1	1324.6	2678.5	158.6	163.2	1377.4	2
4	2584.4	147.5	151.5	1326.4	2681.8	159.0	163.7	1379.2	4
6	2587.6	147.9	151.9	1328.1	2685.0	159.4	164.1	1380.9	6
8	2590.9	148.3	152.3	1329.9	2688.2	159.8	164.5	1382.7	8
10	2594.1	148.7	152.7	1331.6	2691.5	160.2	164.9	1384.5	10
12	2597 4	149.1	153.1	1333.4	2694.7	160.6	165.3	1386.2	12
14	2600 6	149.4	153.5	1335.2	2698.0	161.0	165.7	1388.0	14
16	2603.9	149 8	153.9	1336.9	2701.2	161.4	166 1	1389.8	16
18	2607.1	150 2	154.3	1338.7	2704.4	161.8	166.5	1391.5	18
20	2610.4	150 6	154.7	1340.4	2707.7	162.2	167.0	1393.3	20
22	2613.6	151 0	155.1	1342.2	2710.9	162.6	167.4	1395.0	22
24	2616 9	151.4	155.5	1343 9	2714.1	163 0	167.8	1396.8	24
26	2620.1	151.7	155.9	1345.7	2717.4	163.4	168.2	1398.6	26
28	2623.4	152.1	156.3	1347.4	2720 6	163.8	168.6	1400.3	28
30	2626.6	152.5	156.7	1349.2	2723.8	164.2	169.1	1402.1	30
32	2629.8	152.9	157 1	1351.0	2727.1	164.6	169.5	1403.9	32
34	2633.1	153.3	157.5	1352.7	2730.3	165.0	169.9	1405.6	34
36	2636.3	153 7	157.9	1354 5	2733.6	165.4	170.3	1407.4	36
38	2639.6	154.0	158.3	1356.2	2736.8	165.8	170.8	1409.2	38
40	2642.8	154 4	158.7	1358.0	2740.0	166.2	171.2	1410.9	40
42	2646.1	154.8	159.1	1359.8	2743 3	166.6	171.6	1412.7	42
44	2649.3	155.2	159.5	1361.5	2746.5	167.0	172 0	1414.5	44
46	2652.6	155 6	160.0	1363.3	2749.7	167 4	172.5	1416.3	46
48	2655.8	156.0	160.4	1365.1	2753.0	167.8	172.9	1418.0	48
50	2659.1	156 3	160.8	1366.8	2756.2	168.2	173 3	1419.8	50
52	2662.3	156.7	161.2	1368.6	2759.5	168.6	173.7	1421.6	52
54	2665.6	157 1	161.6	1370.4	2762.7	169.0	174.1	1423.3	54
56	2668.8	157 5	162.0	1372.1	2765.9	169.4	174.6	1425.1	56
58	2672.1	157.9	162.4	1373.9	2769.2	169.8	175.0	1426.9	58
60	2675.3	158.3	162.8	1375.6	2772.4	170.2	175.4	1428.6	60

′	28°				29°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	2772.4	170.2	175.4	1428.6	2869.4	182.5	188.5	1481.9	0
2	2775.6	170.6	175.8	1430.4	2872.6	182.9	189.0	1483.7	2
4	2778.9	171.0	176.3	1432.2	2875.8	183 3	189.4	1485.4	4
6	2782.1	171.4	176 7	1434.0	2879.1	183.7	189.9	1487.2	6
8	2785.3	171.8	177.1	1435.7	2882.3	184.2	190.3	1489.0	8
10	2788.6	172.2	177.6	1437.5	2885.5	184.6	190.8	1490.8	10
12	2791.8	172.6	178.0	1439.3	2888.7	185.0	191.2	1492.6	12
14	2795.0	173 0	178.4	1441.1	2892.0	185.4	191.7	1494.3	14
16	2798.3	173.4	178.9	1442.8	2895.2	185.8	192.1	1496.1	16
18	2801.5	173.8	179.3	1444.6	2898.4	186.3	192.5	1497.9	18
20	2804.7	174.3	179.7	1446.4	2901.6	186.7	193.0	1499.7	20
22	2808.0	174.7	180.2	1448.2	2904.8	187.1	193.5	1501.5	22
24	2811.2	175.1	180.6	1449.9	2908.1	187.5	193.9	1503.2	24
26	2814.4	175.5	181.0	1451.7	2911.3	188.0	194.4	1505.0	26
28	2817.7	175.9	181.5	1453.5	2914.5	188.4	194.8	1506.8	28
30	2820.9	176 3	181.9	1455.2	2917.7	188.8	195.3	1508.6	30
32	2824.1	176.7	182.3	1457.0	2921.0	189.2	195.7	1510.4	32
34	2827 4	177.1	182.8	1458.8	2924.2	189.7	196.2	1512.1	34
36	2830.6	177.5	183.2	1460.6	2927.4	190.1	196.7	1513.9	36
38	2833.8	177.9	183.6	1462.3	2930.6	190.5	197.1	1515.7	38
40	2837.1	178.4	184.1	1464.1	2933.9	190.9	197.6	1517.5	40
42	2840 3	178.8	184.5	1465.9	2937.1	191.4	198.0	1519.3	42
44	2843 5	179 2	185.0	1467.7	2940.3	191.9	198.5	1521.0	44
46	2846.8	179.6	185.4	1469.5	2943.5	192.4	198.9	1522.8	46
48	2850.0	180 0	185.9	1471.2	2946.8	192.8	199.4	1524.6	48
50	2853.2	180.4	186.3	1473.0	2950.0	193.2	199.8	1526.4	50
52	2856.5	180.8	186.8	1474.8	2953.2	193.6	200.3	1528.2	52
54	2859.7	181.2	187.2	1476.6	2956.4	194.0	200.8	1530.0	54
56	2862.9	181.6	187.6	1478.3	2959.6	194.4	201.2	1531.7	56
58	2866.2	182 0	188.1	1480.1	2962.9	194.8	201.7	1533.5	58
60	2869.4	182 5	188.5	1481.9	2966.1	195.2	202.1	1535 3	60

′	30°				31°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	2966.1	195.2	202.1	1535.3	3062.6	208.4	216 3	1589.0	0
2	2969.3	195.6	202.6	1537.1	3065.8	208.8	216 8	1590.8	2
4	2972.5	196.1	203.1	1538.9	3069.0	209.3	217 2	1592.6	4
6	2975.7	196.5	203.5	1540.7	3072.2	209.7	217.7	1594.4	6
8	2979.0	197.0	204.0	1542.5	3075.4	210.2	218 2	1596.2	8
10	2982.2	197.4	204.5	1544.3	3078.6	210.6	218.7	1598.0	10
12	2985.4	197.8	204.9	1546.0	3081.8	211.1	219.2	1599.8	12
14	2988.6	198.2	205.4	1547.8	3085.0	211.5	219.6	1601.6	14
16	2991.8	198.6	205.9	1549.6	3088.3	212.0	220.1	1603.4	16
18	2995.0	199.1	206.3	1551.4	3091.5	212.4	220.6	1605.2	18
20	2998.3	199 5	206.8	1553.2	3094.7	212.9	221.1	1607.0	20
22	3001.5	199.9	207.3	1555.0	3097.9	213.3	221.6	1608.8	22
24	3004.7	200 4	207.7	1556.8	3101.1	213.8	222.1	1610.6	24
26	3007.9	200.8	208.2	1558.6	3104.3	214.2	222.6	1612.4	26
28	3011.1	201.3	208.7	1560.4	3107.5	214.7	223.0	1614.2	28
30	3014.3	201.7	209.1	1562.2	3110.7	215.1	223.5	1616.0	30
32	3017 6	202 1	209 6	1564.0	3113.9	215.6	224.0	1617.8	32
34	3020 8	202 6	210 1	1565.7	3117.1	216.0	224.5	1619.6	34
36	3024.0	203.0	210.5	1567.5	3120.3	216.5	225.0	1621.4	36
38	3027.2	203.5	211.0	1569.3	3123.5	216.9	225.5	1623.2	38
40	3030.4	203.9	211 5	1571.1	3126.7	217.4	226.0	1625.0	40
42	3033.6	204 3	212 0	1572.9	3129.9	217.8	226.5	1626.8	42
44	3036.9	204 8	212.4	1574.7	3133.1	218.3	227.0	1628.6	44
46	3040.1	205.2	212.9	1576.5	3136.4	218.7	227.5	1630.5	46
48	3043.3	205.7	213.4	1578.3	3139.6	219.2	228.0	1632.3	48
50	3046.5	206.1	213.9	1580.1	3142.8	219.6	228.4	1634.1	50
52	3049.7	206.5	214.4	1581.9	3146.0	220.1	228.9	1635.9	52
54	3052.9	207 0	214.8	1583.7	3149.2	220.5	229.4	1637.7	54
56	3056.2	207.4	215.3	1585.5	3152.4	221.0	229.9	1639.5	56
58	3059.4	207.9	215.8	1587.2	3155.6	221.5	230.4	1641.3	58
60	3062 6	208.4	216 3	1589.0	3158.8	222.0	230.9	1643.1	60

′	32°				33°				′
	L. C.	M.	E.	T.	L. C	M.	E.	T.	
0	3158.8	222.0	230.9	1643.1	3254.9	236.0	246.1	1697 3	0
2	3162.0	222 5	231.4	1644 9	3258 1	236.4	246.6	1699.1	2
4	3165.2	222.9	231.9	1646.7	3261.3	236 9	247.1	1700 9	4
6	3168.4	223.4	232.4	1648.5	3264.5	237.4	247.7	1702.7	6
8	3171.6	223 8	232.9	1650.3	3267 7	237.9	248.2	1704.5	8
10	3174.8	224.3	233.4	1652.1	3270.8	238.4	248.7	1706.4	10
12	3178.0	224 8	233.9	1653.9	3274 0	238.9	249.2	1708.2	12
14	3181.2	225.2	234.4	1655 7	3277.2	239 3	249.7	1710.0	14
16	3184.4	225 7	234.9	1657.5	3280.4	239.8	250.2	1711.8	16
18	3187.6	226.1	235.4	1659.3	3283.6	240.3	250.8	1713.6	18
20	3190.8	226.6	235.9	1661.1	3286.8	240.8	251.3	1715.5	20
22	3194.0	227.1	236.4	1662.9	3290 0	241.2	251 8	1717.3	22
24	3197 2	227.5	236 9	1664.7	3293.2	241.7	252.3	1719.1	24
26	3200.4	228.0	237.4	1666 5	3296.4	242.2	252 9	1720.9	26
28	3203.6	228.4	237 9	1668.3	3299.6	242.7	253.4	1722.7	28
30	3206.8	228 9	238.4	1670.1	3302.7	243.2	253.9	1724.6	30
32	3210.0	229.4	239.0	1671.9	3305 9	243.6	254.4	1726.4	32
34	3213.2	229 8	239.5	1673 7	3309 1	244.1	255.0	1728.2	34
36	3216 5	230.3	240.0	1675.5	3312.3	244.6	255.5	1730.0	36
38	3219.7	230.7	240.5	1677.4	3315.5	245.1	256.0	1731.8	38
40	3222 9	231.2	241.0	1679.2	3318.7	245.6	256 5	1733.6	40
42	3226 1	231.7	241.5	1681.0	3321.9	246.0	257.1	1735.5	42
44	3629.3	232.2	242.0	1682 8	3325.1	246.5	257.6	1737.3	44
46	3232.5	232.6	242.5	1684.6	3328.3	247.0	258.1	1739.1	46
48	3235.7	233.1	243.0	1686.4	3331.5	247 5	258.6	1740.9	48
50	3238.9	233.5	243.5	1688.2	3334.6	248.0	259.2	1742.7	50
52	3242.1	234.0	244.1	1690.0	3337.8	248.4	259.7	1744.6	52
54	3245 3	234.5	244.6	1691.8	3341.0	248.9	260 2	1746.4	54
56	3248.5	235.0	245.1	1693.7	3344.2	249.4	260.8	1748.2	56
58	3251.7	235.5	245.6	1695 5	3347.4	249.9	261.3	1750.0	58
60	3254.9	236.0	246.1	1697.3	3350.6	250.4	261.8	1751.8	60

′	34°				35°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	3350.6	250.4	261.8	1751.9	3446.1	265.2	278.1	1806.7	0
2	3353.8	250.8	262.3	1753.7	3449.3	265.7	278.6	1808.5	2
4	3357.0	251.2	262.9	1755.5	3452.5	266.2	279.2	1810.3	4
6	3360.1	251.7	263.4	1757.3	3455.6	266.7	279.7	1812.2	6
8	3363 3	252.2	264.0	1759.1	3458.8	267.2	280.3	1814.0	8
10	3366.5	252.7	264.5	1761.0	3462.0	267.7	280.8	1815.8	10
12	3369.7	253 2	265.0	1762.8	3465.2	268.2	281.4	1817.7	12
14	3372.9	253.7	265.6	1764.6	3468.3	268.7	281.9	1819.5	14
16	3376.1	254.2	266.1	1766.4	3471.5	269.2	282.5	1821.3	16
18	3379.2	254.7	266.7	1768.3	3474.7	269.7	283.0	1823.2	18
20	3382.4	255.2	267.2	1770.1	3477.9	270.2	283.6	1825.0	20
22	3385.6	255.7	267.7	1771.9	3481.0	270.7	284.2	1826.8	22
24	3388.8	256.2	268.3	1773.7	3484.2	271.2	284.7	1828.7	24
26	3392.0	256.7	268.8	1775.6	3487.4	271.7	285.3	1830.5	26
28	3395.2	257.2	269.3	1777.4	3490.6	272.2	285.9	1832.3	28
30	3398.3	257.7	269.9	1779.2	3493.7	272.7	286.4	1834.2	30
32	3401.5	258.2	270.4	1781.0	3496.9	273.2	287.0	1836.0	32
34	3404.7	258.7	271.0	1782.9	3500.1	273.7	287.5	1837.8	34
36	3407 9	259 2	271.5	1784.7	3503.3	274.2	288.1	1839.7	36
38	3411.1	259.7	272.0	1786.5	3506.5	274.7	288.7	1841.5	38
40	3414.3	260.2	272.6	1788.4	3509.6	275.2	289.2	1843.4	40
42	3417.4	260.7	273.1	1790.2	3512.8	275.7	289.8	1845.2	42
44	3420.6	261.2	273.7	1792.0	3516.0	276.2	290.4	1847.1	44
46	3423.8	261.7	274.2	1793.9	3519.2	276.7	290.9	1848.9	46
48	3427.0	262.2	274.8	1795.7	3522.3	277.2	291.5	1850.7	48
50	3430.2	262.7	275.3	1797.5	3525.5	277.7	292.0	1852.6	50
52	3433.4	263.2	275.9	1799.3	3528.7	278.2	292.6	1854.4	52
54	3436.5	263.7	276.4	1801.2	3531.9	278.7	293.2	1856.3	54
56	3439.7	264.2	277.0	1803.0	3535.0	279.2	293.7	1858.1	56
58	3442.9	264.7	277.5	1804.8	3538.2	279.8	294.3	1859.9	58
60	3446 1	265.2	278.1	1806.7	3541.4	280.4	294.9	1861.8	60

′	36°				37°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	3541.4	280.4	294.9	1861.8	3636.3	296.1	312.3	1917.3	0
2	3544.6	280 9	295.4	1863.6	3639.5	296.6	312.8	1919.1	2
4	3547.7	281.4	296.0	1865.5	3642 6	297.1	313.4	1921.0	4
6	3550.9	281.9	296.6	1867.3	3645.8	297.7	314.0	1922.8	6
8	3554.0	282.5	297.2	1869.2	3648.9	298.2	314.6	1924.7	8
10	3557.2	283.0	297.7	1871.0	3652.1	298.7	315.2	1926.5	10
12	3560.4	283.5	298.3	1872.9	3655.2	299.3	315.8	1928.4	12
14	3563 5	284.0	298.9	1874.7	3658.4	299.8	316.4	1930 2	14
16	3566.7	284.6	299.5	1876.5	3661.6	300.3	317.0	1932.1	16
18	3569.9	285.1	300.0	1878.4	3664.7	300.9	317.5	1933.9	18
20	3573.0	285.6	300.6	1880.2	3667.9	301.4	318.1	1935.8	20
22	3576.2	286.1	301.2	1882.1	3671.0	301.9	318.7	1937.6	22
24	3579.4	286.7	301.8	1883.9	3674.2	302.5	319.3	1939.5	24
26	3582.5	287.2	302.3	1885.8	3677.3	303.0	319.9	1941.3	26
28	3585.7	287.7	302.9	1887.6	3680.5	303.5	320.5	1943.2	28
30	3588.8	288.2	303.5	1889.5	3683.6	304.1	321.1	1945.0	30
32	3592 0	288.8	304.1	1891.3	3686.8	304.6	321.7	1946.9	32
34	3595.2	289.3	304.6	1893.2	3690.0	305.1	322.3	1948.8	34
36	3598.3	289 8	305.2	1895.0	3693.1	305.7	322.9	1950.6	36
38	3601.5	290.3	305.8	1896.9	3696.3	306.2	323.5	1952.5	38
40	3604.7	290 9	306.4	1898.7	3699.4	306.7	324.2	1954.4	40
42	3607.8	291.4	307.0	1900.6	3702.6	307.3	324.8	1956.2	42
44	3611.0	291.9	307.5	1902.4	3705.7	307.8	325.4	1958.1	44
46	3614.1	292.4	308.1	1904.3	3708.9	308.3	326.0	1960.0	46
48	3617.3	293.0	308.7	1906.1	3712.1	308.9	326.6	1961.8	48
50	3620 5	293.5	309.3	1908.0	3715.2	309.4	327.2	1963.7	50
52	3623 6	294.0	309.9	1909.8	3718.4	309.9	327.8	1965.5	52
54	3626.8	294.5	310.5	1911.7	3721 5	310.5	328.4	1967.4	54
56	3630.0	295.1	311.1	1913.5	3724.7	311.0	329.0	1969.3	56
58	3633.1	295.6	311.7	1915.4	3727.8	311.6	329.6	1971.1	58
60	3636.3	296.1	312.3	1917.3	3731.0	312.2	330.2	1973.0	60

′	38°				39°				′
	L. C.	M.	E.	T.	L. C.	M	E.	T.	
0	3731.0	312.2	330.2	1973 0	3825 5	328·7	348.7	2029.1	0
2	3734.1	312.7	330.8	1974.9	3828.6	329.2	349 3	2031.0	2
4	3737.3	313 3	331.4	1976·7	3831.8	329.8	349.9	2032 9	4
6	3740 4	313.8	332.0	1978.6	3834 9	330.3	350.6	2034.7	6
8	3743.6	314.4	332.6	1980.5	3838.0	330.9	351.2	2036.6	8
10	3746.7	314.9	333.2	1982.3	3841.2	331 5	351.8	2038.5	10
12	3749.9	315.5	333.8	1984·2	3844 3	332.0	352.4	2040 4	12
14	3753 0	316.0	334.5	1986.1	3847.4	332.6	353.1	2042 3	14
16	3756 2	316.6	335.1	1987.9	3850.6	333.2	353.7	2044.1	16
18	3759.3	317.1	335 7	1989.8	3853 7	333 7	354.3	2046.0	18
20	3762 5	317 7	336 3	1991.7	3856.8	334.3	354.9	2047.9	20
22	3765.6	318 2	336 9	1993·6	3860 0	334.9	355.6	2049.8	22
24	3768 8	318.8	337.5	1995.4	3863.1	335.4	356.2	2051 7	24
26	3771 9	319.3	338.1	1997.3	3866.2	336.0	356.9	2053 5	26
28	3775 1	319·9	338.7	1999.2	3869.4	336.6	357 5	2055.4	28
30	3778.2	320 4	339 4	2001.0	3872 5	337.1	358 1	2057·3	30
32	3781.4	321.0	340.0	2002.9	3875 6	337 7	358.8	2059.2	32
34	3784 5	321.5	340.6	2004.8	3878.8	338 3	359 4	2061.1	34
36	3787.7	322.1	341.2	2006.6	3881.9	338 8	360 1	2063.0	36
38	3790 8	322.6	341.8	2008.5	3885.0	339 4	360.7	2064·8	38
40	3794.0	323 2	342.4	2010 4	3888.2	340.0	361.3	2066.7	40
42	3797 1	323.7	343.1	2012.3	3891 3	340.5	362.0	2068 6	42
44	3800.3	324.3	343.7	2014.1	3894.4	341 1	362.6	2070.5	44
46	3803.4	324.8	344.3	2016 0	3897.6	341.7	363 3	2072.4	46
48	3806 6	325.4	344.9	2017.9	3900 7	342 2	363.9	2074 2	48
50	3809.7	325 9	345.6	2019.7	3903.8	342 8	364.5	2076 1	50
52	3812.9	326 5	346 2	2021.6	3907.0	343 4	365.2	2078 0	52
54	3816.0	327 0	346.8	2023.5	3910.1	343.9	365.8	2079 9	54
56	3819.2	327.6	347.4	2025 4	3913.2	344.5	366 5	2081.8	56
68	3822.3	323.1	348 1	2027.2	3916.4	345 1	367 1	2083.7	58
60	3825.5	328.7	348.7	2029.1	3919 5	345.6	367 7	2085.5	60

′	40°				41°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	3919.5	345.6	367.7	2085.5	4013.4	362.9	387.4	2142.3	0
2	3922.6	346.1	368.4	2087.4	4016.5	363.4	388.1	2144.2	2
4	3925.8	346 7	369 0	2089.3	4019.6	364.0	388.8	2146.1	4
6	3928 9	347.2	369.7	2091 2	4022.7	364.5	389.4	2148.0	6
8	3932 0	347 8	370.3	2093.1	4025.9	365.1	390 1	2149.9	8
10	3935.1	348.4	371.0	2095 0	4029.0	365.6	390.7	2151.9	10
12	3938.3	348.9	371 6	2096 9	4032.1	366.2	391.4	2153.8	12
14	3941.4	349.5	372.3	2098.8	4035.2	366.8	392.1	2155.7	14
16	3944.5	350.1	372.9	2100.7	4038.3	367.4	392.7	2157.6	16
18	3947.7	350.7	373.6	2102 6	4041.4	368.0	393.4	2159.5	18
20	3950.8	351.3	374.3	2104.5	4044.6	368.6	394.1	2161.4	20
22	3953.9	351 8	374.9	2106.3	4047.7	369.2	394.7	2163.3	22
24	3957.1	352.4	375.6	2108.2	4050.8	369.8	395.4	2165.2	24
26	3960.2	353.0	376.2	2110.1	4053.9	370.4	396.1	2167.1	26
28	3963.3	353.6	376.9	2112 0	4057.0	371.0	396.8	2169.0	28
30	3966.4	354.2	377.5	2113 9	4060.1	371.6	397.5	2170 9	30
32	3969.6	354.7	378.2	2115.8	4063.3	372.2	398.1	2172.8	32
34	3972.7	355.3	378.8	2117 7	4066.4	372.8	398.8	2174.7	34
36	3975.8	355.9	379.5	2119 6	4069.5	373.4	399.5	2176.6	36
38	3979.0	356.5	380 1	2121.5	4072.6	374.0	400.2	2178.5	38
40	3982.1	357.1	380.8	2123.4	4075.7	374.6	400.9	2180.4	40
42	3985.2	357 6	381.4	2125.3	4078.8	375.2	401.5	2182.4	42
44	3988.4	358.2	382.1	2127.2	4082.0	375.8	402.2	2184.3	44
46	3991 5	358 8	382 8	2129.1	4085.1	376.4	402.9	2186.2	46
48	3994.6	359.4	383.4	2131.0	4088.2	377.0	403.6	2188.1	48
50	3997 7	360 0	384.1	2132.9	4091.3	377.6	404.3	2190.0	50
52	4000 9	360 5	384.8	2134.7	4094.4	378.2	404.9	2191.9	52
54	4004.0	361 1	385.4	2136.6	4097.5	378.8	405.6	2193.8	54
56	4007 1	361.7	386.1	2138.5	4100.7	379.4	406.3	2195.7	56
58	4010 3	362.3	386 8	2140 4	4103.8	380.0	407.0	2197.6	58
60	4013 4	362.9	387.4	2142.3	4106.9	380.6	407.7	2199.5	60

′	42°				43°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	4106.9	380.6	407.7	2199.5	4200.1	398.7	428.6	2257.1	0
2	4110.0	381.2	408.3	2201.4	4203.2	399.3	429.3	2259.0	2
4	4113.1	381.8	409.0	2203.3	4206.3	399.9	430.0	2261.0	4
6	4116.2	382.4	409.7	2205.3	4209.4	400.5	430.7	2262.9	6
8	4119.3	383.0	410.4	2207.2	4212.5	401.1	431.4	2264.8	8
10	4122.4	383.6	411.1	2209.1	4215.6	401.7	432.1	2266.7	10
12	4125.5	384.2	411.8	2211.0	4218.7	402.4	432.8	2268.7	12
14	4128.6	384.8	412.5	2212.9	4221.8	403.0	433.5	2270.6	14
16	4131.8	385.4	413.2	2214.9	4224.9	403.6	434.2	2272.5	16
18	4134.9	386.0	413.9	2216.8	4228.0	404.2	434.9	2274.5	18
20	4138.0	386.6	414.6	2218.7	4231.1	404.8	435.6	2276.4	20
22	4141.1	387.2	415.3	2220.6	4234.2	405.4	436.3	2278.3	22
24	4144.2	387.8	416.0	2222.5	4237.3	406.1	437.0	2280.2	24
26	4147.3	388.4	416.6	2224.4	4240.4	406.7	437.8	2282.2	26
28	4150.4	389.0	417.3	2226.4	4243.5	407.3	438.5	2284.1	28
30	4153.5	389.6	418.0	2228.3	4246.5	407.9	439.2	2286.0	30
32	4156.6	390.2	418.7	2230.2	4249.6	408.5	439.9	2288.0	32
34	4159.7	390.8	419.4	2232.1	4252.7	409.1	440.6	2289.9	34
36	4162.8	391.4	420.1	2234.0	4255.8	409.8	441.4	2291.8	36
38	4165.9	392.0	420.8	2236.0	4258.9	410.4	442.1	2293.8	38
40	4169.0	392.6	421.5	2237.9	4262.0	411.0	442.8	2295.7	40
42	4172.1	393.2	422.2	2239.8	4265.1	411.6	443.5	2297.7	42
44	4175.2	393.8	422.9	2241.7	4268.2	412.2	444.2	2299.6	44
46	4178.4	394.4	423.6	2243.6	4271.3	412.8	445.0	2301.5	46
48	4181.5	395.0	424.3	2245.6	4274.4	413.5	445.7	2303.5	48
50	4184.6	395.6	425.0	2247.5	4277.5	414.1	446.4	2305.4	50
52	4187.7	396.2	425.7	2249.4	4280.6	414.7	447.1	2307.3	52
54	4190.8	396.8	426.4	2251.3	4283.7	415.3	447.8	2309.3	54
56	4193.9	397.4	427.1	2253.3	4286.8	415.9	448.6	2311.2	56
58	4197.0	398.0	427.8	2255.2	4289.9	416.5	449.3	2313.1	58
60	4200.1	398.7	428.6	2257.1	4293.0	417.2	450.0	2315.1	60

′	44°				45°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	4293.0	417.2	450.0	2315.1	4385.5	436.2	472.1	2373.4	0
2	4296.1	417.8	450.7	2317.0	4388.6	436.8	472.9	2375.4	2
4	4299.2	418.4	451.5	2319.0	4391.7	437.5	473.6	2377.3	4
6	4302.2	419.1	452.2	2320.9	4394.7	438.1	474.4	2379.3	6
8	4305.3	419.7	452.9	2322.8	4397.8	438.8	475.1	2381.2	8
10	4308.4	420.3	453.7	2324.8	4400.9	439.4	475.9	2383.2	10
12	4311.5	421.0	454.4	2326.7	4404.0	440.0	476.6	2385.2	12
14	4314.6	421.6	455.1	2328.7	4407.0	440.7	477.4	2387.1	14
16	4317.7	422.2	455.9	2330.6	4410.1	441.3	478.1	2389.1	16
18	4320.7	422.9	456.6	2332.6	4413.2	442.0	478.9	2391.0	18
20	4323.8	423.5	457.3	2334.5	4416.3	442.6	479.6	2393.0	20
22	4326.9	424.1	458.1	2336.4	4419.3	443.2	480.4	2394.9	22
24	4330.0	424.8	458.8	2338.4	4422.4	443.9	481.1	2396.9	24
26	4333.1	425.4	459.5	2340.3	4425.5	444.5	481.9	2398.8	26
28	4336.2	426.0	460.3	2342.3	4428.6	445.2	482.6	2400.8	28
30	4339.2	426.7	461.0	2344.2	4431.6	445.8	483.4	2402.8	30
32	4342.3	427.3	461.7	2346.1	4434.7	446.4	484.2	2404.7	32
34	4345.4	427.9	462.5	2348.1	4437.8	447.1	484.9	2406.7	34
36	4348.5	428.6	463.2	2350.0	4440.9	447.7	485.7	2408.6	36
38	4351.6	429.2	463.9	2352.0	4444.0	448.3	486.5	2410.6	38
40	4354.7	429.8	464.7	2353.9	4447.0	448.9	487.2	2412.6	40
42	4357.7	430.5	465.4	2355.9	4450.1	449.5	488.0	2414.5	42
44	4360.8	431.1	466.2	2357.8	4453.2	450.2	488.7	2416.5	44
46	4363.9	431.7	466.9	2359.8	4456.3	450.8	489.5	2418.5	46
48	4367.0	432.4	467.7	2361.7	4459.3	451.5	490.3	2420.4	48
50	4370.1	433.0	468.4	2363.7	4462.4	452.1	491.0	2422.4	50
52	4373.2	433.6	469.1	2365.6	4465.5	452.7	491.8	2424.4	52
54	4376.2	434.3	469.9	2367.6	4468.6	453.4	492.5	2426.3	54
56	4379.3	434.9	470.6	2369.5	4471.6	454.1	493.3	2428.3	56
58	4382.4	435.5	471.4	2371.5	4474.7	454.8	494.1	2430.2	58
60	4385.5	436.2	472.1	2373.4	4477.8	455.5	494.8	2432.2	60

′	46°				47°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	4477.8	455.5	494.8	2432.2	4569.7	475.2	518.3	2491.5	0
2	4480.9	456.1	495.6	2434.2	4572.7	475.9	519.0	2493.4	2
4	4483.9	456.8	496.5	2436.1	4575.8	476.5	519.8	2495.4	4
6	4487.0	457.4	497.2	2438.1	4578.8	477.2	520.6	2497.4	6
8	4490.0	458.1	497.9	2440.1	4581.9	477.8	521.4	2499.4	8
10	4493.1	458.7	498.7	2442.1	4584.9	478.5	522.2	2501.4	10
12	4496.2	459.4	499.5	2444.0	4588.0	479.2	523.0	2503.4	12
14	4499.2	460.0	500.3	2446.0	4591.0	479.8	523.8	2505.4	14
16	4502.3	460.7	501.0	2448.0	4594.1	480.5	524.6	2507.3	16
18	4505.4	461.3	501.8	2449.9	4597.1	481.1	525.4	2509.3	18
20	4508.4	462.0	502.6	2451.9	4600.2	481.7	526.2	2511.3	20
22	4511.5	462.7	503.4	2453.9	4603.2	482.3	527.0	2513.3	22
24	4514.6	463.3	504.1	2455.9	4606.3	483.0	527.8	2515.3	24
26	4517.6	464.0	504.9	2457.8	4609.3	483.7	528.6	2517.3	26
28	4520.7	464.6	505.7	2459.8	4612.4	484.3	529.4	2519.3	28
30	4523.7	465.3	506.5	2461.8	4615.4	485.0	530.2	2521.2	30
32	4526.8	466.0	507.3	2463.8	4618.5	485.7	531.0	2523.2	32
34	4529.9	466.6	508.0	2465.7	4621.5	486.3	531.8	2525.2	34
36	4532.9	467.3	508.8	2467.7	4624.6	487.0	532.6	2527.2	36
38	4536.0	467.9	509.6	2469.7	4627.6	487.7	533.4	2529.2	38
40	4539.1	468.6	510.4	2471.7	4630.7	488.4	534.2	2531.2	40
42	4542.1	469.3	511.1	2473.6	4633.7	489.1	535.0	2533.2	42
44	4545.2	469.9	511.9	2475.6	4636.8	489.8	535.8	2535.2	44
46	4548.2	470.6	512.7	2477.6	4639.8	490.5	536.6	2537.2	46
48	4551.3	471.2	513.5	2479.6	4642.9	491.2	537.4	2539.2	48
50	4554.4	471.9	514.3	2481.6	4645.9	491.9	538.2	2541.2	50
52	4557.4	472.6	515.1	2483.5	4649.0	492.6	539.0	2543.1	52
54	4560.5	473.2	515.9	2485.5	4652.0	493.3	539.8	2545.1	54
56	4563.6	473.9	516.7	2487.5	4655.1	494.0	540.6	2547.1	56
58	4566.6	474.5	517.5	2489.5	4658.1	494.7	541.4	2549.1	58
60	4569.7	475.2	518.3	2491.5	4661.2	495.4	542.3	2551.1	60

′	48°				49°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	4661.2	495.4	542.3	2551.1	4752.3	515.9	567.0	2611.3	0
2	4664.2	496.0	543.1	2553.1	4755.3	516.5	567.8	2613.3	2
4	4667.3	496.7	543.9	2555.1	4758.4	517.2	568.7	2615.3	4
6	4670.3	497.4	544.7	2557.1	4761.4	517.9	569.5	2617.3	6
8	4673.3	498.1	545.5	2559.1	4764.4	518.6	570.3	2619.3	8
10	4676.4	498.8	546.4	2561.1	4767.4	519.3	571.2	2621.4	10
12	4679.4	499.4	547.2	2563.1	4770.5	520.0	572.0	2623.4	12
14	4682.5	500.1	548.0	2565.1	4773.5	520.7	572.8	2625.4	14
16	4685.5	500.8	548.8	2567.1	4776.5	521.4	573.7	2627.4	16
18	4688.5	501.5	549.6	2569.1	4779.6	522.1	574.5	2629.4	18
20	4691.6	502.2	550.5	2571.1	4782.6	522.8	575.3	2631.4	20
22	4694.6	502.8	551.3	2573.1	4785.6	523.5	576.2	2633.5	22
24	4697.6	503.5	552.1	2575.1	4788.7	524.2	577.0	2635.5	24
26	4700.7	504.2	552.9	2577.1	4791.7	524.9	577.9	2637.5	26
28	4703.7	504.9	553.7	2579.1	4794.7	525.6	578.7	2639.5	28
30	4706.7	505.6	554.6	2581.1	4797.7	526.3	579.6	2641.5	30
32	4709.8	506.2	555.4	2583.1	4800.8	527.0	580.4	2643.5	32
34	4712.8	506.9	556.2	2585.1	4803.8	527.7	581.3	2645.6	34
36	4715.9	507.6	557.0	2587.2	4806.8	528.4	582.1	2647.6	36
38	4718.9	508.3	557.8	2589.2	4809.9	529.1	583.0	2649.6	38
40	4721.9	509.0	558.7	2591.2	4812.9	529.8	583.8	2651.6	40
42	4725.0	509.6	559.5	2593.2	4815.9	530.5	584.7	2653.7	42
44	4728.0	510.3	560.3	2595.2	4819.0	531.2	585.5	2655.7	44
46	4731.0	511.0	561.2	2597.2	4822.0	531.9	586.4	2657.7	46
48	4734.1	511.7	562.0	2599.2	4825.0	532.6	587.2	2659.7	48
50	4737.1	512.4	562.8	2601.2	4828.0	533.3	588.1	2661.8	50
52	4740.2	513.1	563.7	2603.2	4831.1	534.0	588.9	2663.8	52
54	4743.2	513.8	564.5	2605.2	4834.1	534.7	589.8	2665.8	54
56	4746.2	514.5	565.3	2607.2	4837.1	535.4	590.6	2667.8	56
58	4749.3	515.2	566.2	2609.3	4840.2	536.1	591.5	2669.9	58
60	4752.3	515.9	567.0	2611.3	4843.2	536.8	592.4	2671.9	60

′	50°				51°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	4843.2	536.8	592.4	2671.9	4933 6	558.2	618.5	2733.0	0
2	4846.2	537.5	593 2	2673 9	4936.6	558.9	619.3	2735.1	2
4	4849.2	538 2	594.1	2676.0	4939 6	559.7	620.2	2737.1	4
6	4852.2	538.9	594.9	2678.0	4942 6	560.4	621.1	2739.2	6
8	4855.2	539.6	595.8	2680.0	4945.6	561.1	622.0	2741.2	8
10	4858.3	540.3	596 7	2682.1	4948.6	561.8	622.9	2743.3	10
12	4861.3	541.0	597.5	2684.1	4951.6	562.5	623.7	2745.3	12
14	4864.3	541.7	598.4	2686.1	4954.6	563.3	624.6	2747.4	14
16	4867.3	542.4	599.3	2688.2	4957.6	564.0	625.5	2749.4	16
18	4870.3	543.1	600.1	2690.2	4960.6	564.7	626.4	2751.5	18
20	4873.3	543.9	601.0	2692.3	4963.6	565.4	627.3	2753.5	20
22	4876.3	544.6	601.9	2694.3	4966.6	566.2	628.2	2755.6	22
24	4879.4	545.3	602.7	2696.3	4969.6	566.9	629.9	2757.7	24
26	4882.4	546.0	603.6	2698.4	4972.6	567.6	630.0	2759.7	26
28	4885.4	546.7	604.5	2700.4	4975.6	568.3	630.9	2761.8	28
30	4888.4	547.4	605.3	2702.4	4978.6	569.1	631.8	2763.8	30
32	4891.4	548.1	606.2	2704.5	4981.6	569.8	632.7	2765.9	32
34	4894.4	548.8	607.0	2706.5	4984.6	570.5	633.6	2767.9	34
36	4897.4	549.5	607.9	2708.6	4987.7	571.2	634.5	2770.0	36
38	4900.4	550.2	608.8	2710.6	4990.7	572.0	635.3	2772.0	38
40	4903.5	551.0	609.7	2712.6	4993.7	572.7	636.2	2774.1	40
42	4906.5	551.7	610.5	2714.7	4996.7	573.4	637.1	2776.2	42
44	4909.5	552.4	611.4	2716.7	4999.7	574.1	638.0	2778.2	44
46	4912.5	553.1	612.3	2718.8	5002.7	574.9	638.9	2780.3	46
48	4915.5	553.8	613.2	2720.8	5005.7	575.6	639.8	2782.3	48
50	4918.5	554.5	614.1	2722.8	5008.7	576.3	640.7	2784.4	50
52	4921.5	555.2	614.9	2724.9	5011.7	577.0	641.6	2786.4	52
54	4924.6	555.9	615.8	2726.9	5014.7	577.8	642.5	2788.5	54
56	4927.6	556.6	616.7	2729.0	5017.7	578.5	643.4	2790.6	56
58	4930.6	557.4	617.6	2731.0	5020.7	579.2	644.3	2792.6	58
60	4933.6	558.2	618.5	2733.0	5023.7	579.9	645.2	2794.7	60

′	52°				53°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	5023.7	579.9	645.2	2794.7	5113.5	602.0	672.7	2856.9	0
2	5026.7	580.6	646.1	2796.8	5116.5	602.8	673.7	2858.9	2
4	5029.7	581.3	647.0	2798.8	5119.4	603.5	674.6	2861.0	4
6	5032.7	582.1	647.9	2800.9	5122.4	604.3	675.5	2863.1	6
8	5035.7	582.8	648.9	2803.0	5125.4	605.0	676.4	2865.2	8
10	5038.7	583.5	649.8	2805.0	5128.4	605.8	677.4	2867.3	10
12	5041.7	584.3	650.7	2807.1	5131.3	606.5	678.3	2869.4	12
14	5044.7	585.0	651.6	2809.2	5134.3	607.3	679.2	2871.5	14
16	5047.7	585.7	652.5	2811.2	5137.3	608.0	680.2	2873.5	16
18	5050.7	586.5	653.4	2813.3	5140.3	608.8	681.1	2875.6	18
20	5053.6	587.2	654.3	2815.4	5143.2	609.5	682.0	2877.7	20
23	5056.6	587.9	655.2	2817.4	5146.2	610.3	683.0	2879.8	22
24	5059.6	588.7	656.2	2819.5	5149.2	611.0	683.9	2881.9	24
26	5062.6	589.4	657.1	2821.6	5152.1	611.8	684.9	2884.0	26
28	5065.6	590.1	658.0	2823.6	5155.1	612.5	685.8	2886.1	28
30	5068.6	590.9	658.9	2825.7	5158.1	613.3	686.7	2888.1	30
32	5071.6	591.6	659.8	2827.8	5161.1	614.0	687.7	2890.2	32
34	5074.6	592.3	660.7	2829.8	5164.0	614.8	688.6	2892.3	34
36	5077.6	593.1	661.6	2831.9	5167.0	615.5	689.6	2894.4	36
38	5080.6	593.8	662.5	2834.0	5170.0	616.3	690.5	2896.5	38
40	5083.6	594.5	663.5	2836.1	5173.0	617.0	691.5	2898.6	40
42	5086.6	595.3	664.4	2838.2	5175.9	617.8	692.4	2900.7	42
44	5089.6	596.0	665.3	2840.2	5178.9	618.5	693.4	2902.8	44
46	5092.6	596.7	666.2	2842.3	5181.9	619.3	694.3	2904.9	46
48	5095.6	597.5	667.2	2844.4	5184.9	620.1	695.3	2907.0	48
50	5098.6	598.2	668.1	2846.5	5187.8	620.8	696.2	2909.1	50
52	5101.6	598.9	669.0	2848.5	5190.8	621.5	697.1	2911.2	52
54	5104.6	599.7	669.9	2850.6	5193.8	622.3	698.1	2913.3	54
56	5107.6	600.4	670.9	2852.7	5196.7	623.0	699.0	2915.4	56
58	5110.6	601.2	671.8	2854.8	5199.7	623.8	700.0	2917.5	58
60	5113.5	602.0	672.7	2856.9	5202.7	624.6	700.9	2919.5	60

′	54°				55°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	5202.7	624.6	700.9	2919.5	5291.7	647.4	729.9	2982.8	0
2	5205.7	625.4	701.9	2921.6	5294.6	648.1	730.9	2984.9	2
4	5208.6	626.1	702.8	2923.8	5297.6	648.9	731 9	2987.1	4
6	5211.6	626.9	703.8	2925.9	5300.5	649.6	732.9	2989.2	6
8	5214.6	627.6	704.8	2928.0	5303.5	650 4	733.8	2991.3	8
10	5217.5	628.4	705.7	2930.1	5306 4	651.2	734.8	2993.4	10
12	5220.5	629.2	706.7	2932.2	5309.4	652.0	735.8	2995.5	12
14	5223.5	629.9	707.7	2934.3	5312.3	652.7	736.8	2997.7	14
16	5226.4	630.7	708.6	2936.4	5315.3	653.5	737.8	2999.8	16
18	5229.4	631.4	709.6	2938.5	5318.2	654.3	738.7	3001.9	18
20	5232.4	632.2	710.5	2940.6	5321.2	655.1	739.7	3004.0	20
22	5235.3	633.0	711.5	2942.7	5324.1	655.8	740.7	3006.2	22
24	5238.3	633.7	712.5	2944.8	5327.1	656.6	741.7	3008.3	24
26	5241.3	634.5	713.4	2946.9	5330.0	657.4	742.7	3010.4	26
28	5244.2	635.2	714.4	2949.0	5333.0	658.2	743.7	3012.5	28
30	5247.2	636.0	715.3	2951.1	5335.9	658.9	744.7	3014.7	30
32	5250.2	636.8	716.3	2953.2	5338.8	659.7	745 7	3016.8	32
34	5253.1	637.5	717.3	2955.3	5341.8	660.5	746.7	3018.9	34
36	5256.1	638.3	718.2	2957.5	5344.7	661.3	747.7	3021.1	36
38	5259.1	639.0	719.2	2959.6	5347.7	662.0	748.7	3023.2	38
40	5262.0	639.8	720.2	2961.7	5350.6	662.8	749.7	3025.3	40
42	5265.0	640.6	721.1	2963.8	5353.6	663.6	750.7	3027.5	42
44	5268.0	641.3	722.1	2965.9	5356.5	664.4	751.7	3029.6	44
46	5270.9	642.1	723.1	2968.0	5359.5	665.1	752.6	3031.7	46
48	5273.9	642.8	724.1	2970.1	5362.4	665 9	753.6	3033.8	48
50	5276.9	643.6	725.0	2972.2	5365.4	666.7	754.6	3036.0	50
52	5279.8	644.4	726.0	2974.4	5368.3	667.5	755.6	3038.1	52
54	5282.8	645.1	727.0	2976.5	5371.3	668.3	756.6	3040.2	54
56	5285.8	645.9	728.0	2978.6	5374.2	669.1	757.6	3042.4	56
58	5288.7	646.6	729.0	2980.7	5377.2	669.9	758.6	3044.5	58
60	5291.7	647.4	729 9	2982.8	5380.1	670.7	759.6	3046.6	60

′	56°				57°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	5380.1	670.7	759.6	3046.6	5468.2	694.4	790.2	3111.1	0
2	5383.0	671.4	760.6	3048.8	5471.1	695.2	791.2	3113.3	2
4	5386.0	672.2	761.6	3050.9	5474.0	696.0	792.2	3115.4	4
6	5388.9	672.9	762.7	3053.1	5477.0	696.8	793.3	3117.6	6
8	5391.8	673.7	763.7	3055.2	5479.9	697.6	794.3	3119.7	8
10	5394.8	674.4	764.7	3057.4	5482.8	698.4	795.3	3121.9	10
12	5397.7	675.2	765.7	3059.5	5485.7	699.2	796.3	3124.1	12
14	5400.7	676.0	766.7	3061.6	5488.7	700.0	797.4	3126.2	14
16	5403.6	676.8	767.7	3063.8	5491.6	700.8	798.4	3128.4	16
18	5406.5	677.6	768.7	3065.9	5494.5	701.6	799.4	3130.6	18
20	5409.5	678.4	769.7	3068.1	5497.4	702.4	800.5	3132.7	20
22	5412.4	679.2	770.8	3070.2	5500.3	703.2	801.5	3134.9	22
24	5415.3	680.0	771.8	3072.4	5503.3	704.0	802.6	3137.0	24
26	5418.3	680.8	772.8	3074.5	5506.2	704.8	803.6	3139.2	26
28	5421.2	681.6	773.8	3076.6	5509.1	705.6	804.7	3141.4	28
30	5424.1	682.4	774.8	3078.8	5512.0	706.4	805.7	3143.5	30
32	5427.1	683.2	775.8	3080.9	5515.0	707.2	806.8	3145.7	32
34	5430.0	684.0	776.8	3083.1	5517.9	708.0	807.8	3147.9	34
36	5433.0	684.8	777.8	3085.2	5520.8	708.8	808.8	3150.0	36
38	5435.9	685.6	778.9	3087.4	5523.7	709.6	809.9	3152.2	38
40	5438.8	686.4	779.9	3089.6	5526.7	710.4	810.9	3154.4	40
42	5441.8	687.2	780.9	3091.7	5529.6	711.2	812.0	3156.6	42
44	5444.7	688.0	781.9	3093.9	5532.5	712.0	813.0	3158.7	44
46	5447.6	688.8	783.0	3096.0	5535.4	712.8	814.1	3160.9	46
48	5450.6	689.6	784.0	3098.2	5538.4	713.6	815.1	3163.1	48
50	5453.5	690.4	785.0	3100.3	5541.3	714.4	816.2	3165.3	50
52	5456.5	691.2	786.0	3102.5	5544.2	715.2	817.2	3167.4	52
54	5459.4	692.0	787.1	3104.6	5547.1	716.0	818.3	3169.6	54
56	5462.3	692.8	788.1	3106.8	5550.0	716.8	819.3	3171.8	56
58	5465.3	693.6	789.1	3108.9	5553.0	717.6	820.4	3174.0	58
60	5468.2	694.4	790.2	3111.1	5555.9	718.4	821.4	3176.1	60

′	58°				59°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	5555.9	718.4	821.4	3176.1	5643.1	742.8	853.5	3241.9	0
2	5558.8	719 2	822.5	3178.3	5646.0	743.6	854.6	3244.1	2
4	5561.7	720.0	823.5	3180.5	5648.9	744.4	855.7	3246.3	4
6	5564.6	720.8	824.6	3182.7	5651.8	745.3	856.8	3248.5	6
8	5567.5	721.6	825.7	3184.9	5654.7	746.1	857.9	3250.7	8
10	5570.4	722 4	826.7	3187.1	5657.6	746.9	859.0	3252.9	10
12	5573.3	723.2	827 8	3189.2	5660.5	747.7	860.0	3255.1	12
14	5576.2	724.0	828.9	3191.4	5663.4	748.6	861.1	3257.3	14
16	5579.2	724.8	829.9	3193.6	5666.3	749.4	862.2	3259.5	16
18	5582.1	725.6	831.0	3195 8	5669.2	750.2	863.3	3261.7	18
20	5585.0	726.5	832 1	3198.0	5672.1	751.1	864.4	3263.9	20
22	5587.9	727.3	833.1	3200 2	5675.0	751.9	865.5	3266.1	22
24	5590.8	728.1	834.2	3202.4	5677.9	752.7	866.6	3268 3	24
26	5593.7	728.9	835.3	3204.5	5680.8	753.5	867.7	3270.5	26
28	5596.6	729.7	836.3	3206.7	5683.7	754.4	868.8	3272.7	28
30	5599.5	730 5	837.4	3208.9	5686.5	755.2	869.9	3274.9	30
32	5602.4	731.3	838.4	3211.1	5689.4	756.0	871.0	3277.1	32
34	5605.3	732.1	839.5	3213.3	5692.3	756.9	872.1	3279.4	34
36	5608.2	732.9	840.6	3215.5	5695.2	757.7	873.2	3281.6	36
38	5611.1	733.7	841.6	3217.7	5698.1	758.5	874.3	3283.8	38
40	5614.0	734.6	842.7	3219.9	5701.0	759.4	875.4	3286.0	40
42	5616.9	735.4	843.8	3222.1	5703.9	760.2	876.5	3288.2	42
44	5619.8	736.2	844.9	3224.3	5706.8	761.0	877.6	3290.5	44
46	5622.8	737.0	846.0	3226.5	5709 7	761.9	878.7	3292.7	46
48	5625.7	737 8	847.0	3228.7	5712.6	762.7	879.8	3294.9	48
50	5628.6	738.6	848.1	3230.9	5715.5	763.5	880 9	3297.1	50
52	5631.5	739.4	849.2	3233.1	5718.4	764.4	882.0	3299.3	52
54	5634.4	740.2	850.3	3235.3	5721.3	765.2	883.1	3301 5	54
56	5637.3	741 0	851.4	3237.5	5724.2	766.0	884.2	3303.8	56
58	5640.2	741.9	852.5	3239.7	5727.1	766.8	885.3	3306.0	58
60	5643.1	742.8	853.5	3241.9	5730.0	767.7	886.4	3308.2	60

′	60°				61°				′
	L. C.	M,	E.	T.	L. C.	M.	E.	T.	
0	5730.0	767.7	886.4	3308.2	5816.4	792.9	920.2	3375.2	0
2	5732.9	768.5	887.5	3310.4	5819.3	793.7	921.4	3377.4	2
4	5735.8	769.4	888.7	3312.7	5822.1	794.6	922.5	3379.7	4
6	5738.6	770.2	889.8	3314.9	5825.0	795.4	923.6	3381.9	6
8	5741.5	771.1	890.9	3317.1	5827.9	796.3	924.8	3384.2	8
10	5744.4	771.9	892.0	3319.3	5830.7	797.1	925.9	3386.4	10
12	5747.3	772.7	893.1	3321.6	5833.6	798.0	927.1	3388.7	12
14	5750.2	773.6	894.3	3323.8	5836.5	798.8	928.2	3390.9	14
16	5753.0	774.4	895.4	3326.0	5839.3	799.7	929.3	3393.2	16
18	5755.9	775.3	896.5	3328.3	5842.2	800.5	930.5	3395.4	18
20	5758.8	776.1	897.6	3330.5	5845.1	801.4	931.6	3397.7	20
22	5761.7	776.9	898.8	3332.7	5847.9	802.2	932.8	3399.9	22
24	5764.6	777.8	899.9	3334.9	5850.8	803.1	933.9	3402.2	24
26	5767.4	778.6	901.0	3337.2	5853.7	803.9	935.1	3404.4	26
28	5770.3	779.5	902.1	3339.4	5856.5	804.8	936.3	3406.7	28
30	5773.2	780.3	903.2	3341.6	5859.4	805.6	937.4	3408.9	30
32	5776.1	781.1	904.4	3343.9	5862.3	806.5	938.6	3411.2	32
34	5779.0	782.0	905.5	3346.1	5865.1	807.3	939.7	3413.5	34
36	5781.8	782.8	906.6	3348.3	5868.0	808.2	940.9	3415.7	36
38	5784.7	783.7	907.7	3350.6	5870.9	809.0	942.1	3418.0	38
40	5787.6	784.5	908.8	3352.8	5873.7	809.9	943.2	3420.3	40
42	5790.5	785.3	910.0	3355.0	5876.6	810.7	944.4	3422.5	42
44	5793.4	786.2	911.1	3357.3	5879.5	811.6	945.5	3424.8	44
46	5796.2	787.0	912.3	3359.5	5882.3	812.4	946.7	3427.1	46
48	5799.1	787.9	913.4	3361.8	5885.2	813.3	947.8	3429.3	48
50	5802.0	788.7	914.5	3364.0	5888.1	814.1	949.0	3431.6	50
52	5804.9	789.5	915.7	3366.2	5890.9	815.0	950.2	3433.9	52
54	5807.8	790.4	916.8	3368.5	5893.8	815.8	951.3	3436.1	54
56	5810.6	791.2	918.0	3370.7	5896.7	816.7	952.5	3438.4	56
58	5813.5	792.1	919.1	3373.0	5899.5	817.5	953.6	3440.7	58
60	5816.4	792.9	920.2	3375.2	5902.4	818.4	954.8	3442.9	60

′	62°				63°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	5902.4	818.4	954.8	3442.9	5987.8	844.4	990.3	3511.3	0
2	5905.2	819.3	956.0	3445.2	5990.6	845.3	991.5	3513.6	2
4	5908.1	820.1	957.2	3447.5	5993.5	846.2	992.7	3515.9	4
6	5910.9	821.0	958 3	3449.7	5996.3	847.1	993.9	3518.2	6
8	5913.8	821.8	959.5	3452.0	5999.1	847.9	995.1	3520.5	8
10	5916.6	822.7	960.7	3454.3	6002 0	848.8	996.3	3522.8	10
12	5919.5	823.6	961.9	3456.6	6004.8	849.7	997.5	3525.1	12
14	5922.3	824.4	963.0	3458.8	6007.7	850.6	998.7	3527.4	14
16	5925.2	825.3	964.2	3461.1	6010.5	851.4	999.9	3529.7	16
18	5928.0	826.1	965.4	3463.4	6013.3	852.3	1001.1	3532.0	18
20	5930.9	827.0	966.6	3465.7	6016.2	853.2	1002.3	3534.3	20
22	5933.7	827.9	967.8	3467.9	6019.0	854.1	1003.5	3536.6	22
24	5936.6	828.7	968.9	3470.2	6021.8	854.9	1004.7	3538.9	24
26	5939.4	829.6	970.1	3472.5	6024.7	855.8	1005.9	3541.2	26
28	5942.3	830.4	971.3	3474 7	6027.5	856.7	1007.1	3543.5	28
30	5945.1	831.3	972.5	3477 0	6030.3	857.6	1008.4	3545.8	30
32	5947.9	832.2	973.6	3479.3	6033.2	858.4	1009.6	3548.1	32
34	5950.8	833.0	974.8	3481.6	6036.0	859.3	1010.8	3550.4	34
36	5953.6	833.9	976.0	3483.9	6038.9	860.2	1012.0	3552.7	36
38	5956.5	834.7	977.2	3486.2	6041.7	861.1	1013.2	3555.0	38
40	5959.3	835.6	978.4	3488.5	6044.5	861.9	1014.5	3557.3	40
42	5962.2	836.5	979.6	3490 7	6047.4	862.8	1015.7	3559.6	42
44	5965.0	837.4	980.8	3493.0	6050.2	863.7	1016.9	3562.0	44
46	5967.9	838.3	982.0	3495.3	6053.0	864.6	1018.1	3564.3	46
48	5970.7	839.1	983.2	3497.6	6055.9	865.4	1019.3	3566.6	48
50	5973.6	840.0	984.4	3499 9	6058.7	866.3	1020.6	3568.9	50
52	5976.4	840.9	985.5	3502.2	6061.6	867.2	1021.8	3571.2	52
54	5979.3	841.7	986.7	3504.5	6064.4	868 1	1023.0	3573.5	54
56	5982 1	842.6	987.9	3506.8	6067.2	868.9	1024.2	3575.8	56
58	5985.0	843.5	989.1	3509.0	6070.1	869.8	1025.4	3578.1	58
60	5987.8	844.4	990.3	3511.3	6072.9	870.7	1026.7	3580.4	60

′	64°				65°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	6072.9	870.7	1026.7	3580.4	6157.5	897.3	1064.0	3650.4	0
2	6075.7	871.5	1027.9	3582.8	6160.3	898.2	1065.2	3652.8	2
4	6078.5	872.4	1029.2	3585.1	6163.1	899.1	1066.5	3655.1	4
6	6081.4	873.3	1030.4	3587.4	6165.9	900.0	1067.7	3657.5	6
8	6084.2	874.2	1031.7	3589.7	6168.7	900.9	1069.0	3659.8	8
10	6087.0	875.1	1032.9	3592.1	6171.5	901.8	1070.2	3662.2	10
12	6089.8	875.9	1034.1	3594.4	6174.3	902.7	1071.5	3664.5	12
14	6092.6	876.8	1035.4	3596.7	6177.1	903.6	1072.7	3666.9	14
16	6095.5	877.7	1036.6	3599.1	6179.9	904.5	1074.0	3669.2	16
18	6098.3	878.6	1037.9	3601.4	6182.7	905.4	1075.2	3671.6	18
20	6101.1	879.5	1039.1	3603.7	6185.5	906.3	1076.6	3673.9	20
22	6103.9	880.3	1040.3	3606.0	6188.3	907.2	1077.8	3676.2	22
24	6106.7	881.2	1041.6	3608.4	6191.1	908.1	1079.1	3678.6	24
26	6109.6	882.1	1042.8	3610.7	6193.9	909.0	1080.4	3680.9	26
28	6112.4	883.0	1044.1	3613.0	6196.7	909.9	1081.7	3683.3	28
30	6115.2	883.9	1045.3	3615.3	6199.5	910.8	1083.0	3685.6	30
32	6118.0	884.7	1046.5	3617.7	6202.3	911.7	1084.2	3688.0	32
34	6120.8	885.6	1047.8	3620.0	6205.1	912.6	1085.5	3690.4	34
36	6123.7	886.5	1049.0	3622.3	6208.0	913.5	1086.8	3692.7	36
38	6126.5	887.4	1050.3	3624.7	6210.8	914.4	1088.1	3695.1	38
40	6129.3	888.3	1051.5	3627.0	6213.6	915.3	1089.4	3697.4	40
42	6132.1	889.2	1052.7	3629.4	6216.4	916.2	1090.6	3699.8	42
44	6134.9	890.1	1054.0	3631.7	6219.2	917.1	1091.9	3702.2	44
46	6137.8	891.0	1055.2	3634.0	6222.0	918.0	1093.2	3704.5	46
48	6140.6	891.9	1056.5	3636.4	6224.8	918.9	1094.5	3706.9	48
50	6143.4	892.8	1057.7	3638.7	6227.6	919.8	1095.8	3709.3	50
52	6146.2	893.7	1059.0	3641.1	6230.4	920.7	1097.0	3711.6	52
54	6149.0	894.6	1060.2	3643.4	6233.2	921.6	1098.3	3714.0	54
56	6151.9	895.5	1061.5	3645.7	6236.0	922.5	1099.6	3716.3	56
58	6154.7	896.4	1062.7	3648.1	6238.8	923.4	1100.9	3718.7	58
60	6157.5	897.3	1064.0	3650.4	6241.6	924.3	1102.2	3721.1	60

′	66°				67°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	6241.6	924.3	1102.2	3721.1	6325.2	951.8	1141.5	3792.6	0
2	6244.4	925.2	1103.5	3723.4	6328.0	952.7	1142.8	3795.0	2
4	6247.2	926.1	1104.8	3725.8	6330.7	953.6	1144.1	3797.4	4
6	6250.0	927.0	1106.1	3728.2	6333.5	954.5	1145.4	3799.8	6
8	6252.7	927.9	1107.4	3730.6	6336.3	955.5	1146.7	3802.2	8
10	6255.5	928.8	1108.7	3732.9	6339.0	956.4	1148.1	3804.6	10
12	6258.3	929.8	1110.0	3735.3	6341.8	957.3	1149.4	3807.0	12
14	6261.1	930.7	1111.3	3737.7	6344.6	958.2	1150.7	3809.4	14
16	6263.9	931 6	1112.6	3740.1	6347.4	959.2	1152.0	3811.8	16
18	6266.7	932.5	1113.9	3742.4	6350.1	960.1	1153.3	3814.2	18
20	6269.5	933.4	1115.2	3744.8	6352.9	961.0	1154.7	3816.6	20
22	6272.3	934.3	1116.5	3747.2	6355.7	961.9	1156.0	3819.0	22
24	6275 0	935.3	1117.8	3749.6	6358.4	962.9	1157.4	3821.4	24
26	6277.8	936.2	1119.1	3751.9	6361.2	963.8	1158.7	3823.8	26
28	6280.6	937.1	1120.4	3754.3	6364.0	964.7	1160.1	3826.2	28
30	6283.4	938.0	1121.7	3756.7	6366.7	965.6	1161.4	3828.6	30
32	6286.2	938.9	1123.0	3759.1	6369.5	966.6	1162.8	3831.0	32
34	6289.0	939.8	1124.3	3761.5	6372.3	967.5	1164.1	3833.4	34
36	6291.8	940.8	1125.6	3763.9	6375.1	968.4	1165.5	3835.9	36
38	6294.5	941.7	1126.9	3766.3	6377.8	969.3	1166.8	3838.3	38
40	6297.3	942 6	1128.3	3768.7	6380.6	970.3	1168.2	3840.7	40
42	6300.1	943.5	1129.6	3771.0	6383.4	971.2	1169.5	3843.1	42
44	6302.9	944.4	1130.9	3773.4	6386.1	972.1	1170.9	3845.5	44
46	6305.7	945.3	1132.2	3775.8	6388.9	973.0	1172.2	3847.9	46
48	6308.5	946.3	1133.5	3778.2	6391.7	974.0	1173.6	3850.4	48
50	6311.3	947.2	1134.9	3780.6	6394.4	974.9	1174.9	3852.8	50
52	6314.1	948.1	1136.2	3783.0	6397.2	975.8	1176.3	3855.2	52
54	6316.8	949.0	1137.5	3785.4	6400.0	976.8	1177.6	3857.6	54
56	6319.6	949.9	1138.8	3787.8	6402.8	977.7	1179.0	3860.0	56
58	6322.4	950.8	1140.1	3790.2	6405.5	978.6	1180.3	3862 5	58
60	6325.2	951.8	1141.5	3792.6	6408.3	979.6	1181.6	3864.9	60

′	68°				69°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	6408.3	979.6	1181.6	3864.9	6491.1	1007.7	1222.9	3938.1	0
2	6411.1	980.5	1183.0	3867.3	6493.8	1008.7	1224.3	3940.6	2
4	6413.8	981.4	1184.4	3869.7	6496.6	1009.6	1225.7	3943.0	4
6	6416.6	982.4	1185.7	3872.2	6499.3	1010.6	1227.1	3945.5	6
8	6419.8	983.3	1187.1	3874.6	6502.1	1011.5	1228.5	3947.9	8
10	6422.1	984.2	1188.5	3877.0	6504.8	1012.5	1229.9	3950.4	10
12	6424.9	985.2	1189.8	3879.5	6507.5	1013.4	1231.3	3952.9	12
14	6427.6	986.1	1191.2	3881.9	6510.3	1014.4	1232.7	3955.3	14
16	6430.4	987.0	1192.6	3884.3	6513.0	1015.3	1234.1	3957.8	16
18	6433.1	988.0	1193.9	3886.8	6515.8	1016.3	1235.5	3960.2	18
20	6435.9	988.9	1195.3	3889.2	6518.5	1017.2	1236.9	3962.7	20
22	6438.7	989.8	1196.7	3891.6	6521.2	1018.2	1238.3	3965.2	22
24	6441.4	990.8	1198.0	3894.1	6524.0	1019.1	1239.7	3967.6	24
26	6444.2	991.7	1199.4	3896.5	6526.7	1020.1	1241.1	3970.1	26
28	6446.9	992.6	1200.8	3898.9	6529.5	1021.0	1242.5	3972.5	28
30	6449.7	993.6	1202.1	3901.4	6532.2	1022.0	1243.9	3975.0	30
32	6452.5	994.5	1203.5	3903.8	6534.9	1022.9	1245.3	3977.5	32
34	6455.2	995.4	1204.9	3906.3	6537.7	1023.9	1246.7	3980.0	34
36	6458.0	996.4	1206.2	3908.7	6540.4	1024.8	1248.1	3982.4	36
38	6460.7	997.3	1207.6	3911.2	6543.2	1025.8	1249.5	3984.9	38
40	6463.5	998.2	1209.0	3913.6	6545.9	1026.7	1250.9	3987.4	40
42	6466.3	999.2	1210.3	3916.1	6548.6	1027.7	1252.3	3989.9	42
44	6469.0	1000.1	1211.7	3918.5	6551.4	1028.6	1253.7	3992.3	44
46	6471.8	1001.0	1213.1	3921.0	6554.1	1029.6	1255.1	3994.8	46
48	6474.5	1002.0	1214.5	3923.4	6556.9	1030.5	1256.5	3997.3	48
50	6477.3	1002.9	1215.9	3925.9	6559.6	1031.5	1257.9	3999.8	50
52	6480.1	1003.8	1217.3	3928.3	6562.3	1032.4	1259.3	4002.2	52
54	6482.8	1004.8	1218.7	3930.8	6565.1	1033.4	1260.7	4004.7	54
56	6485.6	1005.7	1220.1	3933.2	6567.8	1034.3	1262.1	4007.2	56
58	6488.3	1006.7	1221.5	3935.7	6570.6	1035.3	1263.5	4009.7	58
60	6491.1	1007.7	1222.9	3938.1	6573.3	1036.3	1265.0	4012.1	60

′	70°				71°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	6573.3	1036.3	1265.0	4012.1	6654.9	1065.1	1308.4	4087.1	0
2	6576.0	1037.3	1266.4	4014.6	6657.6	1066.1	1309.9	4089.7	2
4	6578.7	1038.2	1267.9	4017.1	6660.3	1067.0	1311.3	4092.2	4
6	6581.5	1039.2	1269.3	4019.6	6663.0	1068.0	1312.8	4094.7	6
8	6584.2	1040.1	1270.8	4022.1	6665.7	1068.9	1314.2	4097.2	8
10	6586.9	1041.1	1272.2	4024.6	6668.4	1069.9	1315.7	4099.8	10
12	6589.6	1042.1	1273.6	4027.1	6671.1	1070.9	1317.2	4102.3	12
14	6592.3	1043.0	1275.1	4029.6	6673.8	1071.9	1318.6	4104.8	14
16	6595.1	1044.0	1276.5	4032.1	6676.6	1072.9	1320.1	4107.3	16
18	6597.8	1044.9	1278.0	4034.6	6679.3	1073.8	1321.5	4109.8	18
20	6600.5	1045.9	1279.4	4037.1	6682.0	1074.8	1323.0	4112.4	20
22	6603.2	1046.9	1280.8	4039.6	6684.7	1075.8	1324.4	4114.9	22
24	6605.9	1047.8	1282.3	4042.1	6687.4	1076.8	1325.9	4117.4	24
26	6608.7	1048.8	1283.7	4044.6	6690.1	1077.7	1327.4	4119.9	26
28	6611.4	1049.7	1285.2	4047.1	6692.8	1078.7	1328.9	4122.4	28
30	6614.1	1050.7	1286.6	4049.6	6695.5	1079.7	1330.4	4125.0	30
32	6616.8	1051.7	1288.0	4052.1	6698.2	1080.7	1331.8	4127.5	32
34	6619.5	1052.6	1289.5	4054.6	6700.9	1081.6	1333.3	4130.1	34
36	6622.3	1053.6	1290.9	4057.1	6703.6	1082.6	1334.8	4132.6	36
38	6625.0	1054.5	1292.4	4059.6	6706.3	1083.6	1336.3	4135.1	38
40	6627.7	1055.5	1293.8	4062.1	6709.0	1084.5	1337.8	4137.7	40
42	6630.4	1056.5	1295.3	4064.6	6711.7	1085.5	1339.2	4140.2	42
44	6633.1	1057.4	1296.7	4067.1	6714.4	1086.5	1340.7	4142.7	44
46	6635.9	1058.4	1298.2	4069.6	6717.2	1087.5	1342.2	4145.3	46
48	6638.6	1059.3	1299.6	4072.1	6719.9	1088.4	1343.7	4147.8	48
50	6641.3	1060.3	1301.1	4074.6	6722.6	1089.4	1345.2	4150.4	50
52	6644.0	1061.3	1302.6	4077.1	6725.3	1090.4	1346.7	4152.9	52
54	6646.7	1062.2	1304.0	4079.6	6728.0	1091.3	1348.2	4155.4	54
56	6649.5	1063.2	1305.5	4082.1	6730.7	1092.3	1349.7	4158.0	56
58	6652.2	1064.1	1306.9	4084.6	6733.4	1093.3	1351.2	4160.5	58
60	6654.9	1065.1	1308.4	4087.1	6736.1	1094.3	1352.7	4163.1	60

′	72°				73°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	6736.1	1094.3	1352.7	4163.1	6816.6	1123.9	1398.1	4240.0	0
2	6738.8	1095.2	1354.2	4165.6	6819.3	1124.8	1399.6	4242.6	2
4	6741.5	1096.2	1355.7	4168.2	6821.9	1125.8	1401.2	4245.1	4
6	6744.1	1097.2	1357.2	4170.7	6824.6	1126.8	1402.7	4247.7	6
8	6746.8	1098.2	1358.7	4173.3	6827.3	1127.8	1404.2	4250.3	8
10	6749.5	1099.2	1360.2	4175.8	6830.0	1128.8	1405.8	4252.9	10
12	6752.2	1100.1	1361.7	4178.4	6832.6	1129.8	1407.3	4255.5	12
14	6754.9	1101.1	1363.2	4181.0	6835.3	1130.8	1408.8	4258.1	14
16	6757.6	1102.1	1364.7	4183.5	6838.0	1131.8	1410.4	4260.7	16
18	6760.2	1103.1	1366.2	4186.1	6840.7	1132.8	1411.9	4263.2	18
20	6762.9	1104.1	1367.7	4188.6	6843.3	1133.8	1413.5	4265.8	20
22	6765.6	1105.1	1369.2	4191.2	6846.0	1134.8	1415.1	4268.4	22
24	6768.3	1106.0	1370.7	4193.7	6848.7	1135.8	1416.6	4271.0	24
26	6771.0	1107.0	1372.2	4196.3	6851.3	1136.8	1418.2	4273.6	26
28	6773.7	1108.0	1373.7	4198.8	6854.0	1137.8	1419.7	4276.2	28
30	6776.3	1109.0	1375.2	4201.4	6856.7	1138.8	1421.3	4278.8	30
32	6779.0	1109.9	1376.7	4204.0	6859.4	1139.8	1422.9	4281.4	32
34	6781.7	1110.9	1378.2	4206.5	6862.0	1140.8	1424.4	4284.0	34
36	6784.4	1111.9	1379.7	4209.1	6864.7	1141.8	1426.0	4286.6	36
38	6787.1	1112.9	1381.2	4211.7	6867.4	1142.8	1427.5	4289.2	38
40	6789.8	1113.9	1382.8	4214.3	6870.1	1143.8	1429.1	4291.8	40
42	6792.4	1114.9	1384.3	4216.8	6872.7	1144.8	1430.7	4294.4	42
44	6795.1	1115.9	1385.8	4219.4	6875.4	1145.8	1432.2	4297.0	44
46	6797.8	1116.9	1387.4	4222.0	6878.1	1146.8	1433.8	4299.6	46
48	6800.5	1117.9	1388.9	4224.5	6880.8	1147.8	1435.3	4302.2	48
50	6803.2	1118.9	1390.4	4227.1	6883.4	1148.8	1436.9	4304.8	50
52	6805.9	1119.9	1392.0	4229.7	6886.1	1149.8	1438.5	4307.4	52
54	6808.5	1120.9	1393.5	4232.3	6888.8	1150.8	1440.0	4310.0	54
56	6811.2	1121.9	1395.0	4234.8	6891.4	1151.8	1441.6	4312.6	56
58	6813.9	1122.9	1396.6	4237.4	6894.1	1152.8	1443.1	4315.2	58
60	6816.6	1123.9	1398.1	4240.0	6896.8	1153.8	1444.7	4317.8	60

	74°				75°				
	L. C.	M.	E.	T.	L. C.	M	E.	T.	
0	6896.8	1153.8	1444.7	4317.8	6976.4	1184.1	1492.5	4396.7	0
2	6899.4	1154.8	1446.2	4320.5	6979.0	1185.1	1494.1	4399.4	2
4	6902.1	1155.8	1447.8	4323.1	6981.7	1186.1	1495.7	4402.1	4
6	6904.8	1156.8	1449.4	4325.7	6984.3	1187.1	1497.3	4404.7	6
8	6907.4	1157.8	1451.0	4328.3	6986.9	1188.1	1499.0	4407.4	8
10	6910.1	1158.8	1452.6	4330.9	6989.6	1189.2	1500.6	4410.0	10
12	6912.7	1159.8	1454.1	4333.6	6992.2	1190.2	1502.2	4412.7	12
14	6915.4	1160.8	1455.7	4336.2	6994.9	1191.2	1503.8	4415.3	14
16	6918.0	1161.8	1457.3	4338.8	6997.5	1192.2	1505.4	4418.0	16
18	6920.7	1162.8	1458.9	4341.4	7000.1	1193.2	1507.0	4420.7	18
20	6923.3	1163.9	1460.5	4344.0	7002.8	1194.3	1508.7	4423.3	20
22	6926.0	1164.9	1462.0	4346.7	7005.4	1195.3	1510.3	4426.0	22
24	6928.6	1165.9	1463.6	4349.3	7008.0	1196.3	1512.0	4428.6	24
26	6931.3	1166.9	1465.2	4351.9	7010.7	1197.3	1513.6	4431.3	26
28	6933.9	1167.9	1466.8	4354.5	7013.3	1198.3	1515.3	4434.0	28
30	6936.6	1168.9	1468.4	4357.1	7015.9	1199.4	1516.9	4436.6	30
32	6939.2	1169.9	1469.9	4359.8	7018.6	1200.4	1518.5	4439.3	32
34	6941.9	1170.9	1471.5	4362.4	7021.2	1201.4	1520.2	4442.0	34
36	6944.6	1171.9	1473.1	4365.1	7023.9	1202.4	1521.8	4444.6	36
38	6947.2	1172.9	1474.7	4367.7	7026.5	1203.4	1523.5	4447.3	38
40	6949.9	1174.0	1476.4	4370.3	7029.1	1204.5	1525.1	4450.0	40
42	6952.5	1175.0	1478.0	4373.0	7031.8	1205.5	1526.7	4452.7	42
44	6955.2	1176.0	1479.6	4375.6	7034.4	1206.5	1528.4	4455.3	44
46	6957.8	1177.0	1481.2	4378.3	7037.0	1207.5	1530.0	4458.0	46
48	6960.5	1178.0	1482.8	4380.9	7039.7	1208.5	1531.7	4460.7	48
50	6963.1	1179.0	1484.4	4383.5	7042.3	1209.6	1533.3	4463.4	50
52	6965.8	1180.0	1486.0	4386.2	7045.0	1210.6	1534.9	4466.0	52
54	6968.4	1181.0	1487.7	4388.8	7047.6	1211.6	1536.6	4468.7	54
56	6971.1	1182.0	1489.3	4391.5	7050.2	1212.6	1538.2	4471.4	56
58	6973.7	1183.0	1490.9	4294.1	7052.9	1213.6	1539.9	4474.1	58
60	6976.4	1184.1	1492.5	4396.7	7055.5	1214.7	1541.5	4476.7	60

′	76°				77°				′
	L. C.	M.	E.	T.	L. C.	M	E.	T.	
0	7055.5	1214.7	1541.5	4476.7	7134.0	1245.6	1591.7	4557.8	0
2	7058.1	1215.7	1543.2	4479.4	7136.6	1246.6	1593.4	4560.5	2
4	7060.7	1216.7	1544.9	4482.1	7139.2	1247.7	1595.1	4563.3	4
6	7063.3	1217.8	1546.5	4484.8	7141.8	1248.7	1596.8	4566.0	6
8	7066.0	1218.8	1548.2	4487.5	7144.4	1249.8	1598.5	4568.7	8
10	7068.6	1219.8	1549.9	4490.2	7147.0	1250.8	1600.2	4571.5	10
12	7071.2	1220.9	1551.5	4492.9	7149.6	1251.8	1601.9	4574.2	12
14	7073.8	1221.9	1553.2	4495.6	7152.2	1252.9	1603.6	4576.9	14
16	7076.4	1222.9	1554.9	4498.3	7154.8	1253 9	1605.3	4579.7	16
18	7079.0	1224.0	1556.5	4501.0	7157.4	1255 0	1607.0	4582.4	18
20	7081.7	1225.0	1558.2	4503.7	7160.0	1256.0	1608.7	4585.1	20
22	7084.3	1226.0	1559.9	4506.3	7162.6	1257.0	1610.4	4587.9	22
24	7086.9	1227.1	1561.5	4509.0	7165.2	1258.1	1612.1	4590.6	24
26	7089.5	1228.1	1563.2	4511.7	7167.8	1259.1	1613.8	4593.3	26
28	7092.1	1229.1	1564.9	4514.4	7170.4	1260.2	1615.5	4596.0	28
30	7094.7	1230.2	1566.5	4517.1	7173.0	1261.2	1617.3	4598 8	30
32	7097.4	1231.2	1568.2	4519.8	7175.6	1262.2	1619.0	4601.5	32
34	7100.0	1232.2	1569.9	4522.5	7178.2	1263.3	1620.7	4604.3	34
36	7102.6	1233.3	1571.5	4525.3	7180.8	1264.3	1622.4	4607.0	36
38	7105.2	1234.3	1573.2	4528.0	7183.4	1265.4	1624.1	4609.8	38
40	7107.8	1235.3	1574.8	4530.7	7186.0	1266.4	1625.9	4612.5	40
42	7110.4	1236.4	1576.4	4533.4	7188.6	1267.4	1627.6	4615.3	42
44	7113.1	1237.4	1578.1	4536.1	7191.2	1268.5	1629.3	4618.0	44
46	7115.7	1238 4	1579.8	4538.8	7193.8	1269.5	1631.0	4620.8	46
48	7118.3	1239 5	1581.5	4541.5	7196.4	1270 6	1632.7	4623.5	48
50	7120.9	1240.5	1583.2	4544.2	7199.0	1271.6	1634 5	4626.3	50
52	7123.5	1241.5	1584.9	4547.0	7201.6	1272.7	1636.2	4629.0	52
54	7126.1	1242.6	1586.6	4549.7	7204 2	1273.7	1637.9	4631.8	54
56	7128.8	1243.6	1588.3	4552.4	7206.8	1274.8	1639.6	4634 5	56
58	7131.4	1244.6	1590.0	4555.1	7209.4	1275.8	1641.3	4637.3	58
60	7134 0	1245.6	1591.7	4557.8	7212.0	1276.9	1643.1	4640.0	60

′	78°				79°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	7212.0	1276.9	1643.1	4640.0	7289.5	1308.5	1696.0	4723.4	0
2	7214.6	1278.0	1644.8	4642.8	7292.1	1309.5	1697.7	4726.2	2
4	7217.2	1279.0	1646.6	4645.6	7294.6	1310.6	1699.5	4729.0	4
6	7219.7	1280.1	1648.3	4648.3	7297.2	1311.7	1701.3	4731.8	6
8	7222.3	1281.1	1650.1	4651.1	7299.7	1312.7	1703.1	4734.7	8
10	7224.9	1282.2	1651.8	4653.9	7302.3	1313.8	1704.9	4737.5	10
12	7227.5	1283.2	1653.6	4656.7	7304.9	1314.9	1706.6	4740.3	12
14	7230.1	1284.3	1655.3	4659.4	7307.4	1315.9	1708.4	4743.1	14
16	7232.7	1285.3	1657.1	4662.2	7310.0	1317.0	1710.2	4745.9	16
18	7235.2	1286.4	1658.8	4665.0	7312.6	1318.1	1712.0	4748.7	18
20	7237.8	1287.4	1660.6	4667.7	7315.1	1319.1	1713.8	4751.5	20
22	7240.4	1288.5	1662.3	4670.5	7317.7	1320.2	1715.6	4754.3	22
24	7243.0	1289.5	1664.1	4673.3	7320.3	1321.3	1717.4	4757.1	24
26	7245.6	1290.6	1665.8	4676.0	7322.8	1322.3	1719.2	4760.0	26
28	7248.2	1291.6	1667.6	4678.8	7325.4	1323.4	1721.0	4762.8	28
30	7250.7	1292.7	1669.3	4681.6	7327.9	1324.5	1722.8	4765.6	30
32	7253.3	1293.7	1671.1	4684.4	7330.5	1325.5	1724.6	4768.4	32
34	7255.9	1294.8	1672.8	4687.2	7333.1	1326.6	1726.4	4771.2	34
36	7258.5	1295.8	1674.6	4689.9	7335.6	1327.7	1728.2	4774.1	36
38	7261.1	1296.9	1676.3	4692.7	7338.2	1328.7	1730.0	4776.9	38
40	7263.7	1297.9	1678.2	4695.5	7340.8	1329.8	1731.9	4779.7	40
42	7266.2	1299.0	1679.9	4698 3	7343.3	1330.8	1733.7	4782.6	42
44	7268.8	1300.0	1681.7	4701.1	7345.9	1331.9	1735.5	4785.4	44
46	7271.4	1301.1	1683.5	4703.9	7348.4	1333.0	1737.3	4788.2	46
48	7274.0	1302.1	1685.3	4706.7	7351.0	1334.1	1739.1	4791.0	48
50	7276.6	1303.2	1687.1	4709.5	7353.6	1335.2	1740.9	4793.9	50
52	7279.2	1304.2	1688.8	4712.2	7356.1	1336.2	1742.7	4796.7	52
54	7281.7	1305.3	1690.6	4715.0	7358.7	1337.3	1744.5	4799.5	54
56	7284.3	1306.3	1692.4	4717.8	7361.3	1338.4	1746.3	4802.4	56
58	7286.9	1307.4	1694.2	4720.6	7363.8	1339.5	1748.1	4805.2	58
60	7289.5	1308.5	1696.0	4723.4	7366.4	1340.6	1750.0	4808.0	60

′	80°				81°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	7366.4	1340.6	1750 0	4808.0	7442.7	1372.8	1805.5	4893.9	0
2	7368 9	1341.7	1751.8	4810.9	7445.2	1373.9	1807.3	4896.8	2
4	7371.5	1342.7	1753.7	4813.7	7447.7	1375.0	1809.2	4899.7	4
6	7374.0	1343.8	1755.5	4816.6	7450.3	1376.1	1811.1	4902.6	6
8	7376.6	1344.9	1757.4	4819.4	7452.8	1377.1	1813.0	4905.4	8
10	7379.1	1346.0	1759.2	4822.3	7455.3	1378.2	1814.9	4908.3	10
12	7381.7	1347.0	1761.0	4825.1	7457.8	1379.3	1816.8	4911.2	12
14	7384.2	1348.1	1762.9	4828.0	7460.4	1380.4	1818.6	4914.1	14
16	7386.7	1349.2	1764.7	4830.8	7462.9	1381.4	1820.5	4917.0	16
18	7389.3	1350.3	1766.6	4833.7	7465.4	1382.5	1822.4	4919.9	18
20	7391.8	1351.3	1768.4	4836.5	7467.9	1383.6	1824.2	4922.8	20
22	7394.4	1352.4	1770.2	4839.4	7470.4	1384.7	1826.1	4925.7	22
24	7396.9	1353.5	1772.1	4842.2	7473.0	1385.7	1828.0	4928.6	24
26	7399.5	1354.6	1773.9	4845.1	7475.5	1386.8	1829.9	4931.5	26
28	7402.0	1355.6	1775.8	4847.9	7478.0	1387.9	1831.8	4934.4	28
30	7404.5	1356.7	1777.6	4850.8	7480.5	1389.0	1833.7	4937.2	30
32	7407.1	1357.8	1779.4	4853.7	7483.1	1390.1	1835.6	4940.2	32
34	7409.6	1358.9	1781.3	4856.5	7485.6	1391.2	1837.5	4943.1	34
36	7412.2	1359.9	1783.1	4859.4	7488.1	1392.3	1839.4	4946.0	36
38	7414.7	1361.0	1785.0	4862.3	7490.6	1393.4	1841.3	4948.9	38
40	7417.3	1362.1	1786.8	4865.1	7493.2	1394.5	1843.2	4951.8	40
42	7419 8	1363 2	1788.6	4868.0	7495.7	1395.6	1845.1	4954.7	42
44	7422.3	1364.2	1790.5	4870.9	7498.2	1396.7	1847.0	4957.6	44
46	7424.9	1365.3	1792 4	4873.8	7500.7	1397.8	1848.9	4960.6	46
48	7427.4	1366.4	1794 3	4876.6	7503.3	1398.9	1850.8	4963.5	48
50	7430.0	1367.5	1796.2	4879.5	7505.8	1400.0	1852 7	4966.4	50
52	7432.5	1368.5	1798.0	4882.4	7508.3	1401.1	1854 6	4969.3	52
54	7435.1	1369.6	1799.9	4885 3	7510.8	1402.2	1856.5	4972.2	54
56	7437.6	1370 7	1801.8	4888.1	7513.3	1403.3	1858.4	4975.1	56
58	7440.1	1371.8	1803.7	4891.0	7515.9	1404 4	1860.3	4978.0	58
60	7442.7	1372.8	1805.5	4893.9	7518.4	1405.5	1862 3	4981.0	60

′	82°				83°				′
	L. C.	M.	E.	T	L. C.	M.	E.	T.	
0	7518.4	1405.5	1862.3	4981.0	7593.6	1438.5	1920.6	5069.4	0
2	7520.9	1406.6	1864.2	4983.9	7596.1	1439 6	1922.6	5072.4	2
4	7523.4	1407.7	1866.1	4986.8	7598.6	1440 7	1924.6	5075.4	4
6	7525.9	1408 8	1868.1	4989.8	7601.1	1441.8	1926.5	5078.4	6
8	7528.4	1409.9	1870.0	4992 7	7603.6	1442.9	1928.5	5081.4	8
10	7530.9	1411.0	1871 9	4995.7	7606.0	1444 0	1930.5	5084.4	10
12	7533.4	1412.1	1873 9	4998.6	7608.5	1445 1	1932.4	5087.3	12
14	7535.9	1413.2	1875 8	5001.5	7611.0	1446.2	1934.4	5090.3	14
16	7538.5	1414.3	1877.7	5004.5	7613.5	1447.3	1936.4	5093.3	16
18	7541.0	1415.4	1879.7	5007.4	7616.0	1448.4	1938.4	5096.3	18
20	7543.5	1416.5	1881.6	5010.3	7618.5	1449.6	1940.4	5099.3	20
22	7546.0	1417.6	1883.5	5013.3	7621.0	1450.7	1942.4	5102.3	22
24	7548.5	1418.7	1885.5	5016.2	7623.5	1451.8	1944.4	5105.2	24
26	7551.0	1419.8	1887.4	5019.2	7626.0	1452.9	1946.4	5108.2	26
28	7553.5	1420.9	1889.3	5022.1	7628.5	1454.0	1948 4	5111.2	28
30	7556.0	1422.0	1891.3	5025.0	7630.9	1455.1	1950.4	5114 2	30
32	7558.5	1423.1	1893.2	5028.0	7633.4	1456.2	1952.4	5117 2	32
34	7561.0	1424.2	1895.1	5031.0	7635.9	1457.3	1954.4	5120.2	34
36	7563.5	1425.3	1897.1	5033.9	7638.4	1458.4	1956.4	5123.2	36
38	7566.0	1426.4	1899.0	5036.9	7640.9	1459.5	1958.4	5126.2	38
40	7568.5	1427.5	1901.0	5039.8	7643.4	1460.7	1960.4	5129.2	40
42	7571.0	1428 6	1902.9	5042.8	7645.9	1461.8	1962.4	5132.2	42
44	7573.5	1429.7	1904.9	5045.8	7648.4	1462.9	1964.4	5135.2	44
46	7576.1	1430.8	1906.9	5048.7	7650.9	1464.0	1966.4	5138.2	46
48	7578.6	1431.9	1908.8	5051.7	7653.4	1465.1	1968.4	5141.2	48
50	7581.1	1433.0	1910.8	5054.6	7655.8	1466.2	1970 4	5144 3	50
52	7583.6	1434.1	1912.8	5057.6	7658.3	1467.3	1972.4	5147.3	52
54	7586.1	1435.2	1914.7	5060.6	7660.8	1468.4	1974.4	5150.3	54
56	7588.6	1436.3	1916.7	5063.5	7663.3	1469.5	1976.4	5153.3	56
58	7591.1	1437.4	1918.7	5066.5	7665.8	1470.6	1978.4	5156 3	58
60	7593.6	1438.5	1920.6	5069.4	7668.3	1471.8	1980.5	5159.3	60

′	84°				85°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T	
0	7668.3	1471.8	1980.5	5159.3	7742.4	1505.4	2041.8	5250.6	0
2	7670.8	1472.9	1982.5	5162.3	7744.8	1506.5	2043.9	5253 6	2
4	7673.2	1474.0	1984.5	5165.3	7747.3	1507.6	2046.0	5256.7	4
6	7675.7	1475.1	1986.6	5168.4	7749.7	1508.8	2048.0	5259.8	6
8	7678.2	1476.2	1988.6	5171.4	7752.2	1509.9	2050.1	5262.9	8
10	7680 6	1477.4	1990.6	5174.4	7754.6	1511.0	2052.2	5266 0	10
12	7683.1	1478.5	1992.7	5177.5	7757.1	1512 2	2054.2	5269.0	12
14	7685 6	1479.6	1994.7	5180.5	7759 5	1513.3	2056.3	5272.1	14
16	7688.1	1480.7	1996.7	5183.5	7762.0	1514.4	2058.4	5275.2	16
18	7690.5	1481.8	1998.8	5186.6	7764 4	1515.6	2060.5	5278.3	18
20	7693.0	1483.0	2000.8	5189.6	7766.9	1516.7	2062.6	5281.4	20
22	7695.5	1484 1	2002.8	5192.6	7769.3	1517.8	2064.7	5284.4	22
24	7697.9	1485.2	2004.9	5195.6	7771.8	1519.0	2066.8	5287.5	24
26	7700.4	1486.3	2006.9	5198.7	7774.2	1520.1	2068.9	5290.6	26
28	7702.9	1487.4	2008.9	5201.7	7776.7	1521.2	2071.0	5293.7	28
30	7705.3	1488 6	2011.0	5204.7	7779.1	1522.4	2073.1	5296.7	30
32	7707.8	1489.7	2013.0	5207.8	7781.5	1523.5	2075 2	5299 8	32
34	7710.3	1490.8	2015.0	5210 8	7784.0	1524.6	2077 3	5302.9	34
36	7712.8	1491.9	2017.0	5213.9	7786.4	1525.8	2079.4	5306.1	36
38	7715.2	1493.0	2019.1	5216.9	7788.9	1526.9	2081.5	5309.2	38
40	7717.7	1494.2	2021.2	5220.0	7791 3	1528 0	2083.7	5312.3	40
42	7720.2	1495.3	2023.2	5223.1	7793.8	1529.2	2085.8	5315.4	42
44	7722 6	1496.4	2025.3	5226.1	7796.2	1530.3	2087.9	5318.5	44
46	7725.1	1497.5	2027.4	5229.2	7798 7	1531.4	2090.0	5321.6	46
48	7727.6	1498.6	2029.4	5232.2	7801.1	1532.6	2092.1	5324.7	48
50	7730.0	1499 8	2031.5	5235.3	7803.6	1533.7	2094.2	5327.8	50
52	7732 5	1500.9	2033.6	5238.3	7806.0	1534 8	2096.3	5330 9	52
54	7735.0	1502.0	2035.6	5241 4	7808.5	1536.0	2098.4	5334.0	54
56	7737 5	1503.1	2037 7	5244 5	7810.9	1537.1	2100.6	5337.1	56
58	7739 9	1504.2	2039.8	5247.5	7813 4	1538.2	2102.7	5340.2	58
60	7742.4	1505 4	2041.8	5250.6	7815.8	1539.3	2104.8	5343.3	60

′	86°				87°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	7815.8	1539.3	2104.8	5343.3	7888.5	1573.6	2169.5	5437.5	0
2	7818.2	1540.4	2106.9	5346.4	7890.9	1574.8	2171.6	5440.7	2
4	7820.6	1541.6	2109.1	5349.5	7893.3	1575.9	2173.8	5443.9	4
6	7823.1	1542.7	2111.2	5352.7	7895.7	1577.1	2176.0	5447.1	6
8	7825.5	1543.9	2113.4	5355.8	7898.1	1578.2	2178.2	5450.3	8
10	7827.9	1545.0	2115.5	5358.9	7900.5	1579.4	2180.4	5453.4	10
12	7830.3	1546.1	2117.6	5362.0	7903.0	1580.5	2182.5	5456.6	12
14	7832.8	1547.3	2119.8	5365.2	7905.4	1581.7	2184.7	5459.8	14
16	7835.2	1548.4	2121.9	5368.3	7907.8	1582.9	2186.9	5463.0	16
18	7837.6	1549.6	2124.1	5371.4	7910.2	1584.0	2189.1	5466.2	18
20	7840.0	1550.7	2126.2	5374.6	7912.6	1585.1	2191.3	5469.4	20
22	7842.4	1551.8	2128.3	5377.7	7915.0	1586.3	2193.5	5472.5	22
24	7844.9	1553.0	2130.5	5 80.8	7917.4	1587.4	2195.7	5475.7	24
26	7847.3	1554.1	2132.6	5383.9	7919.8	1588.6	2197.9	5478.9	26
28	7849.7	1555.3	2134.8	5387.1	7922.2	1589.7	2200.1	5482.1	28
30	7852.1	1556.4	2136.9	5390.2	7924.6	1590.9	2202.3	5485.3	30
32	7854.6	1557.5	2139.0	5393.4	7927.1	1592.0	2204.5	5488.5	32
34	7857.0	1558.7	2141.2	5396.5	7929.5	1593.2	2206.8	5491.7	34
36	7859.4	1559.8	2143.3	5399.7	7931.9	1594.3	2209.0	5494.9	36
38	7861.8	1561.0	2145.5	5402.8	7934.3	1595.5	2211.2	5498.1	38
40	7864.3	1562.1	2147.7	5406.0	7936.7	1596.6	2213.4	5501.3	40
42	7866.7	1563.2	2149.8	5409.1	7939.1	1597.8	2215.6	5504.5	42
44	7869.1	1564.4	2152.0	5412.3	7941.5	1598.9	2217.8	5507.7	44
46	7871.5	1565.5	2154.2	5415.4	7943.9	1600.1	2220.0	5510.9	46
48	7874.0	1566.7	2156.4	5418.6	7946.3	1601.2	2222.3	5514.1	48
50	7876.4	1567.8	2158.6	5421.8	7948.7	1602.4	2224.5	5517.3	50
52	7878.8	1568.9	2160.7	5424.9	7951.2	1603.5	2226.7	5520.5	52
54	7881.2	1570.1	2162.9	5428.1	7953.6	1604.7	2228.9	5523.7	54
56	7883.6	1571.2	2165.1	5431.2	7956.0	1605.8	2231.1	5526.9	56
58	7886.1	1572.4	2167.3	5434.4	7958.4	1607.0	2233.3	5530.1	58
60	7888.5	1573.6	2169.5	5437.5	7960.8	1608.2	2235.6	5533.3	60

′	88°				89°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	7960.8	1608.2	2235.6	5533.3	8032.4	1643.0	2303.6	5630.8	0
2	7963.2	1609.4	2237.8	5536.6	8034.8	1644.1	2305.9	5634.1	2
4	7965.6	1610.5	2240.1	5539.8	8037.1	1645.3	2308.2	5637.4	4
6	7968.0	1611.7	2242.3	5543.1	8039.5	1646.5	2310.5	5640.7	6
8	7970.3	1612.8	2244.6	5546.3	8041.9	1647.7	2312.8	5644.0	8
10	7972.7	1614.0	2246.8	5549.5	8044.2	1648.9	2315.1	5647.3	10
12	7975.1	1615.2	2249.1	5552.8	8046.6	1650.0	2317.4	5650.6	12
14	7977.5	1616.3	2251.3	5556.0	8049.0	1651.2	2319.7	5653.9	14
16	7979.9	1617.5	2253.6	5559.2	8051.4	1652.4	2322.0	5657.1	16
18	7982.3	1618.6	2255.8	5562.5	8053.7	1653.6	2324.3	5660.4	18
20	7984.7	1619.8	2258.1	5565.7	8056.1	1654.8	2326.7	5663.7	20
22	7987.1	1621.0	2260.4	5568.9	8058.5	1655.9	2329.0	5667.0	22
24	7989.4	1622.1	2262.7	5572.2	8060.8	1657.1	2331.3	5670.3	24
26	7991.8	1623.3	2264.9	5575.4	8063.2	1658.3	2333.7	5673.6	26
28	7994.2	1624.4	2267.2	5578.6	8065.6	1659.5	2336.0	5676.9	28
30	7996.6	1625.6	2269.5	5581.9	8067.9	1660.7	2338.3	5680.2	30
32	7999.0	1626.8	2271.7	5585.1	8070.3	1661.8	2340.7	5683.5	32
34	8001.4	1627.9	2273.9	5588.4	8073.7	1663.0	2343.0	5686.8	34
36	8003.8	1629.1	2276.2	5591.7	8075.1	1664.2	2345.3	5690.2	36
38	8006.1	1630.2	2278.5	5594.9	8077.4	1665.4	2347.7	5693.5	38
40	8008.5	1631.4	2280.8	5598.2	8079.8	1666.6	2350.0	5696.8	40
42	8010.9	1632.6	2283.0	5601.4	8082.2	1667.7	2352.3	5700.1	42
44	8013.3	1633.7	2285.3	5604.7	8084.5	1668.8	2354.7	5703.4	44
46	8015.7	1634.9	2287.6	5608.0	8086.9	1670.0	2357.0	5706.8	46
48	8018.1	1636.0	2289.9	5611.2	8089.3	1671.2	2359.3	5710.1	48
50	8020.5	1637.2	2292.2	5614.5	8091.6	1672.4	2361.7	5713.4	50
52	8022.9	1638.4	2294.4	5617.8	8094.0	1673.5	2364.0	5716.7	52
54	8025.2	1639.5	2296.7	5621.0	8096.4	1674.7	2366.3	5720.0	54
56	8027.6	1640.7	2299.0	5624.3	8098.8	1675.9	2368.7	5723.4	56
58	8030.0	1641.8	2301.3	5627.5	8101.1	1677.1	2371.0	5726.7	58
60	8032.4	1643.0	2303.6	5630.8	8103.5	1678.3	2373.4	5730.0	60

′	90°				91°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	8103.5	1678.3	2373.4	5730.0	8173.9	1713.8	2445.1	5830.9	0
2	8105.8	1679.5	2375.8	5733.3	8176.2	1715.0	2447.5	5834.3	2
4	8108.2	1680.6	2378.2	5736.7	8178.5	1716.2	2450.0	5837.7	4
6	8110.5	1681.8	2380.5	5740.0	8180.9	1717.4	2452.4	5841.1	6
8	8112.9	1683.0	2382.9	5743.4	8183.2	1718.6	2454.8	5844.5	8
10	8115.2	1684.2	2385.3	5746.7	8185.5	1719.7	2457.2	5847.9	10
12	8117.6	1685.4	2387.6	5750.0	8187.9	1720.9	2459.7	5851.3	12
14	8119.9	1686.5	2390.0	5753.4	8190.2	1722.1	2462.1	5854.7	14
16	8122.3	1687.7	2392.4	5756.7	8192.5	1723.3	2464.5	5858.1	16
18	8124.6	1688.9	2394.7	5760.1	8194.8	1724.5	2467.0	5861.5	18
20	8127.0	1690.1	2397.1	5763.4	8197.2	1725.7	2469.4	5864.9	20
22	8129.3	1691.3	2399.5	5766.8	8199.5	1726.9	2471.9	5868.3	22
24	8131.7	1692.5	2401.9	5770.1	8201.8	1728.1	2474.3	5871.8	24
26	8134.0	1693.6	2404.3	5773.5	8204.2	1729.3	2476.7	5875.2	26
28	8136.4	1694.8	2406.6	5776.9	8206.5	1730.5	2479.2	5878.6	28
30	8138.7	1696.0	2409.0	5780.2	8208.8	1731.7	2481.6	5882.0	30
32	8141.1	1697.2	2411.4	5783.6	8211.1	1732.9	2484.1	5885.4	32
34	8143.4	1698.4	2413.8	5787.0	8213.5	1734.1	2486.5	5888.9	34
36	8145.8	1699.6	2416.2	5790.3	8215.8	1735.3	2489.0	5892.3	36
38	8148.1	1700.7	2418.6	5793.7	8218.1	1736.4	2491.5	5895.7	38
40	8150.4	1701.9	2421.0	5797.1	8220.4	1737.6	2493.9	5899.2	40
42	8152.8	1703.1	2423.4	5800.4	8222.8	1738.8	2496.4	5902.6	42
44	8155.1	1704.3	2425.8	5803.8	8225.1	1740.0	2498.9	5906.0	44
46	8157.5	1705.5	2428.2	5807.2	8227.4	1741.2	2501.3	5909.4	46
48	8159.8	1706.7	2430.6	5810.6	8229.7	1742.4	2503.8	5912.9	48
50	8162.2	1707.9	2433.0	5814.0	8232.0	1743.6	2506.3	5916.3	50
52	8164.5	1709.0	2435.4	5817.3	8234.3	1744.8	2508.7	5919.8	52
54	8166.8	1710.2	2437.9	5820.7	8236.7	1746.0	2511.2	5923.2	54
56	8169.2	1711.4	2440.3	5824.1	8239.0	1747.2	2513.7	5926.7	56
58	8171.5	1712.6	2442.7	5827.5	8241.3	1748.4	2516.2	5930.1	58
60	8173.9	1713.8	2445.1	5830.9	8243.6	1749.6	2518.7	5933.6	60

′	92°				93°				′
	L. C	M.	E.	T.	L. C.	M.	E.	T.	
0	8243.6	1749.6	2518.7	5933.6	8312.8	1785.7	2594.2	6038.2	0
2	8245.9	1750.8	2521.2	5937.0	8315.1	1786.9	2596.8	6041.7	2
4	8248.2	1752.0	2523.6	5940.5	8317.4	1788.2	2599.3	6045.2	4
6	8250.6	1753.2	2526.1	5944.0	8319.7	1789.4	2601.9	6048.7	6
8	8252.9	1754.4	2528.6	5947.4	8322.0	1790.6	2604.4	6052.2	8
10	8255.2	1755.6	2531.1	5950.9	8324.3	1791.8	2607.0	6055.8	10
12	8257.5	1756.8	2533.6	5954.4	8326.6	1793.0	2609.6	6059.3	12
14	8259.8	1758.0	2536.1	5957.8	8328.8	1794.2	2612.1	6062.8	14
16	8262.2	1759.2	2538.6	5961.3	8331.1	1795.4	2614.7	6066.4	16
18	8264.5	1760.4	2541.1	5964.8	8333.3	1796.6	2617.3	6069.9	18
20	8266.8	1761.6	2543.6	5968.2	8335.6	1797.8	2619.8	6073.4	20
22	8269.1	1762.8	2546.1	5971.7	8337.9	1799.1	2622.4	6077.0	22
24	8271.4	1764.0	2548.6	5975.2	8340.2	1800.3	2625.0	6080.5	24
26	8273.7	1765.2	2551.2	5978.7	8342.5	1801.5	2627.6	6084.1	26
28	8276.0	1766.4	2553.7	5982.2	8344.8	1802.7	2630.2	6087.6	28
30	8278.3	1767.6	2556.2	5985.6	8347.1	1803.9	2632.7	6091.2	30
32	8280.6	1768.8	2558.7	5989.1	8349.4	1805.1	2635.3	6094.7	32
34	8282.9	1770.0	2561.2	5992.6	8351.7	1806.3	2637.9	6098.3	34
36	8285.2	1771.2	2563.8	5996.1	8354.0	1807.6	2640.5	6101.8	36
38	8287.5	1772.5	2566.3	5999.6	8356.3	1808.8	2643.1	6105.4	38
40	8289.8	1773.7	2568.8	6003.1	8358.5	1810.0	2645.7	6109.0	40
42	8292.1	1774.9	2571.3	6006.6	8360.8	1811.2	2648.3	6112.5	42
44	8294.4	1776.1	2573.9	6010.1	8363.1	1812.4	2650.9	6116.1	44
46	8296.7	1777.3	2576.4	6013.6	8365.4	1813.6	2653.5	6119.7	46
48	8299.0	1778.5	2578.9	6017.1	8367.7	1814.9	2656.1	6123.2	48
50	8301.3	1779.7	2581.5	6020.6	8369.9	1816.1	2658.7	6126.8	50
52	8303.6	1780.9	2584.0	6024.1	8372.2	1817.3	2661.3	6130.4	52
54	8305.9	1782.1	2586.6	6027.6	8374.5	1818.5	2663.9	6133.9	54
56	8308.2	1783.3	2589.1	6031.1	8376.8	1819.7	2666.6	6137.5	56
58	8310.5	1784.5	2591.7	6034.6	8379.1	1820.9	2669.2	6141.1	58
60	8312.8	1785.7	2594.2	6038.2	8381.3	1822.2	2671.8	6144.7	60

′	94°				95°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	8381.3	1822.2	2671.8	6144.7	8449.2	1858.9	2751.5	6253.2	0
2	8383.6	1823.4	2674.4	6148.3	8451.5	1860.1	2754.2	6256.9	2
4	8385.9	1824.6	2677.0	6151.9	8453.7	1861.3	2756.9	6260.5	4
6	8388.1	1825.8	2679.7	6155.4	8456.0	1862.6	2759.6	6264.2	6
8	8390.4	1827.0	2682.3	6159.0	8458.2	1863.8	2762.3	6267.8	8
10	8392.7	1828.3	2684.9	6162.6	8460.4	1865.0	2765 0	6271.5	10
12	8395.0	1829.5	2687.6	6166.2	8462.7	1866.3	2767.7	6275.2	12
14	8397.2	1830.7	2690.2	6169.8	8464.9	1867.5	2770.4	6278.8	14
16	8399.5	1831.9	2692.8	6173.4	8467.2	1868.7	2773.1	6282.5	16
18	8401.7	1833.1	2695.6	6177.0	8469.4	1869.9	2775.8	6286.2	18
20	8404.0	1834.4	2698.1	6180.6	8471.7	1871.2	2778.5	6289.8	20
22	8406.3	1835.6	2700.8	6184.2	8473.9	1872.4	2781.2	6293.5	22
24	8408.5	1836.8	2703.4	6187.8	8476.2	1873.6	2784.0	6297.2	24
26	8410.8	1838.0	2706.1	6191.5	8478.4	1874.9	2786.7	6300.9	26
28	8413.1	1839.3	2708.7	6195.1	8480.7	1876.1	2789.4	6304.6	28
30	8415.3	1840.5	2711.4	6198.7	8482.9	1877.3	2792.1	6308.2	30
32	8417.6	1841.7	2714.0	6202.3	8485.1	1878.6	2794.9	6311.9	32
34	8419.9	1842.9	2716.7	6205.9	8487.4	1879.8	2797.6	6315.6	34
36	8422.1	1844.2	2719.3	6209.5	8489.6	1881.0	2800.3	6319.3	36
38	8424.4	1845.4	2722.0	6213.2	8491.9	1882.3	2803.1	6323.0	38
40	8426.6	1846.6	2724.7	6216.8	8494.1	1883.5	2805.8	6326.7	40
42	8428.9	1847.8	2727.3	6220.4	8496.3	1884.8	2808.6	6330.4	42
44	8431.2	1849.1	2730.0	6224.1	8498.6	1886.0	2811.3	6334.1	44
46	8433.4	1850.3	2732.7	6227.7	8500.8	1887.2	2814.1	6337.8	46
48	8435.7	1851.5	2735.4	6231.3	8503.0	1888.5	2816.8	6341.5	48
50	8437.9	1852.7	2738.0	6235.0	8505.3	1889.7	2819.6	6345.2	50
52	8440.2	1854.0	2740.7	6238.6	8507.5	1890.9	2822.3	6349.0	52
54	8442.4	1855.2	2743.4	6242.3	8509.8	1892.2	2825.1	6352.7	54
56	8444.7	1856.4	2746.1	6245.9	8512.0	1893.4	2827.8	6356.4	56
58	8447.0	1857.6	2748.8	6249.6	8514.2	1894.6	2830.6	6360.1	58
60	8449.2	1858.9	2751.5	6253.2	8516.4	1895.9	2833.4	6363.8	60

′	96°				97°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	8516.4	1895.9	2833.4	6363.8	8583.0	1933.2	2917.5	6476 6	0
2	8518.7	1897.1	2836.1	6367.5	8585.2	1934.4	2920.3	6480.4	2
4	8520.9	1898.4	2838.9	6371.3	8587.5	1935.7	2923.2	6484.2	4
6	8523.1	1899.6	2841.7	6375.0	8589 7	1936.9	2926.0	6488.0	6
8	8525.4	1900.8	2844.5	6378.7	8591.9	1938.2	2928.9	6491.8	8
10	8527.6	1902.1	2847.2	6382.5	8594 1	1939.4	2931.7	6495.6	10
12	8529.8	1903.3	2850.0	6386.2	8596.3	1940.7	2934.6	6499.4	12
14	8532.0	1904 6	2852.8	6389.9	8598.5	1941.9	2937.5	6503.2	14
16	8534.3	1905.8	2855 6	6393.7	8600.7	1943.2	2940.3	6507.1	16
18	8536.5	1907.0	2858.4	6397.4	8602.9	1944.4	2943.2	6510.9	18
20	8538.7	1908.3	2861.2	6401.2	8605 1	1945.7	2946.1	6514.7	20
22	8540.9	1909.5	2864.0	6404.9	8607.3	1946.9	2948.9	6518.5	22
24	8543.2	1910.8	2866.7	6408.7	8609.5	1948.2	2951.8	6522.3	24
26	8545.4	1912.0	2869.5	6412.4	8611.7	1949.4	2954.7	6526.2	26
28	8547.6	1913 3	2872.3	6416.2	8613.9	1950.7	2957.6	6530.0	28
30	8549.8	1914 5	2875.1	6419.9	8616.1	1952.0	2960.4	6533.8	30
32	8552.0	1915.7	2877.9	6423.7	8618.3	1953.2	2963.3	6537.7	32
34	8554.3	1917.0	2880 8	6427.5	8620.5	1954.5	2966.2	6541.5	34
36	8556.5	1918.2	2883.6	6431.2	8622.7	1955.7	2969.1	6545.3	36
38	8558.7	1919.5	2886.4	6435.0	8624.9	1957.0	2972.0	6549.2	38
40	8560.9	1920.7	2889.2	6438.8	8627.1	1958.2	2974.9	6553.0	40
42	8563.1	1922.0	2892.0	6442 5	8629.3	1959 5	2977.8	6556.9	42
44	8565 3	1923.2	2894.8	6446.3	8631.5	1960.7	2980.7	6560.7	44
46	8567 6	1924 5	2897.7	6450.1	8633.7	1962.0	2983.6	6564.6	46
48	8569 8	1925.7	2900.5	6453.9	8635.8	1963.2	2986.5	6568.4	48
50	8572.0	1927.0	2903.3	6457.6	8638.0	1964.5	2989.4	6572.3	50
52	8574.2	1928.2	2906.1	6461.4	8640.2	1965.8	2992.3	6576.2	52
54	8576.4	1929 4	2909.0	6465.2	8642.4	1967.0	2995.2	6580.0	54
56	8578.6	1930.7	2911.8	6469.0	8644.6	1968.3	2998.1	6583.9	56
58	8580.8	1931.9	2914.7	6472.8	8646.8	1969.5	3001.1	6587.7	58
60	8583.0	1933.2	2917.5	6476.6	8649 0	1970.8	3004 0	6591.6	60

′	98°				99°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	8649.0	1970.8	3004.0	6591.6	8714.3	2008.7	3092.9	6709.0	0
2	8651 2	1972.0	3006.9	6595.5	8716.4	2009.9	3095.9	6712.9	2
4	8653.3	1973.3	3009.8	6599.4	8718.6	2011.2	3098.9	6716.9	4
6	8655 5	1974.6	3012.8	6603.2	8720.7	2012.5	3101.9	6720.8	6
8	8657.7	1975.8	3015.7	6607.1	8722.9	2013.7	3104.9	6724.8	8
10	8659.9	1977.1	3018.6	6611.0	8725.1	2015.0	3107.9	6728.8	10
12	8662.1	1978.3	3021.6	6614.9	8727.2	2016.3	3111.0	6732.7	12
14	8664.3	1979.6	3024.5	6618.8	8729.4	2017.5	3114.0	6736.7	14
16	8666.4	1980.9	3027.5	6622.7	8731.5	2018.8	3117.0	6740 7	16
18	8668.6	1982.1	3030.4	6626.6	8733.7	2020.1	3120.0	6744.6	18
20	8670.8	1983.4	3033.3	6630.5	8735.9	2021.4	3123.1	6748 6	20
22	8673.0	1984.6	3036.3	6634.4	8738.0	2022.6	3126 1	6752 6	22
24	8675.2	1985.9	3039.3	6638.3	8740.2	2023.9	3129.1	6756.6	24
26	8677.3	1987.2	3042.2	6642.2	8742.3	2025.2	3132.2	6760 6	26
28	8679.5	1988.4	3045.2	6646.1	8744.5	2026.4	3135.2	6764.6	28
30	8681.7	1989.7	3048.1	6650.0	8746.6	2027.7	3138.3	6768.6	30
32	8683.9	1991.0	3051 1	6653.9	8748.8	2029.0	3141.3	6772.6	32
34	8686.0	1992.2	3054.1	6657.8	8750.9	2030.3	3144.4	6776.6	34
36	8688.2	1993.5	3057.0	6661.7	8753.1	2031.5	3147.4	6780 6	36
38	8690.4	1994.7	3060.0	6665.7	8755.3	2032.8	3150.5	6784.6	38
40	8692.6	1996.0	3063.0	6669.6	8757.4	2034.1	3153.5	6788.6	40
42	8694.7	1997.3	3066.0	6673.5	8759.5	2035.4	3156.6	6792.6	42
44	8696.9	1998.5	3068.9	6677.4	8761.7	2036.6	3159.7	6796 6	44
46	8699.1	1999.8	3071.9	6681.4	8763.8	2037.9	3162.7	6800 6	46
48	8701.2	2001.1	3074.9	6685.3	8766.0	2039.2	3165.8	6804.6	48
50	8703.4	2002.3	3077.9	6689.2	8768.1	2040.5	3168.9	6808 6	50
52	8705.6	2003.6	3080.9	6693.2	8770.3	2041.7	3172 0	6812 6	52
54	8707.8	2004.9	3083.9	6697.1	8772.4	2043.0	3175 1	6816.7	54
56	8709.9	2006.1	3086.9	6701.1	8774.5	2044.3	3178.1	6820 7	56
58	8712.1	2007.4	3089.9	6705.2	8776.7	2045.6	3181.2	6824.7	58
60	8714.3	2008.7	3092.9	6709.0	8778.9	2046.8	3184.3	6828.8	60

′	100°				101°				′
	L C.	M.	E.	T.	L. C.	M.	E.	T.	
0	8778.9	2046.8	3184.3	6828.8	8842.8	2085 3	3278.8	6951.0	0
2	8781.0	2048.1	3187.4	6832.8	8844.9	2086 6	3281 5	6955.2	2
4	8783.1	2049.4	3190.5	6836.8	8847.0	2087.8	3284.7	6959 3	4
6	8785.3	2050.7	3193.6	6840.9	8849.2	2089.1	3287.9	6963.4	6
8	8787.4	2051.9	3196.7	6844.9	8851.3	2090.4	3291.1	6967.6	8
10	8789.6	2053 2	3199.8	6849.0	8853.4	2091.7	3294.3	6971.7	10
12	8791.7	2054.5	3202.9	6853.0	8855.5	2093.0	3297.5	6975.8	12
14	8793.9	2055.8	3206.0	6857.1	8857.6	2094.3	3300.7	6980.0	14
16	8796.0	2057.1	3209.1	6861.1	8859.8	2095.6	3303.9	6984.1	16
18	8798.9	2058.3	3212.2	6865.2	8861.9	2096.9	3307.1	6988.2	18
20	8800.3	2059.6	3215.4	6869.2	8864.0	2098.2	3310.3	6992.4	20
22	8802.4	2060.9	3218.5	6873.3	8866.1	2099.4	3313 5	6996.6	22
24	8804.5	2062.2	3221.6	6877.4	8868.2	2100.7	3316 7	7000.7	24
26	8806.7	2063.5	3224.7	6881.4	8870.3	2102.0	3319.9	7004.9	26
28	8808.8	2064.7	3227.9	6885.5	8872.4	2103.3	3323.1	7009 0	28
30	8810 9	2066.0	3231 0	6889.6	8874.5	2104.6	3326.4	7013.2	30
32	8813.1	2067.3	3234 1	6893.7	8876.7	2105.9	3329.6	7017.3	32
34	8815.2	2068.6	3237 3	6897.8	8878.8	2107.2	3332.8	7021.5	34
36	8817.3	2069 9	3240 4	6901.8	8880.9	2108.5	3336 0	7025.7	36
38	8819.5	2071.1	3243.5	6905.9	8883.0	2109.8	3339.3	7029.9	38
40	8821.6	2072.4	3246.7	6910.0	8885.1	2111.1	3342.5	7034.0	40
42	8823.7	2073.7	3249.8	6914.1	8887.2	2112.4	3345.8	7038.2	42
44	8825.8	2075.0	3253 C	6918.2	8889.3	2113.6	3349.0	7042.4	44
46	8828.0	2076.3	3256.2	6922.3	8891 4	2114 9	3352.3	7046.6	46
48	8830.1	2077.6	3259.3	6926.4	8893.5	2116.2	3355.5	7050.8	48
50	8832.2	2078.9	3262.5	6930.5	8895.6	2117.5	3358.8	7055.0	50
52	8834.3	2080.1	3265.7	6934.6	8897.7	2118.8	3362 0	7059.2	52
54	8836.4	2081.4	3268.8	6938.7	8899.8	2120.1	3365 5	7063 4	54
56	8838 6	2082.7	3272.0	6942.8	8901.9	2121.4	3368.7	7067.6	56
58	8840.7	2084.0	3275.2	6946.9	8904.0	2122.7	3372.0	7071.8	58
60	8842 8	2085.3	3278.3	6951.0	8906.1	2124.0	3375.1	7076.0	60

′	102°				103°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	8906.1	2124.0	3375.1	7076.0	8968.7	2163.0	3474.6	7203.6	0
2	8908.2	2125.3	3378.3	7080.2	8970.8	2164.3	3478.0	7207.9	2
4	8910.3	2126.6	3381.6	7084.4	8972.9	2165.6	3481.4	7212.2	4
6	8912.4	2127.9	3384.9	7088 6	8974.9	2166.9	3484.7	7216 5	6
8	8914.5	2129.2	3388.2	7092.8	8977.0	2168.2	3488.1	7220.8	8
10	8916.6	2130.5	3391.5	7097.1	8979.1	2169.5	3491.5	7225.1	10
12	8918.7	2131.8	3394 7	7101.3	8981.1	2170.8	3494.9	7229.5	12
14	8920.8	2133.1	3398.0	7105.5	8983.2	2172.1	3498.3	7233.8	14
16	8922.9	2134.4	3401.3	7109.7	8985.3	2173.4	3501.6	7238.1	16
18	8925.0	2135.7	3404.6	7114.0	8987.3	2174.7	3505.3	7242.4	18
20	8927.0	2137.0	3407.9	7118 2	8989 4	2176.1	3508.4	7246.8	20
22	8929.1	2138.3	3411.2	7122.4	8991.5	2177.4	3511.8	7251.1	22
24	8931.2	2139.6	3414.5	7126.7	8993.5	2178.7	3515.2	7255.4	24
26	8933.3	2140.9	3417.9	7130.9	8995.6	2180.0	3518.7	7259.8	26
28	8935 4	2142.2	3421.2	7135 2	8997.7	2181.3	3522.1	7264.1	28
30	8937.5	2143.5	3424.5	7139.4	8999.7	2182.6	3525.5	7268.5	30
32	8939.6	2144 8	3427.8	7143.7	9001.8	2183.9	3528.9	7272.8	32
34	8941.6	2146.1	3431.1	7148.0	9003.9	2185.2	3532.3	7277.2	34
36	8943.7	2147.4	3434.5	7152.2	9005.9	2186.5	3535.7	7281.5	36
38	8945.8	2148.7	3437.8	7156.5	9008.0	2187.8	3539.2	7285.9	38
40	8947.9	2150.0	3441.1	7160.7	9010.0	2189.1	3542.6	7290.3	40
42	8950.0	2151.3	3444.4	7165.0	9012.1	2190.5	3546.0	7294.6	42
44	8952.1	2152.6	3447.8	7169.3	9014.2	2191.8	3549.5	7299.0	44
46	8954.1	2153.9	3451.1	7173.6	9016.2	2193.1	3552.9	7303.4	46
48	8956.2	2155.2	3454.5	7177.9	9018.3	2194.4	3556.3	7307.7	48
50	8958.3	2156.5	3457.8	7182.1	9020.3	2195 7	3559.8	7312.1	50
52	8960.4	2157.8	3461.2	7186.4	9022.4	2197.0	3563.2	7316.5	52
54	8962.5	2159.1	3464.5	7190.7	9024.5	2198.3	3566.7	7320.9	54
56	8964.5	2160.4	3467.9	7195.0	9026 5	2199.6	3570.2	7325.3	56
58	8966.6	2161.7	3471.2	7199.3	9028.6	2200.9	3573.6	7329.7	58
60	8968.7	2163.0	3474.6	7203.6	9030.6	2202.3	3577.1	7334.1	60

′	104°				105°				′
	L. C.	M.	E.	T.	L. C.	M.	E.	T.	
0	9030.6	2202.3	3577.1	7334.1	9091.8	2241.8	3682.6	7467.5	0
2	9032.7	2203.6	3580.5	7338.5	9093.9	2243.1	3686.1	7472.0	2
4	9034.7	2204.9	3584.0	7342.9	9095.9	2244.4	3689.7	7476.5	4
6	9036.8	2206.2	3587.5	7347.3	9097.9	2245.8	3693.3	7481.0	6
8	9038.8	2207.5	3591.0	7351.7	9099.9	2247.1	3696.9	7485.5	8
10	9040.9	2208 8	3594.4	7356.1	9102.0	2248.4	3700.4	7490.0	10
12	9042.9	2210.2	3597.9	7360.5	9104.0	2249.7	3704.0	7494.5	12
14	9045.0	2211.5	3601.4	7364.9	9106.0	2251.1	3707.6	7499.1	14
16	9047.0	2212.8	3604.9	7369.4	9108.0	2252.4	3711.2	7503.6	16
18	9049.1	2214.1	3608.4	7373.8	9110.1	2253.7	3714.8	7508.1	18
20	9051.1	2215.4	3611.9	7378.2	9112.1	2255.0	3718.4	7512.6	20
22	9053.1	2216.7	3615.4	7382.6	9114.1	2256.4	3722.0	7517.2	22
24	9055.2	2218.0	3618.9	7387.1	9116.1	2257.7	3725.6	7521.7	24
26	9057.2	2219.4	3622.4	7391.5	9118.1	2259.0	3729.3	7526.3	26
28	9059.3	2220.7	3625.9	7396.0	9120.2	2260.3	3732.9	7530.8	28
30	9061.3	2222.0	3629.4	7400.4	9122.2	2261.7	3736.5	7535.3	30
32	9063.3	2223.3	3633.0	7404.8	9124.2	2263.0	3740.1	7539 9	32
34	9065.4	2224.6	3636.5	7409.3	9126.2	2264.3	3743.7	7544.4	34
36	9067.4	2226 0	3640.0	7413 8	9128 2	2265.7	3747.4	7549.0	36
38	9069.5	2227.3	3643.5	7418.2	9130.2	2267.0	3751.0	7553.6	38
40	9071.5	2228.6	3647.1	7422.7	9132.3	2268.3	3754.6	7558.1	40
42	9073.5	2229.9	3650.6	7427.1	9134.3	2269.6	3758.3	7562.7	42
44	9075.6	2231 2	3654.1	7431.6	9136.3	2271.0	3761.9	7567.3	44
46	9077.6	2232.6	3657.7	7436.1	9138.3	2272.3	3765.6	7571.8	46
48	9079.6	2233.9	3661.2	7440.6	9140.3	2273 6	3769.2	7576.4	48
50	9081.7	2235.2	3664.8	7445.0	9142.3	2275.0	3772.9	7581.0	50
52	9083.7	2236.5	3668.3	7449.5	9144.3	2276.3	3776.5	7585.6	52
54	9085.7	2237.8	3671.9	7454.0	9146.3	2277.6	3780.2	7590.2	54
56	9087.8	2239.2	3675.4	7458 5	9148.3	2278.9	3783.9	7594.8	56
58	9089.8	2240.5	3679.0	7463.0	9150.4	2280.3	3787.5	7599.4	58
60	9091.8	2241.8	3682.6	7467 5	9152.4	2281.6	3791.2	7604 0	60

N	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
1	0432	0475	0518	0561	0604	0647	0689	0732	0775	0817
2	0860	0903	0945	0988	1030	1072	1115	1157	1199	1242
3	1284	1326	1368	1410	1452	1494	1536	1578	1620	1662
4	1703	1745	1787	1828	1870	1912	1953	1995	2036	2078
5	2119	2160	2202	2243	2284	2325	2366	2407	2449	2490
6	2531	2572	2612	2653	2694	2735	2776	2816	2857	2898
7	2938	2979	3019	3060	3100	3141	3181	3222	3262	3302
8	3342	3383	3423	3463	3503	3543	3583	3623	3663	3703
9	3743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
1	4532	4571	4610	4650	4689	4727	4766	4805	4844	4883
2	4922	4961	4999	5038	5077	5115	5154	5192	5231	5269
3	5308	5346	5385	5423	5461	5500	5538	5576	5614	5652
4	5690	5729	5767	5805	5843	5881	5918	5956	5994	6032
5	6070	6108	6145	6183	6221	6258	6296	6333	6371	6408
6	6446	6483	6521	6558	6595	6633	6670	6707	6744	6781
7	6819	6856	6893	6930	6967	7004	7041	7078	7115	7151
8	7188	7225	7262	7298	7335	7372	7408	7445	7482	7518
9	7555	7591	7628	7664	7700	7737	7773	7809	7846	7882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
1	8279	8314	8350	8386	8422	8458	8493	8529	8565	8600
2	8636	8672	8707	8743	8778	8814	8849	8884	8920	8955
3	8991	9026	9061	9096	9132	9167	9202	9237	9272	9307
4	9342	9377	9412	9447	9482	9517	9552	9587	9621	9656
5	9691	9726	9760	9795	9830	9864	9899	9934	9968	10003
6	10037	10072	10106	10140	10175	10209	10243	10278	10312	0346
7	0380	0415	0449	0483	0517	0551	0585	0619	0653	0687
8	0721	0755	0789	0823	0857	0890	0924	0958	0992	1025
9	1059	1093	1126	1160	1193	1227	1261	1294	1327	1361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
1	1727	1760	1793	1826	1860	1893	1926	1959	1992	2024
2	2057	2090	2123	2156	2189	2222	2254	2287	2320	2352
3	2385	2418	2450	2483	2516	2548	2581	2613	2646	2678
4	2710	2743	2775	2808	2840	2872	2905	2937	2969	3001
5	3033	3066	3098	3130	3162	3194	3226	3258	3290	3322
6	3354	3386	3418	3450	3481	3513	3545	3577	3609	3640
7	3672	3704	3735	3767	3799	3830	3862	3893	3925	3956
8	3988	4019	4051	4082	4114	4145	4176	4208	4239	4270
9	4301	4333	4364	4395	4426	4457	4489	4520	4551	4582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
1	4922	4953	4983	5014	5045	5076	5106	5137	5168	5198
2	5229	5259	5290	5320	5351	5381	5412	5442	5473	5503
3	5534	5564	5594	5625	5655	5685	5715	5746	5776	5806
4	5836	5866	5897	5927	5957	5987	6017	6047	6077	6107
5	6137	6167	6197	6227	6256	6286	6316	6346	6376	6406
6	6435	6465	6495	6524	6554	6584	6613	6643	6673	6702
7	6732	6761	6791	6820	6850	6879	6909	6938	6967	6997
8	7026	7056	7085	7114	7143	7173	7202	7231	7260	7289
9	7319	7348	7377	7406	7435	7464	7493	7522	7551	7580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869

N	0	1	2	3	4	5	6	7	8	9
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
1	7898	7926	7955	7984	8013	8041	8070	8099	8127	8156
2	8184	8213	8241	8270	8298	8327	8355	8384	8412	8441
3	8469	8498	8526	8554	8583	8611	8639	8667	8696	8724
4	8752	8780	8808	8837	8865	8893	8921	8949	8977	9005
5	9033	9061	9089	9117	9145	9173	9201	9229	9257	9285
6	9312	9340	9368	9396	9424	9451	9479	9507	9535	9562
7	9590	9618	9645	9673	9700	9728	9756	9783	9811	9838
8	9866	9893	9921	9948	9976	20003	20030	20058	20085	20112
9	20140	20167	20194	20222	20249	0276	0303	0330	0358	0385
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
1	0683	0710	0737	0763	0790	0817	0844	0871	0898	0925
2	0952	0978	1005	1032	1059	1085	1112	1139	1165	1192
3	1219	1245	1272	1299	1325	1352	1378	1405	1431	1458
4	1484	1511	1537	1564	1590	1617	1643	1669	1696	1722
5	1748	1775	1801	1827	1854	1880	1906	1932	1958	1985
6	2011	2037	2063	2089	2115	2141	2167	2194	2220	2246
7	2272	2298	2324	2350	2376	2401	2427	2453	2479	2505
8	2531	2557	2583	2608	2634	2660	2686	2712	2737	2763
9	2789	2814	2840	2866	2891	2917	2943	2968	2994	3019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
1	3300	3325	3350	3376	3401	3426	3452	3477	3502	3528
2	3553	3578	3603	3629	3654	3679	3704	3729	3754	3779
3	3805	3830	3855	3880	3905	3930	3955	3980	4005	4030
4	4055	4080	4105	4130	4155	4180	4204	4229	4254	4279
5	4304	4329	4353	4378	4403	4428	4452	4477	4502	4527
6	4551	4576	4601	4625	4650	4674	4699	4724	4748	4773
7	4797	4822	4846	4871	4895	4920	4944	4969	4993	5018
8	5042	5066	5091	5115	5139	5164	5188	5212	5237	5261
9	5285	5310	5334	5358	5382	5406	5431	5455	5479	5503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
1	5768	5792	5816	5840	5864	5888	5912	5935	5959	5983
2	6007	6031	6055	6079	6102	6126	6150	6174	6198	6221
3	6245	6269	6293	6316	6340	6364	6387	6411	6435	6458
4	6482	6505	6529	6553	6576	6600	6623	6647	6670	6694
5	6717	6741	6764	6788	6811	6834	6858	6881	6905	6928
6	6951	6975	6998	7021	7045	7068	7091	7114	7138	7161
7	7184	7207	7231	7254	7277	7300	7323	7346	7370	7393
8	7416	7439	7462	7485	7508	7531	7554	7577	7600	7623
9	7646	7669	7692	7715	7738	7761	7784	7807	7830	7852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
1	8103	8126	8149	8171	8194	8217	8240	8262	8285	8307
2	8330	8353	8375	8398	8421	8443	8466	8488	8511	8533
3	8556	8578	8601	8623	8646	8668	8691	8713	8735	8758
4	8780	8803	8825	8847	8870	8892	8914	8937	8959	8981
5	9003	9026	9048	9070	9092	9115	9137	9159	9181	9203
6	9226	9248	9270	9292	9314	9336	9358	9380	9403	9425
7	9447	9469	9491	9513	9535	9557	9579	9601	9623	9645
8	9667	9688	9710	9732	9754	9776	9798	9820	9842	9863
9	9885	9907	9929	9951	9973	9994	30016	30038	30060	30081
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298

N	0	1	2	3	4	5	6	7	8	9
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298
1	0320	0341	0363	0384	0406	0428	0449	0471	0492	0514
2	0535	0557	0578	0600	0621	0643	0664	0685	0707	0728
3	0750	0771	0792	0814	0835	0856	0878	0899	0920	0942
4	0963	0984	1006	1027	1048	1069	1091	1112	1133	1154
5	1175	1197	1218	1239	1260	1281	1302	1323	1345	1366
6	1387	1408	1429	1450	1471	1492	1513	1534	1555	1576
7	1597	1618	1639	1660	1681	1702	1723	1744	1765	1785
8	1806	1827	1848	1869	1890	1911	1931	1952	1973	1994
9	2015	2035	2056	2077	2098	2118	2139	2160	2181	2201
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408
1	2428	2449	2469	2490	2510	2531	2552	2572	2593	2613
2	2634	2654	2675	2695	2715	2736	2756	2777	2797	2818
3	2838	2858	2879	2899	2919	2940	2960	2980	3001	3021
4	3041	3062	3082	3102	3122	3143	3163	3183	3203	3224
5	3244	3264	3284	3304	3325	3345	3365	3385	3405	3425
6	3445	3465	3486	3506	3526	3546	3566	3586	3606	3626
7	3646	3666	3686	3706	3726	3746	3766	3786	3806	3826
8	3846	3866	3885	3905	3925	3945	3965	3985	4005	4025
9	4044	4064	4084	4104	4124	4143	4163	4183	4203	4223
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420
1	4439	4459	4479	4498	4518	4537	4557	4577	4596	4616
2	4635	4655	4674	4694	4713	4733	4753	4772	4792	4811
3	4830	4850	4869	4889	4908	4928	4947	4967	4986	5005
4	5025	5044	5064	5083	5102	5122	5141	5160	5180	5199
5	5218	5238	5257	5276	5295	5315	5334	5353	5372	5392
6	5411	5430	5449	5468	5488	5507	5526	5545	5564	5583
7	5603	5622	5641	5660	5679	5698	5717	5736	5755	5774
8	5793	5813	5832	5851	5870	5889	5908	5927	5946	5965
9	5984	6003	6021	6040	6059	6078	6097	6116	6135	6154
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342
1	6361	6380	6399	6418	6436	6455	6474	6493	6511	6530
2	6549	6568	6586	6605	6624	6642	6661	6680	6698	6717
3	6736	6754	6773	6791	6810	6829	6847	6866	6884	6903
4	6922	6940	6959	6977	6996	7014	7033	7051	7070	7088
5	7107	7125	7144	7162	7181	7199	7218	7236	7254	7273
6	7291	7310	7328	7346	7365	7383	7401	7420	7438	7457
7	7475	7493	7511	7530	7548	7566	7585	7603	7621	7639
8	7658	7676	7694	7712	7731	7749	7767	7785	7803	7822
9	7840	7858	7876	7894	7912	7931	7949	7967	7985	8003
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184
1	8202	8220	8238	8256	8274	8292	8310	8328	8346	8364
2	8382	8399	8417	8435	8453	8471	8489	8507	8525	8543
3	8561	8578	8596	8614	8632	8650	8668	8686	8703	8721
4	8739	8757	8775	8792	8810	8828	8846	8863	8881	8899
5	8917	8934	8952	8970	8987	9005	9023	9041	9058	9076
6	9094	9111	9129	9146	9164	9182	9199	9217	9235	9252
7	9270	9287	9305	9322	9340	9358	9375	9393	9410	9428
8	9445	9463	9480	9498	9515	9533	9550	9568	9585	9602
9	9620	9637	9655	9672	9690	9707	9724	9742	9759	9777
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6	8046	8050	8055	8059	8064	8068	8073	8078	8082	8087
7	8091	8096	8100	8105	8109	8114	8118	8123	8127	8132
8	8137	8141	8146	8150	8155	8159	8164	8168	8173	8177
9	8182	8186	8191	8195	8200	8204	8209	8214	8218	8223
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268
1	8272	8277	8281	8286	8290	8295	8299	8304	8308	8313
2	8318	8322	8327	8331	8336	8340	8345	8349	8354	8358
3	8363	8367	8372	8376	8381	8385	8390	8394	8399	8403
4	8408	8412	8417	8421	8426	8430	8435	8439	8444	8448
5	8453	8457	8462	8466	8471	8475	8480	8484	8489	8493
6	8498	8502	8507	8511	8516	8520	8525	8529	8534	8538
7	8543	8547	8552	8556	8561	8565	8570	8574	8579	8583
8	8588	8592	8597	8601	8605	8610	8614	8619	8623	8628
9	8632	8637	8641	8646	8650	8655	8659	8664	8668	8673
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717
1	8722	8726	8731	8735	8740	8744	8749	8753	8758	8762
2	8767	8771	8776	8780	8784	8789	8793	8798	8802	8807
3	8811	8816	8820	8825	8829	8834	8838	8843	8847	8851
4	8856	8860	8865	8869	8874	8878	8883	8887	8892	8896
5	8900	8905	8909	8914	8918	8923	8927	8932	8936	8941
6	8945	8949	8954	8958	8963	8967	8972	8976	8981	8985
7	8989	8994	8998	9003	9007	9012	9016	9021	9025	9029
8	9034	9038	9043	9047	9052	9056	9061	9065	9069	9074
9	9078	9083	9087	9092	9096	9100	9105	9109	9114	9118
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162
1	9167	9171	9176	9180	9185	9189	9193	9198	9202	9207
2	9211	9216	9220	9224	9229	9233	9238	9242	9247	9251
3	9255	9260	9264	9269	9273	9277	9282	9286	9291	9295
4	9300	9304	9308	9313	9317	9322	9326	9330	9335	9339
5	9344	9348	9352	9357	9361	9366	9370	9374	9379	9383
6	9388	9392	9396	9401	9405	9410	9414	9419	9423	9427
7	9432	9436	9441	9445	9449	9454	9458	9463	9467	9471
8	9476	9480	9484	9489	9493	9498	9502	9506	9511	9515
9	9520	9524	9528	9533	9537	9542	9546	9550	9555	9559
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603
1	9607	9612	9616	9621	9625	9629	9634	9638	9642	9647
2	9651	9656	9660	9664	9669	9673	9677	9682	9686	9691
3	9695	9699	9704	9708	9712	9717	9721	9726	9730	9734
4	9739	9743	9747	9752	9756	9760	9765	9769	9774	9778
5	9782	9787	9791	9795	9800	9804	9808	9813	9817	9822
6	9826	9830	9835	9839	9843	9848	9852	9856	9861	9865
7	9870	9874	9878	9883	9887	9891	9896	9900	9904	9909
8	9913	9917	9922	9926	9930	9935	9939	9944	9948	9952
9	9957	9961	9965	9970	9974	9978	9983	9987	9991	9996
000	00000	00004	00009	00013	00017	00022	00026	00030	00035	00039

′	0°		1°		2°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	−∞	10.00000	8.24186	9.99993	8.54282	9 99974	60
1	6.46373	00000	24903	99993	54642	99973	59
2	76476	00000	25609	99993	54999	99973	58
3	94085	00000	26304	99993	55354	99972	57
4	7.06579	00000	26988	99992	55705	99972	56
5	16270	00000	27661	99992	56054	69971	55
6	24188	00000	28324	99992	56400	99971	54
7	30882	00000	28977	99992	56743	99970	53
8	36682	00000	29621	99992	57084	99970	52
9	41797	00000	30255	99991	57421	99969	51
10	7.46373	10.00000	8.30879	9.99991	8.57757	9.99969	50
11	50512	00000	31495	99991	58089	99968	49
12	54291	00000	32103	99990	58419	99968	48
13	57767	00000	32702	99990	58747	99967	47
14	60985	00000	33292	99990	59072	99967	46
15	63982	00000	33875	99990	59395	99967	45
16	66784	00000	34450	99989	59715	99966	44
17	69417	9.99999	35018	99989	60033	99966	43
18	71900	99999	35578	99989	60349	99965	42
19	74248	99999	36131	99989	60662	99964	41
20	7.76475	9.99999	8.36678	9.99988	8.60973	9.99964	40
21	78594	99999	37217	99988	61282	99963	39
22	80615	99999	37750	99988	61589	99963	38
23	82545	99999	38276	99987	61894	99962	37
24	84393	99999	38796	99987	62196	99962	36
25	86166	99999	39310	99987	62497	99961	35
26	87870	99999	39818	99986	62795	99961	34
27	89509	99999	40320	99986	63091	99960	33
28	91088	99999	40816	99986	63385	99960	32
29	92612	99998	41307	99985	63678	99959	31
30	7.94084	9.99998	8.41792	9.99985	8.63968	9.99959	30
31	95508	99998	42272	99985	64256	99958	29
32	96887	99998	42746	99984	64543	99958	28
33	98223	99998	43216	99984	64827	99957	27
34	99520	99998	43680	99984	65110	99956	26
35	8.00779	99998	44139	99983	65391	99956	25
36	02002	99998	44594	99983	65670	99955	24
37	03192	99997	45044	99983	65947	99955	23
38	04350	99997	45489	99982	66223	99954	22
39	05478	99997	45930	99982	66497	99954	21
40	8.06578	9.99997	8.46366	9.99982	8.66769	9.99953	20
41	07650	99997	46799	99981	67039	99952	19
42	08696	99997	47226	99981	67308	99952	18
43	09718	99997	47650	99981	67575	99951	17
44	10717	99996	48069	99980	67841	99951	16
45	11693	99996	48485	99980	68104	99950	15
46	12647	99996	48896	99979	68367	99949	14
47	13581	99996	49304	99979	68627	99949	13
48	14495	99996	49708	99979	68886	99948	12
49	15391	99996	50108	99978	69144	99948	11
50	8.16268	9.99995	8.50504	9.99978	8.69400	9.99947	10
51	17128	99995	50897	99977	69654	99946	9
52	17971	99995	51287	99977	69907	99946	8
53	18798	99995	51673	99977	70159	99945	7
54	19610	99995	52055	99976	70409	99944	6
55	20407	99994	52434	99976	70658	99944	5
56	21189	99994	52810	99975	70905	99943	4
57	21958	99994	53183	99975	71151	99942	3
58	22713	99994	53552	99974	71395	99942	2
59	23456	99994	53919	99974	71638	99941	1
60	24186	99993	54282	99974	71880	99940	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		

′	3°		4°		5°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	8.71880	9.99940	8.84358	9.99894	8.94030	9.99834	60
1	72120	99940	84539	99893	94174	99833	59
2	72359	99939	84718	99892	94317	99832	58
3	72597	99938	84897	99891	94461	99831	57
4	72834	99938	85075	99891	94603	99830	56
5	73069	99937	85252	99890	94746	99829	55
6	73303	99936	85429	99889	94887	99828	54
7	73535	99936	85605	99888	95029	99827	53
8	73767	99935	85780	99887	95170	99825	52
9	73997	99934	85955	99886	95310	99824	51
10	8.74226	9.99934	8.86128	9.99885	8.95450	9.99823	50
11	74454	99933	86301	99884	95589	99822	49
12	74680	99932	86474	99883	95728	99821	48
13	74906	99932	86645	99882	95867	99820	47
14	75130	99931	86816	99881	96005	99819	46
15	75353	99930	86987	99880	96143	99817	45
16	75575	99929	87156	99879	96280	99816	44
17	75795	99929	87325	99879	96417	99815	43
18	76015	99928	87494	99878	96553	99814	42
19	76234	99927	87661	99877	96689	99813	41
20	8.76451	9.99926	8.87829	9.99876	8.96825	9.99812	40
21	76667	99926	87995	99875	96960	99810	39
22	76883	99925	88161	99874	97095	99809	38
23	77097	99924	88326	99873	97229	99808	37
24	77310	99923	88490	99872	97363	99807	36
25	77522	99923	88654	99871	97496	99806	35
26	77733	99922	88817	99870	97629	99804	34
27	77943	99921	88980	99869	97762	99803	33
28	78152	99920	89142	99868	97894	99802	32
29	78360	99920	89304	99867	98026	99801	31
30	8.78568	9.99919	8.89464	9.99866	8.98157	9.99800	30
31	78774	99918	89625	99865	98288	99798	29
32	78979	99917	89784	99864	98419	99797	28
33	79183	99917	89943	99863	98549	99796	27
34	79386	99916	90102	99862	98679	99795	26
35	79588	99915	90260	99861	98808	99793	25
36	79789	99914	90417	99860	98937	99792	24
37	79990	99913	90574	99859	99066	99791	23
38	80189	99913	90730	99858	99194	99790	22
39	80388	99912	90885	99857	99322	99788	21
40	8.80585	9.99911	8.91040	9.99856	8.99450	9.99787	20
41	80782	99910	91195	99855	99577	99786	19
42	80978	99909	91349	99854	99704	99785	18
43	81173	99909	91502	99853	99830	99783	17
44	81367	99908	91655	99852	99956	99782	16
45	81560	99907	91807	99851	9.00082	99781	15
46	81752	99906	91959	99850	00207	99780	14
47	81944	99905	92110	99848	00332	99778	13
48	82134	99904	92261	99847	00456	99777	12
49	82324	99904	92411	99846	00581	99776	11
50	8.82513	9.99903	8.92561	9.99845	9.00704	9.99775	10
51	82701	99902	92710	99844	00828	99773	9
52	82888	99901	92859	99843	00951	99772	8
53	83075	99900	93007	99842	01074	99771	7
54	83261	99899	93154	99841	01196	99769	6
55	83446	99898	93301	99840	01318	99768	5
56	83630	99898	93448	99839	01440	99767	4
57	83813	99897	93594	99838	01561	99765	3
58	83996	99896	93740	99837	01682	99764	2
59	84177	99895	93885	99836	01803	99763	1
60	84358	99894	94030	99834	01923	99761	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	86°		85°		84°		

′	6°		7°		8°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.01923	9.99761	9.08589	9.99675	9.14856	9.99575	60
1	02043	99760	08692	99674	14945	99574	59
2	02163	99759	08795	99672	14535	99572	58
3	02283	99757	08897	99670	14624	99570	57
4	02402	99756	08999	99669	14714	99568	56
5	02520	99755	09101	99667	14803	99566	55
6	02639	99753	09202	99666	14891	99565	54
7	02757	99752	09304	99664	14980	99563	53
8	02874	99751	09405	99663	15069	99561	52
9	02992	99749	09506	99661	15157	99559	51
10	9.03109	9.99748	9.09606	9.99659	9.15245	9.99557	50
11	03226	99747	09707	99658	15333	99556	49
12	03342	99745	09807	99656	15421	99554	48
13	03458	99744	09907	99655	15508	99552	47
14	03574	99742	10006	99653	15596	99550	46
15	03690	99741	10106	99651	15683	99548	45
16	03805	99740	10205	99650	15770	99546	44
17	03920	99738	10304	99648	15857	99545	43
18	04034	99737	10402	99647	15944	99543	42
19	04149	99736	10501	99645	16030	99541	41
20	9.04262	9.99734	9.10599	9.99643	9.16116	9.99539	40
21	04376	99733	10697	99642	16203	99537	39
22	04490	99731	10795	99640	16289	99535	38
23	04603	99730	10893	99638	16374	99533	37
24	04715	99728	10990	99637	16460	99532	36
25	04828	99727	11087	99635	16545	99530	35
26	04940	99726	11184	99633	16631	99528	34
27	05052	99724	11281	99632	16716	99526	33
28	05164	99723	11377	99630	16801	99524	32
29	05275	99721	11474	99629	16886	99522	31
30	9.05386	9.99720	9.11570	9.99627	9.16970	9.99520	30
31	05497	99718	11666	99625	17055	99518	29
32	05607	99717	11761	99624	17139	99517	28
33	05717	99716	11857	99622	17223	99515	27
34	05827	99714	11952	99620	17307	99513	26
35	05937	99713	12047	99618	17391	99511	25
36	06046	99711	12142	99617	17474	99509	24
37	06155	99710	12236	99615	17558	99507	23
38	06264	99708	12331	99613	17641	99505	22
39	06372	99707	12425	99612	17724	99503	21
40	9.06481	9.99705	9.12519	9.99610	9.17807	9.99501	20
41	06589	99704	12612	99608	17890	99499	19
42	06696	99702	12706	99607	17973	99497	18
43	06804	99701	12799	99605	18055	99495	17
44	06911	99699	12892	99603	18137	99494	16
45	07018	99698	12985	99601	18220	99492	15
46	07124	99696	13078	99600	18302	99490	14
47	07231	99695	13171	99598	18383	99488	13
48	07337	99693	13263	99596	18465	99486	12
49	07442	99692	13355	99595	18547	99484	11
50	9.07548	9.99690	9.13447	9.99593	9.18628	9.99482	10
51	07653	99689	13539	99591	18709	99480	9
52	07758	99687	13630	99589	18790	99478	8
53	07863	99686	13722	99588	18871	99476	7
54	07968	99684	13813	99586	18952	99474	6
55	08072	99683	13904	99584	19033	99472	5
56	08176	99681	13994	99582	19113	99470	4
57	08280	99680	14085	99581	19193	99468	3
58	08383	99678	14175	99579	19273	99466	2
59	08486	99677	14266	99577	19353	99464	1
60	08589	99675	14356	99575	19433	99462	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	83°		82°		81°		

′	9°		10°		11°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.19433	9.99482	9.23967	9.99335	9.28060	9.99195	60
1	19513	99460	24039	99333	28125	99192	59
2	19592	99458	24110	99331	28190	99190	58
3	19672	99456	24181	99328	28254	99187	57
4	19751	99454	24253	99326	28319	99185	56
5	19830	99452	24324	99324	28384	99182	55
6	19909	99450	24395	99322	28448	99180	54
7	19988	99448	24466	99319	28512	99177	53
8	20067	99446	24536	99317	28577	99175	52
9	20145	99444	24607	99315	28641	99172	51
10	9.20223	9.99442	9.24677	9.99313	9.28705	9.99170	50
11	20302	99440	24748	99310	28769	99167	49
12	20380	99438	24818	99308	28833	99165	48
13	20458	99436	24888	99306	28896	99162	47
14	20535	99434	24958	99304	28960	99160	46
15	20613	99432	25028	99301	29024	99157	45
16	20691	99429	25098	99299	29087	99155	44
17	20768	99427	25168	99297	29150	99152	43
18	20845	99425	25237	99294	29214	99150	42
19	20922	99423	25307	99292	29277	99147	41
20	9.20999	9.99421	9.25376	9.99290	9.29340	9.99145	40
21	21076	99419	25445	99288	29403	99142	39
22	21153	99417	25514	99285	29466	99140	38
23	21229	99415	25583	99283	29529	99137	37
24	21306	99413	25652	99281	29591	99135	36
25	21382	99411	25721	99278	29654	99132	35
26	21458	99409	25790	99276	29716	99130	34
27	21534	99407	25858	99274	29779	99127	33
28	21610	99404	25927	99271	29841	99124	32
29	21685	99402	25995	99269	29903	99122	31
30	9.21761	9.99400	9.26063	9.99267	9.29966	9.99119	30
31	21836	99398	26131	99264	30028	99117	29
32	21912	99396	26199	99262	30090	99114	28
33	21987	99394	26267	99260	30151	99112	27
34	22062	99392	26335	99257	30213	99109	26
35	22137	99390	26403	99255	30275	99106	25
36	22211	99388	26470	99252	30336	99104	24
37	22286	99385	26538	99250	30398	99001	23
38	22361	99383	26605	99248	30459	99099	22
39	22435	99381	26672	99245	30521	99096	21
40	9.22509	9.99379	9.26739	9.99243	9.30582	9.99093	20
41	22583	99377	26806	99241	30643	99091	19
42	22657	99375	26873	99238	30704	99088	18
43	22731	99372	26940	99236	30765	99086	17
44	22805	99370	27007	99233	30826	99083	16
45	22878	99368	27073	99231	30887	99080	15
46	22952	99366	27140	99229	30947	99078	14
47	23025	99364	27206	99226	31008	99075	13
48	23098	99362	27273	99224	31068	99072	12
49	23171	99359	27339	99221	31129	99070	11
50	9.23244	9.99357	9.27405	9.99219	9.31189	9.99067	10
51	23317	99355	27471	99217	31250	99064	9
52	23390	99353	27537	99214	31310	99062	8
53	23462	99351	27602	99212	31370	99059	7
54	23535	99348	27668	99209	31430	99056	6
55	23607	99346	27734	99207	31490	99054	5
56	23679	99344	27799	99204	31549	99051	4
57	23752	99342	27864	99202	31609	99048	3
58	23823	99340	27930	99200	31669	99046	2
59	23895	99337	27995	99197	31728	99043	1
60	23967	99335	28060	99195	31788	99040	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	80°		79°		78°		

′	12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.31788	9.99040	9.35209	9.98872	9.38368	9.98690	60
1	31847	99038	35263	98869	38418	98687	59
2	31907	99035	35318	98867	38469	98684	58
3	31966	99032	35373	98864	38519	98681	57
4	32025	99030	35427	98861	38570	98678	56
5	32084	99027	35481	98858	38620	98675	55
6	32143	99024	35536	98855	38670	98671	54
7	32202	99022	35590	98852	38721	98668	53
8	32261	99019	35644	98849	38771	98665	52
9	32319	99016	35698	98846	38821	98662	51
10	9.32378	9.99013	9.35752	9.98843	9.38871	9.98659	50
11	32437	99011	35806	98840	38921	98656	49
12	32495	99008	35860	98837	38971	98652	48
13	32553	99005	35914	98834	39021	98649	47
14	32612	99002	35968	98831	39071	98646	46
15	32670	99000	36022	98828	39121	98643	45
16	32728	98997	36075	98825	39170	98640	44
17	32786	98994	36129	98822	39220	98636	43
18	32844	98991	36182	98819	39270	98633	42
19	32902	98989	36236	98816	39319	98630	41
20	9.32960	9.98986	9.36289	9.98813	9.39369	9.98627	40
21	33018	98983	36342	98810	39418	98623	39
22	33075	98980	36395	98807	39467	98620	38
23	33133	98978	36449	98804	39517	98617	37
24	33190	98975	36502	98801	39566	98614	36
25	33248	98972	36555	98798	39615	98610	35
26	33305	98969	36608	98795	39664	98607	34
27	33362	98967	36660	98792	39713	98604	33
28	33420	98964	36713	98789	39762	98601	32
29	33477	98961	36766	98785	39811	98597	31
30	9.33534	9.98958	9.36819	9.98783	9.39860	9.98594	30
31	33591	98955	36871	98780	39909	98591	29
32	33647	98953	36924	98777	39958	98588	28
33	33704	98950	36976	98774	40006	98584	27
34	33761	98947	37028	98771	40055	98581	26
35	33818	98944	37081	98768	40103	98578	25
36	33874	98941	37133	98765	40152	98574	24
37	33931	98938	37185	98762	40200	98571	23
38	33987	98936	37237	98759	40249	98568	22
39	34043	98933	37289	98756	40297	98565	21
40	9.34100	9.98930	9.37341	9.98753	9.40346	9.98561	20
41	34156	98927	37393	98750	40394	98558	19
42	34212	98924	37445	98746	40442	98555	18
43	34268	98921	37497	98743	40490	98551	17
44	34324	98919	37549	98740	40538	98548	16
45	34380	98916	37600	98737	40586	98545	15
46	34436	98913	37652	98734	40634	98541	14
47	34491	98910	37703	98731	40682	98538	13
48	34547	98907	37755	98728	40730	98535	12
49	34602	98904	37806	98725	40778	98531	11
50	9.34658	9.98901	9.37858	9.98722	9.40825	9.98528	10
51	34713	98898	37909	98719	40873	98525	9
52	34769	98896	37960	98715	40921	98521	8
53	34824	98893	38011	98712	40968	98518	7
54	34879	98890	38062	98709	41016	98515	6
55	34934	98887	38113	98706	41063	98511	5
56	34989	98884	38164	98703	41111	98508	4
57	35044	98881	38215	98700	41158	98505	3
58	35099	98878	38266	98697	41205	98501	2
59	35154	98875	38317	98694	41252	98498	1
60	35209	98872	38368	98690	41300	98494	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	77°		76°		75°		

′	15°		16°		17°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.41300	9.98494	9.44034	9.98284	9.46594	9.98060	60
1	41347	98491	44078	98281	46635	98056	59
2	41394	98488	44122	98277	46676	98052	58
3	41441	98484	44166	98273	46717	98048	57
4	41488	98481	44210	98270	46758	98044	56
5	41535	98477	44253	98266	46800	98040	55
6	41582	98474	44297	98262	46841	98036	54
7	41628	98471	44341	98259	46882	98032	53
8	41675	98467	44385	98255	46923	98029	52
9	41722	98464	44428	98251	46964	98025	51
10	9.41768	9.98460	9.44472	9.98248	9.47005	9.98021	50
11	41815	98457	44516	98244	47045	98017	49
12	41861	98453	44559	98240	47086	98013	48
13	41908	98450	44602	98237	47127	98009	47
14	41954	98447	44646	98233	47168	98005	46
15	42001	98443	44689	98229	47209	98001	45
16	42047	98440	44733	98226	47249	97997	44
17	42093	98436	44776	98222	47290	97993	43
18	42140	98433	44819	98218	47330	97989	42
19	42186	98429	44862	98215	47371	97986	41
20	9.42232	9.98426	9.44905	9.98211	9.47411	9.97982	40
21	42278	98422	44948	98207	47452	97978	39
22	42324	98419	44992	98204	47492	97974	38
23	42370	98415	45035	98200	47533	97970	37
24	42416	98412	45077	98196	47573	97966	36
25	42461	98409	45120	98192	47613	97962	35
26	42507	98405	45163	98189	47654	97958	34
27	42553	98402	45206	98185	47694	97954	33
28	42599	98398	45249	98181	47734	97950	32
29	42644	98395	45292	98177	47774	97946	31
30	9.42690	9.98391	9.45334	9.98174	9.47814	9.97942	30
31	42735	98388	45377	98170	47854	97938	29
32	42781	98384	45419	98166	47894	97934	28
33	42826	98381	45462	98162	47934	97930	27
34	42872	98377	45504	98159	47974	97926	26
35	42917	98373	45547	98155	48014	97922	25
36	42962	98370	45589	98151	48054	97918	24
37	43008	98366	45632	98147	48094	97914	23
38	43053	98363	45674	98144	48133	97910	22
39	43098	98359	45716	98140	48173	97906	21
40	9.43143	9.98356	9.45758	9.98136	9.48213	9.97902	20
41	43188	98352	45801	98132	48252	97898	19
42	43233	98349	45843	98129	48292	97894	18
43	43278	98345	45885	98125	48332	97890	17
44	43323	98342	45927	98121	48371	97886	16
45	43367	98338	45969	98117	48411	97882	15
46	43412	98334	46011	98113	48450	97878	14
47	43457	98331	46053	98110	48490	97874	13
48	43502	98327	46095	98106	48529	97870	12
49	43546	98324	46136	98102	48568	97866	11
50	9.43591	9.98320	9.46178	9.98098	9.48607	9.97861	10
51	43635	98317	46220	98094	48647	97857	9
52	43680	98313	46262	98090	48686	97853	8
53	43724	98309	46303	98087	48725	97849	7
54	43769	98306	46345	98083	48764	97845	6
55	43813	98302	46386	98079	48803	97841	5
56	43857	98299	46428	98075	48842	97837	4
57	43901	98295	46469	98071	48881	97833	3
58	43946	98291	46511	98067	48920	97829	2
59	43990	98288	46552	98063	48959	97825	1
60	44034	98284	46594	98060	48998	97821	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		

′	18°		19°		20°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.48998	9.97821	9.51264	9.97567	9.53405	9.97299	60
1	49037	97817	51301	97563	53440	97294	59
2	49076	97812	51338	97558	53475	97289	58
3	49115	97808	51374	97554	53509	97285	57
4	49153	97804	51411	97550	53544	97280	56
5	49192	97800	51447	97545	53578	97276	55
6	49231	97796	51484	97541	53613	97271	54
7	49269	97792	51520	97536	53647	97266	53
8	49308	97788	51557	97532	53682	97262	52
9	49347	97784	51593	97528	53716	97257	51
10	9.49385	9.97779	9.51629	9.97523	9.53751	9.97252	50
11	49424	97775	51666	97519	53785	97248	49
12	49462	97771	51702	97515	53819	97243	48
13	49500	97767	51738	97510	53854	97238	47
14	49539	97763	51774	97506	53888	97234	46
15	49577	97759	51811	97501	53922	97229	45
16	49615	97754	51847	97497	53957	97224	44
17	49654	97750	51883	97492	53991	97220	43
18	49692	97746	51919	97488	54025	97215	42
19	49730	97742	51955	97484	54059	97210	41
20	9.49768	9.97738	9.51991	9.97479	9.54093	9.97206	40
21	49806	97734	52027	97475	54127	97201	39
22	49844	97729	52063	97470	54161	97196	38
23	49882	97725	52099	97466	54195	97192	37
24	49920	97721	52135	97461	54229	97187	36
25	49958	97717	52171	97457	54263	97182	35
26	49996	97713	52207	97453	54297	97178	34
27	50034	97708	52242	97448	54331	97173	33
28	50072	97704	52278	97444	54365	97168	32
29	50110	97700	52314	97439	54399	97163	31
30	9.50148	9.97696	9.52350	9.97435	9.54433	9.97159	30
31	50185	97691	52385	97430	54466	97154	29
32	50223	97687	52421	97426	54500	97149	28
33	50261	97683	52456	97421	54534	97145	27
34	50298	97679	52492	97417	54567	97140	26
35	50336	97674	52527	97412	54601	97135	25
36	50374	97670	52563	97408	54635	97130	24
37	50411	97666	52598	97403	54668	97126	23
38	50449	97662	52634	97399	54702	97121	22
39	50486	97657	52669	97394	54735	97116	21
40	9.50523	9.97653	9.52705	9.97390	9.54769	9.97111	20
41	50561	97649	52740	97385	54802	97107	19
42	50598	97645	52775	97381	54836	97102	18
43	50635	97640	52811	97376	54869	97097	17
44	50673	97636	52846	97372	54903	97092	16
45	50710	97632	52881	97367	54936	97087	15
46	50747	97628	52916	97363	54969	97083	14
47	50784	97623	52951	97358	55003	97078	13
48	50821	97619	52986	97353	55036	97073	12
49	50858	97615	53021	97349	55069	97068	11
50	9.50896	9.97610	9.53056	9.97344	9.55102	9.97063	10
51	50933	97606	53092	97340	55136	97059	9
52	50970	97602	53126	97335	55169	97054	8
53	51007	97597	53161	97331	55202	97049	7
54	51043	97593	53196	97326	55235	97044	6
55	51080	97589	53231	97322	55268	97039	5
56	51117	97584	53266	97317	55301	97035	4
57	51154	97580	53301	97312	55334	97030	3
58	51191	97576	53336	97308	55367	97025	2
59	51227	97571	53370	97303	55400	97020	1
60	51264	97567	53405	97299	55433	97015	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	71°		70°		69°		

′	21°		22°		23°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.55433	9.97015	9.57358	9.96717	9.59188	9.96403	60
1	55466	97010	57389	96711	59218	96397	59
2	55499	97005	57420	96706	59247	96392	58
3	55532	97001	57451	96701	59277	96387	57
4	55564	96996	57482	96696	59307	96381	56
5	55597	96991	57514	96691	59336	96376	55
6	55630	96986	57545	96686	59366	96370	54
7	55663	96981	57576	96681	59396	96365	53
8	55695	96976	57607	96676	59425	96360	52
9	55728	96971	57638	96670	59455	96354	51
10	9.55761	9.96966	9.57669	9.96665	9.59484	9.96349	50
11	55793	96962	57700	96660	59514	96343	49
12	55826	96957	57731	96655	59543	96338	48
13	55858	96952	57762	96650	59573	96333	47
14	55891	96947	57793	96645	59602	96327	46
15	55923	96942	57824	96640	59632	96322	45
16	55956	96937	57855	96634	59661	96316	44
17	55988	96932	57885	96629	59690	96311	43
18	56021	96927	57916	96624	59720	96305	42
19	56053	96922	57947	96619	59749	96300	41
20	9.56085	9.96917	9.57978	9.96614	9.59778	9.96294	40
21	56118	96912	58008	96608	59808	96289	39
22	56150	96907	58039	96603	59837	96284	38
23	56182	96903	58070	96598	59866	96278	37
24	56215	96898	58101	96593	59895	96273	36
25	56247	96893	58131	96588	59924	96267	35
26	56279	96888	58162	96582	59954	96262	34
27	56311	96883	58192	96577	59983	96256	33
28	56343	96878	58223	96572	60012	96251	32
29	56375	96873	58253	96567	60041	96245	31
30	9.56408	9.96868	9.58284	9.96562	9.60070	9.96240	30
31	56440	96863	58314	96556	60099	96234	29
32	56472	96858	58345	96551	60128	96229	28
33	56504	96853	58375	96546	60157	96223	27
34	56536	96848	58406	96541	60186	96218	26
35	56568	96843	58436	96535	60215	96212	25
36	56599	96838	58467	96530	60244	96207	24
37	56631	96833	58497	96525	60273	96201	23
38	56663	96828	58527	96520	60302	96196	22
39	56695	96823	58557	96514	60331	96190	21
40	9.56727	9.96818	9.58588	9.96509	9.60359	9.96185	20
41	56759	96813	58618	96504	60388	96179	19
42	56790	96808	58648	96498	60417	96174	18
43	56822	96803	58678	96493	60446	96168	17
44	56854	96798	58709	96488	60474	96162	16
45	56886	96793	58739	96483	60503	96157	15
46	56917	96788	58769	96477	60532	96151	14
47	56949	96783	58799	96472	60561	96146	13
48	56980	96778	58829	96467	60589	96140	12
49	57012	96772	58859	96461	60618	96135	11
50	9.57044	9.96767	9.58889	9.96456	9.60646	9.96129	10
51	57075	96762	58919	96451	60675	96123	9
52	57107	96757	58949	96445	60704	96118	8
53	57138	96752	58979	96440	60732	96112	7
54	57169	96747	59009	96435	60761	96107	6
55	57201	96742	59039	96429	60789	96101	5
56	57232	96737	59069	96424	60818	96095	4
57	57264	96732	59098	96419	60846	96090	3
58	57295	96727	59128	96413	60875	96084	2
59	57326	96722	59158	96408	60903	96079	1
60	57358	96717	59188	96403	60931	96073	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	68°		67°		66°		

′	24°		25°		26°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.60931	9.96073	9.62595	9.95728	9.64184	9.95866	60
1	60960	96067	62622	95722	64210	95860	59
2	60988	96062	62649	95716	64236	95854	58
3	61016	96056	62676	95710	64262	95848	57
4	61045	96050	62703	95704	64288	95841	56
5	61073	96045	62730	95698	64313	95835	55
6	61101	96039	62757	95692	64339	95829	54
7	61129	96034	62784	95686	64365	95823	53
8	61158	96028	62811	95680	64391	95817	52
9	61186	96022	62838	95674	64417	95810	51
10	9.61214	9.96017	9.62865	9.95668	9.64442	9.95304	50
11	61242	96011	62892	95663	64468	95298	49
12	61270	96005	62918	95657	64494	95292	48
13	61298	96000	62945	95651	64519	95286	47
14	61326	95994	62972	95645	64545	95279	46
15	61354	95988	62999	95639	64571	95273	45
16	61382	95982	63026	95633	64596	95267	44
17	61411	95977	63052	95627	64622	95261	43
18	61438	95971	63079	95621	64647	95254	42
19	61466	95965	63106	95615	64673	95248	41
20	9.61494	9.95960	9.63133	9.95609	9.64698	9.95242	40
21	61522	95954	63159	95603	64724	95236	39
22	61550	95948	63186	95597	64749	95229	38
23	61578	95942	63213	95591	64775	95223	37
24	61606	95937	63239	95585	64800	95217	36
25	61634	95931	63266	95579	64826	95211	35
26	61662	95925	63292	95573	64851	95204	34
27	61689	95920	63319	95567	64877	95198	33
28	61717	95914	63345	95561	64902	95192	32
29	61745	95908	63372	95555	64927	95185	31
30	9.61773	9.95902	9.63398	9.95549	9.64953	9.95179	30
31	61800	95897	63425	95543	64978	95173	29
32	61828	95891	63451	95537	65003	95167	28
33	61856	95885	63478	95531	65029	95160	27
34	61883	95879	63504	95525	65054	95154	26
35	61911	95873	63531	95519	65079	95148	25
36	61939	95868	63557	95513	65104	95141	24
37	61966	95862	63583	95507	65130	95135	23
38	61994	95856	63610	95500	65155	95129	22
39	62021	95850	63636	95494	65180	95122	21
40	9.62049	9.95844	9.63662	9.95488	9.65205	9.95116	20
41	62076	95839	63689	95482	65230	95110	19
42	62104	95833	63715	95476	65255	95103	18
43	62131	95827	63741	95470	65281	95097	17
44	62159	95821	63767	95464	65306	95090	16
45	62186	95815	63794	95458	65331	95084	15
46	62214	95810	63820	95452	65356	95078	14
47	62241	95804	63846	95446	65381	95071	13
48	62268	95798	63872	95440	65406	95065	12
49	62296	95792	63898	95434	65431	95059	11
50	9.62323	9.95786	9.63924	9.95427	9.65456	9.95052	10
51	62350	95780	63950	95421	65481	95046	9
52	62377	95775	63976	95415	65506	95039	8
53	62405	95769	64002	95409	65531	95033	7
54	62432	95763	64028	95403	65555	95027	6
55	62459	95757	64054	95397	65580	95020	5
56	62486	95751	64080	95391	65605	95014	4
57	62513	95745	64106	95384	65630	95007	3
58	62541	95739	64132	95378	65655	95001	2
59	62568	95733	64158	95372	65680	94995	1
60	62595	95728	64184	95366	65705	94988	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	65°		64°		63°		

′	27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.65705	9.94988	9.67161	9.94593	9.68557	9.94182	60
1	65729	94982	67186	94587	68580	94175	59
2	65754	94975	67208	94580	68603	94168	58
3	65779	94969	67232	94573	68625	94161	57
4	65804	94962	67256	94567	68648	94154	56
5	65828	94956	67280	94560	68671	94147	55
6	65853	94949	67303	94553	68694	94140	54
7	65878	94943	67327	94546	68716	94133	53
8	65902	94936	67350	94540	68739	94126	52
9	65927	94930	67374	94533	68762	94119	51
10	9.65952	9.94923	9.67398	9.94526	9.68784	9.94112	50
11	65976	94917	67421	94519	68807	94105	49
12	66001	94911	67445	94513	68829	94098	48
13	66025	94904	67468	94506	68852	94090	47
14	66050	94898	67492	94499	68875	94083	46
15	66075	94891	67515	94492	68897	94076	45
16	66099	94885	67539	94485	68920	94069	44
17	66124	94878	67562	94479	68942	94062	43
18	66148	94871	67586	94472	68965	94055	42
19	66173	94865	67609	94465	68987	94048	41
20	9.66197	9.94858	9.67633	9.94458	9.69010	9.94041	40
21	66221	94852	67656	94451	69032	94034	39
22	66246	94845	67680	94445	69055	94027	38
23	66270	94839	67703	94438	69077	94020	37
24	66295	94832	67726	94431	69100	94012	36
25	66319	94826	67750	94424	69122	94005	35
26	66343	94819	67773	94417	69144	93998	34
27	66368	94813	67796	94410	69167	93991	33
28	66392	94806	67820	94404	69189	93984	32
29	66416	94799	67843	94397	69212	93977	31
30	9.66441	9.94793	9.67866	9.94390	9.69234	9.93970	30
31	66465	94786	67890	94383	69256	93963	29
32	66489	94780	67913	94376	69279	93955	28
33	66513	94773	67936	94369	69301	93948	27
34	66537	94767	67959	94362	69323	93941	26
35	66562	94760	67982	94355	69345	93934	25
36	66586	94753	68006	94349	69368	93927	24
37	66610	94747	68029	94342	69390	93920	23
38	66634	94740	68052	94335	69412	93912	22
39	66658	94734	68075	94328	69434	93905	21
40	9.66682	9.94727	9.68098	9.94321	9.69456	9.93898	20
41	66706	94720	68121	94314	69479	93891	19
42	66731	94714	68144	94307	69501	93884	18
43	66755	94707	68167	94300	69523	93876	17
44	66779	94700	68190	94293	69545	93869	16
45	66803	94694	68213	94286	69567	93862	15
46	66827	94687	68237	94279	69589	93855	14
47	66851	94680	68260	94273	69611	93847	13
48	66875	94674	68283	94266	69633	93840	12
49	66899	94667	68305	94259	69655	93833	11
50	9.66922	9.94660	9.68328	9.94252	9.69677	9.93826	10
51	66946	94654	68351	94245	69699	93819	9
52	66970	94647	68374	94238	69721	93811	8
53	66994	94640	68397	94231	69743	93804	7
54	67018	94634	68420	94224	69765	93797	6
55	67042	94627	68443	94217	69787	93789	5
56	67066	94620	68466	94210	69809	93782	4
57	67090	94614	68489	94203	69831	93775	3
58	67113	94607	68512	94196	69853	93768	2
59	67137	94600	68534	94189	69875	93760	1
60	67161	94593	68557	94182	69897	93753	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	62°		61°		60°		

′	30° Sine	30° Cosine	31° Sine	31° Cosine	32° Sine	32° Cosine	′
0	9.69897	9.93753	9.71184	9.93307	9.72421	9.92842	60
1	69919	93746	71205	93299	72441	92834	59
2	69941	93738	71226	93291	72461	92826	58
3	69963	93731	71247	93284	72482	92818	57
4	69984	93724	71268	93276	72502	92810	56
5	70006	93717	71289	93269	72522	92803	55
6	70028	93709	71310	93261	72542	92795	54
7	70050	93702	71331	93253	72562	92787	53
8	70072	93695	71352	93246	72582	92779	52
9	70093	93687	71373	93238	72602	92771	51
10	9.70115	9.93680	9.71393	9.93230	9.72622	9.92763	50
11	70137	93673	71414	93223	72643	92755	49
12	70159	93665	71435	93215	72663	92747	48
13	70180	93658	71456	93207	72683	92739	47
14	70202	93650	71477	93200	72703	92731	46
15	70224	93643	71498	93192	72723	92723	45
16	70245	93636	71519	93184	72743	92715	44
17	70267	93628	71539	93177	72763	92707	43
18	70288	93621	71560	93169	72783	92699	42
19	70310	93614	71581	93161	72803	92691	41
20	9.70332	9.93606	9.71602	9.93154	9.72823	9.92683	40
21	70353	93599	71622	93146	72843	92675	39
22	70375	93591	71643	93138	72863	92667	38
23	70396	93584	71664	93131	72883	92659	37
24	70418	93577	71685	93123	72902	92651	36
25	70439	93569	71705	93115	72922	92643	35
26	70461	93562	71726	93108	72942	92635	34
27	70482	93554	71747	93100	72962	92627	33
28	70504	93547	71767	93092	72982	92619	32
29	70525	93539	71788	93084	73002	92611	31
30	9.70547	9.93532	9.71809	9.93077	9.73022	9.92603	30
31	70568	93525	71829	93069	73041	92595	29
32	70590	93517	71850	93061	73061	92587	28
33	70611	93510	71870	93053	73081	92579	27
34	70633	93502	71891	93046	73101	92571	26
35	70654	93495	71911	93038	73121	92563	25
36	70675	93487	71932	93030	73140	92555	24
37	70697	93480	71952	93022	73160	92546	23
38	70718	93472	71973	93014	73180	92538	22
39	70739	93465	71994	93007	73200	92530	21
40	9.70761	9.93457	9.72014	9.92999	9.73219	9.92522	20
41	70782	93450	72034	92991	73239	92514	19
42	70803	93442	72055	92983	73259	92506	18
43	70824	93435	72075	92976	73278	92498	17
44	70846	93427	72096	92968	73298	92490	16
45	70867	93420	72116	92960	73318	92482	15
46	70888	93412	72137	92952	73337	92473	14
47	70909	93405	72157	92944	73357	92465	13
48	70931	93397	72177	92936	73377	92457	12
49	70952	93390	72198	92929	73396	92449	11
50	9.70973	9.93382	9.72218	9.92921	9.73416	9.92441	10
51	70994	93375	72238	92913	73435	92433	9
52	71015	93367	72259	92905	73455	92425	8
53	71036	93360	72279	92897	73474	92416	7
54	71058	93352	72299	92889	73494	92408	6
55	71079	93344	72320	92881	73513	92400	5
56	71100	93337	72340	92874	73533	92392	4
57	71121	93329	72360	92866	73552	92384	3
58	71142	93322	72381	92858	73572	92376	2
59	71163	93314	72401	92850	73591	92367	1
60	71184	93307	72421	92842	73611	92359	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		

′	33°		34°		35°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.73611	9.92359	9.74756	9.91857	9.75859	9.91336	60
1	73630	92351	74775	91849	75877	91328	59
2	73650	92343	74794	91840	75895	91319	58
3	73669	92335	74812	91832	75913	91310	57
4	73689	92326	74831	91823	75931	91301	56
5	73708	92318	74850	91815	75949	91292	55
6	73727	92310	74868	91806	75967	91283	54
7	73747	92302	74887	91798	75985	91274	53
8	73766	92293	74906	91789	76003	91266	52
9	73785	92285	74924	91781	76021	91257	51
10	9.73805	9.92277	9.74943	9.91772	9.76039	9.91248	50
11	73824	92269	74961	91763	76057	91239	49
12	73843	92260	74980	91755	76075	91230	48
13	73863	92252	74999	91746	76093	91221	47
14	73882	92244	75017	91738	76111	91212	46
15	73901	92235	75036	91729	76129	91203	45
16	73921	92227	75054	91720	76146	91194	44
17	73940	92219	75073	91712	76164	91185	43
18	73959	92211	75091	91703	76182	91176	42
19	73978	92202	75110	91695	76200	91167	41
20	9.73997	9.92194	9.75128	9.91686	9.76218	9.91158	40
21	74017	92186	75147	91677	76236	91149	39
22	74036	92177	75165	91669	76253	91141	38
23	74055	92169	75184	91660	76271	91132	37
24	74074	92161	75202	91651	76289	91123	36
25	74093	92152	75221	91643	76307	91114	35
26	74113	92144	75239	91634	76324	91105	34
27	74132	92136	75258	91625	76342	91096	33
28	74151	92127	75276	91617	76360	91087	32
29	74170	92119	75294	91608	76378	91078	31
30	9.74189	9.92111	9.75313	9.91599	9.76395	9.91069	30
31	74208	92102	75331	91591	76413	91060	29
32	74227	92094	75350	91582	76431	91051	28
33	74246	92086	75368	91573	76448	91042	27
34	74265	92077	75386	91565	76466	91033	26
35	74284	92069	75405	91556	76484	91023	25
36	74303	92060	75423	91547	76501	91014	24
37	74322	92052	75441	91538	76519	91005	23
38	74341	92044	75459	91530	76537	90996	22
39	74360	92035	75478	91521	76554	90987	21
40	9.74379	9.92027	9.75496	9.91512	9.76572	9.90978	20
41	74398	92018	75514	91504	76590	90969	19
42	74417	92010	75533	91495	76607	90960	18
43	74436	92002	75551	91486	76625	90951	17
44	74455	91993	75569	91477	76642	90942	16
45	74474	91985	75587	91469	76660	90933	15
46	74493	91976	75605	91460	76677	90924	14
47	74512	91968	75624	91451	76695	90915	13
48	74531	91959	75642	91442	76712	90906	12
49	74549	91951	75660	91433	76730	90896	11
50	9.74568	9.91942	9.75678	9.91425	9.76747	9.90887	10
51	74587	91934	75696	91416	76765	90878	9
52	74606	91925	75714	91407	76782	90869	8
53	74625	91917	75733	91398	76800	90860	7
54	74644	91908	75751	91389	76817	90851	6
55	74662	91900	75769	91381	76835	90842	5
56	74681	91891	75787	91372	76852	90832	4
57	74700	91883	75805	91363	76870	90823	3
58	74719	91874	75823	91354	76887	90814	2
59	74737	91866	75841	91345	76904	90805	1
60	74756	91857	75859	91336	76922	90796	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	56°		55°		54°		

′	36°		37°		38°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.76922	9.90796	9.77946	9.90235	9.78934	9.89653	60
1	76939	90787	77963	90225	78950	89643	59
2	76957	90777	77980	90216	78967	89633	58
3	76974	90768	77997	90206	78983	89624	57
4	76991	90759	78013	90197	78999	89614	56
5	77009	90750	78030	90187	79015	89604	55
6	77026	90741	78047	90178	79031	89594	54
7	77043	90731	78063	90168	79047	89584	53
8	77061	90722	78080	90159	79063	89574	52
9	77078	90713	78097	90149	79079	89564	51
10	9.77095	9.90704	9.78113	9.90139	9.79095	9.89554	50
11	77112	90694	78130	90130	79111	89544	49
12	77130	90685	78147	90120	79126	89534	48
13	77147	90676	78163	90111	79144	89524	47
14	77164	90667	78180	90101	79160	89514	46
15	77181	90657	78197	90091	79176	89504	45
16	77199	90648	78213	90082	79192	89495	44
17	77216	90639	78230	90072	79208	89485	43
18	77233	90630	78246	90063	79224	89475	42
19	77250	90620	78263	90053	79240	89465	41
20	9.77268	9.90611	9.78280	9.90043	9.79256	9.89455	40
21	77285	90602	78296	90034	79272	89445	39
22	77302	90592	78313	90024	79288	89435	38
23	77319	90583	78329	90014	79304	89425	37
24	77336	90574	78346	90005	79319	89415	36
25	77353	90565	78362	89995	79335	89405	35
26	77370	90555	78379	89985	79351	89395	34
27	77387	90546	78395	89976	79367	89385	33
28	77405	90537	78412	89966	79383	89375	32
29	77422	90527	78428	89956	79399	89364	31
30	9.77439	9.90518	9.78445	9.89947	9.79415	9.89354	30
31	77456	90509	78461	89937	79431	89344	29
32	77473	90499	78478	89927	79447	89334	28
33	77490	90490	78494	89918	79463	89324	27
34	77507	90480	78510	89908	79478	89314	26
35	77524	90471	78527	89898	79494	89304	25
36	77541	90462	78543	89888	79510	89294	24
37	77558	90452	78560	89879	79526	89284	23
38	77575	90443	78576	89869	79542	89274	22
39	77592	90434	78592	89859	79558	89264	21
40	9.77609	9.90424	9.78609	9.89849	9.79573	9.89254	20
41	77626	90415	78625	89840	79589	89244	19
42	77643	90405	78642	89830	79605	89233	18
43	77660	90396	78658	89820	78621	89223	17
44	77677	90386	78674	89810	79636	89213	16
45	77694	90377	78691	89801	79652	89203	15
46	77711	90368	78707	89791	79668	89193	14
47	77728	90358	78723	89781	79684	89183	13
48	77744	90349	78739	89771	79699	89173	12
49	77761	90339	78756	89761	79715	89162	11
50	9.77778	9.90330	9.78772	9.89752	9.79731	9.89152	10
51	77795	90320	78788	89742	79746	89142	9
52	77812	90311	78805	89732	79762	89132	8
53	77829	90301	78821	89722	79778	89122	7
54	77846	90292	78837	89712	79793	89112	6
55	77862	90282	78853	89702	79809	89101	5
56	77879	90273	78869	89693	79825	89091	4
57	77896	90263	78886	89683	79840	89081	3
58	77913	90254	78902	89673	79856	89071	2
59	77930	90244	78918	89663	79872	89060	1
60	77946	90235	78934	89653	79887	89050	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	53°		52°		51°		

′	39°		40°		41°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.79887	9.89050	9.80807	9.88425	9.81694	9.87778	60
1	79903	89040	80822	88415	81709	87767	59
2	79918	89030	80837	88404	81723	87756	58
3	79934	89020	80852	88394	81738	87745	57
4	79950	89009	80867	88383	81752	87734	56
5	79965	88999	80882	88372	81767	87723	55
6	79981	88989	80897	88362	81781	87712	54
7	79996	88978	80912	88351	81796	87701	53
8	80012	88968	80927	88340	81810	87690	52
9	80027	88958	80942	88330	81825	87679	51
10	9.80043	9.88948	9.80957	9.88319	9.81839	9.87668	50
11	80058	88937	80972	88308	81854	87657	49
12	80074	88927	80987	88298	81868	87646	48
13	80089	88917	81002	88287	81882	87635	47
14	80105	88906	81017	88276	81897	87624	46
15	80120	88896	81032	88266	81911	87613	45
16	80136	88886	81047	88255	81926	87601	44
17	80151	88875	81061	88244	81940	87590	43
18	80166	88865	81076	88234	81955	87579	42
19	80182	88855	81091	88223	81969	87568	41
20	9.80197	9.88844	9.81106	9.88212	9.81983	9.87557	40
21	80213	88834	81121	88201	81998	87546	39
22	80228	88824	81136	88191	82012	87535	38
23	80244	88813	81151	88180	82026	87524	37
24	80259	88803	81166	88169	82041	87513	36
25	80274	88793	81180	88158	82055	87501	35
26	80290	88782	81195	88148	82069	87490	34
27	80305	88772	81210	88137	82084	87479	33
28	80320	88761	81225	88126	82098	87468	32
29	80336	88751	81240	88115	82112	87457	31
30	9.80351	9.88741	9.81254	9.88105	9.82126	9.87446	30
31	80366	88730	81269	88094	82141	87434	29
32	80382	88720	81284	88083	82155	87423	28
33	80397	88709	81299	88072	82169	87412	27
34	80412	88699	81314	88061	82184	87401	26
35	80428	88688	81328	88051	82198	87390	25
36	80443	88678	81343	88040	82212	87378	24
37	80458	88668	81358	88029	82226	87367	23
38	80473	88657	81372	88018	82240	87356	22
39	80489	88647	81387	88007	82255	87345	21
40	9.80504	9.88636	9.81402	9.87996	9.82269	9.87334	20
41	80519	88626	81417	87985	82283	87322	19
42	80534	88615	81431	87975	82297	87311	18
43	80550	88605	81446	87964	82311	87300	17
44	80565	88594	81461	87953	82326	87288	16
45	80580	88584	81475	87942	82340	87277	15
46	80595	88573	81490	87931	82354	87266	14
47	80610	88563	81505	87920	82368	87255	13
48	80625	88552	81519	87909	82382	87243	12
49	80641	88542	81534	87898	82396	87232	11
50	9.80656	9.88531	9.81549	9.87887	9.82410	9.87221	10
51	80671	88521	81563	87877	82424	87209	9
52	80686	88510	81578	87866	82439	87198	8
53	80701	88499	81592	87855	82453	87187	7
54	80716	88489	81607	87844	82467	87175	6
55	80731	88478	81622	87833	82481	87164	5
56	80746	88468	81636	87822	82495	87153	4
57	80762	88457	81651	87811	82509	87141	3
58	80777	88447	81665	87800	82523	87130	2
59	80792	88436	81680	87789	82537	87119	1
60	80807	88425	81694	87778	82551	87107	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	50°		49°		48°		

′	42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.82551	9.87107	9.83378	9.86413	9.84177	9.85693	60
1	82565	87096	83392	86401	84190	85681	59
2	82579	87085	83405	86389	84203	85669	58
3	82593	87073	83419	86377	84216	85657	57
4	82607	87062	83432	86366	84229	85645	56
5	82621	87050	83446	86354	84242	85632	55
6	82635	87039	83459	86342	84255	85620	54
7	82649	87028	83473	86330	84269	85608	53
8	82663	87016	83486	86318	84282	85596	52
9	82677	87005	83500	86306	84295	85583	51
10	9.82691	9.86993	9.83513	9.86295	9.84308	9.85571	50
11	82705	86982	83527	86283	84321	85559	49
12	82719	86970	83540	86271	84334	85547	48
13	82733	86959	83554	86259	84347	85534	47
14	82747	86947	83567	86247	84360	85522	46
15	82761	86936	83581	86235	84373	85510	45
16	82775	86924	83594	86223	84385	85497	44
17	82788	86913	83608	86211	84398	85485	43
18	82802	86902	83621	86200	84411	85473	42
19	82816	86890	83634	86188	84424	85460	41
20	9.82830	9.86879	9.83648	9.86176	9.84437	9.85448	40
21	82844	86867	83661	86164	84450	85436	39
22	82858	86855	83674	86152	84463	85423	38
23	82872	86844	83688	86140	84476	85411	37
24	82885	86832	83701	86128	84489	85399	36
25	82899	86821	83715	86116	84502	85386	35
26	82913	86809	83728	86104	84515	85374	34
27	82927	86798	83741	86092	84528	85361	33
28	82941	86786	83755	86080	84540	85349	32
29	82955	86775	83768	86068	84553	85337	31
30	9.82968	9.86763	9.83781	9.86056	9.84566	9.85324	30
31	82982	86752	83795	86044	84579	85312	29
32	82996	86740	83808	86032	84592	85299	28
33	83010	86728	83821	86020	84605	85287	27
34	83023	86717	83834	86008	84618	85274	26
35	83037	86705	83848	85996	84630	85262	25
36	83051	86694	83861	85984	84643	85250	24
37	83065	86682	83874	85972	84656	85237	23
38	83078	86670	83887	85960	84669	85225	22
39	83092	86659	83901	85948	84682	85212	21
40	9.83106	9.86647	9.83914	9.85936	9.84694	9.85200	20
41	83120	86635	83927	85924	84707	85187	19
42	83133	86624	83940	85912	84720	85175	18
43	83147	86612	83954	85900	84733	85162	17
44	83161	86600	83967	85888	84745	85150	16
45	83174	86589	83980	85876	84758	85137	15
46	83188	86577	83993	85864	84771	85125	14
47	83202	86565	84006	85851	84784	85112	13
48	83215	86554	84020	85839	84796	85100	12
49	83229	86542	84033	85827	84809	85087	11
50	9.83242	9.86530	9.84046	9.85815	9.84822	9.85074	10
51	83256	86518	84059	85803	84835	85062	9
52	83270	86507	84072	85791	84847	85049	8
53	83283	86495	84085	85779	84860	85037	7
54	83297	86483	84098	85766	84873	85024	6
55	83310	86472	84112	85754	84885	85012	5
56	83324	86460	84125	85742	84898	84999	4
57	83338	86448	84138	85730	84911	84986	3
58	83351	86436	84151	85718	84923	84974	2
59	83365	86425	84164	85706	84936	84961	1
60	83378	86413	84177	85693	84949	84949	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	47°		46°		45°		

′	0°		1°		2°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	−∞	∞	8.24192	11.75808	8.54308	11.45692	60
1	6.46373	13.53627	24910	75090	54669	45331	59
2	76476	23524	25616	74384	55027	44973	58
3	94085	05915	26312	73688	55382	44618	57
4	7.06579	12.93421	26996	73004	55734	44266	56
5	16270	83730	27669	72331	56083	43917	55
6	24188	75812	28332	71668	56429	43571	54
7	30882	69118	28986	71014	56773	43227	53
8	36682	63318	29629	70371	57114	42886	52
9	41797	58203	30263	69737	57452	42548	51
10	7.46373	12.53627	8.30888	11.69112	8.57788	11.42212	50
11	50512	49488	31505	68495	58121	41879	49
12	54291	45709	32112	67888	58451	41549	48
13	57767	42233	32711	67289	58779	41221	47
14	60986	39014	33302	66698	59105	40895	46
15	63982	36018	33886	66114	59428	40572	45
16	66785	33215	34461	65539	59749	40251	44
17	69418	30582	35029	64971	60068	39932	43
18	71900	28100	35590	64410	60384	39616	42
19	74248	25752	36143	63857	60698	39302	41
20	7 76476	12.23524	8.36689	11.63311	8.61009	11.38991	40
21	78595	21405	37229	62771	61319	38681	39
22	80615	19385	37762	62238	61626	38374	38
23	82546	17454	38289	61711	61931	38069	37
24	84394	15606	38809	61191	62234	37766	36
25	86167	13833	39323	60677	62535	37465	35
26	87871	12129	39832	60168	62834	37166	34
27	89510	10490	40334	59666	63131	36869	33
28	91089	08911	40830	59170	63426	36574	32
29	92613	07387	41321	58679	63718	36282	31
30	7.94086	12.05914	8.41807	11.58193	8.64009	11.35991	30
31	95510	04490	42287	57713	64298	35702	29
32	96889	03111	42762	57238	64585	35415	28
33	98225	01775	43232	56768	64870	35130	27
34	99522	00478	43696	56304	65154	34846	26
35	8.00781	11.99219	44156	55844	65435	34565	25
36	02004	97996	44611	55389	65715	34285	24
37	03194	96806	45061	54939	65993	34007	23
38	04353	95647	45507	54493	66269	33731	22
39	05481	94519	45948	54052	66543	33457	21
40	8.06581	11.93419	8.46385	11.53615	8.66816	11.33184	20
41	07653	92347	46817	53183	67087	32913	19
42	08700	91300	47245	52755	67356	32644	18
43	09722	90278	47669	52331	67624	32376	17
44	10720	89280	48089	51911	67890	32110	16
45	11696	88304	48505	51495	68154	31846	15
46	12651	87349	48917	51083	68417	31583	14
47	13585	86415	49325	50675	68678	31322	13
48	14500	85500	49729	50271	68938	31062	12
49	15395	84605	50130	49870	69196	30804	11
50	8.16273	11.83727	8.50527	11.49473	8.69453	11.30547	10
51	17133	82867	50920	49080	69708	30292	9
52	17976	82024	51310	48690	69962	30038	8
53	18804	81196	51696	48304	70214	29786	7
54	19616	80384	52079	47921	70465	29535	6
55	20413	79587	52459	47541	70714	29286	5
56	21195	78805	52835	47165	70962	29038	4
57	21964	78036	53208	46792	71208	28792	3
58	22720	77280	53578	46422	71453	28547	2
59	23462	76538	53945	46055	71697	28303	1
60	24192	75808	54308	45692	71940	28060	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	89°		88°		87°		

′	3°		4°		5°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	8.71940	11.28060	8.84464	11.15536	8.94195	11.05805	60
1	72181	27819	84646	15354	94340	05660	59
2	72420	27580	84826	15174	94485	05515	58
3	72659	27341	85006	14994	94630	05370	57
4	72896	27104	85185	14815	94773	05227	56
5	73132	26868	85363	14637	94917	05083	55
6	73366	26634	85540	14460	95060	04940	54
7	73600	26400	85717	14283	95202	04798	53
8	73832	26168	85893	14107	95344	04656	52
9	74063	25937	86069	13931	95486	04514	51
10	8.74292	11.25708	8.86243	11.13757	8.95627	11.04373	50
11	74521	25479	86417	13583	95767	04233	49
12	74748	25252	86591	13409	95908	04092	48
13	74974	25026	86763	13237	96047	03953	47
14	75199	24801	86935	13065	96187	03813	46
15	75423	24577	87106	12894	96325	03675	45
16	75645	24355	87277	12723	96464	03536	44
17	75867	24133	87447	12553	96602	03398	43
18	76087	23913	87616	12384	96739	03261	42
19	76306	23694	87785	12215	96877	02123	41
20	8.76525	11.23475	8.87953	11.12047	8.97013	11.02987	40
21	76742	23258	88120	11880	97150	02850	39
22	76958	23042	88287	11713	97285	02715	38
23	77173	22827	88453	11547	97421	02579	37
24	77387	22613	88618	11382	97556	02444	36
25	77600	22400	88783	11217	97691	02309	35
26	77811	22189	88948	11052	97825	02175	34
27	78022	21978	89111	10889	97959	02041	33
28	78232	21768	89274	10726	98092	01908	32
29	78441	21559	89437	10563	98225	01775	31
30	8.78649	11.21351	8.89598	11.10402	8.98358	11.01642	30
31	78855	21145	89760	10240	98490	01510	29
32	79061	20939	89920	10080	98622	01378	28
33	79266	20734	90080	09920	98753	01247	27
34	79470	20530	90240	09760	98884	01116	26
35	79673	20327	90399	09601	99015	00985	25
36	79875	20125	90557	09443	99145	00855	24
37	80076	19924	90715	09285	99275	00725	23
38	80277	19723	90872	09128	99405	00595	22
39	80476	19524	91029	08971	99534	00466	21
40	8.80674	11.19326	8.91185	11.08815	8.99662	11.00338	20
41	80872	19128	91340	08660	99791	00209	19
42	81068	18932	91495	08505	99919	00081	18
43	81264	18736	91650	08350	9.00046	10.99954	17
44	81459	18541	91803	08197	00174	99826	16
45	81653	18347	91957	08043	00301	99699	15
46	81846	18154	92110	07890	00427	99573	14
47	82038	17962	92262	07738	00553	99447	13
48	82230	17770	92414	07586	00679	99321	12
49	82420	17580	92565	07435	00805	99195	11
50	8.82610	11.17390	8.92716	11.07284	9.00930	10.99070	10
51	82799	17201	92866	07134	01055	98945	9
52	82987	17013	93016	06984	01179	98821	8
53	83175	16825	93165	06835	01303	98697	7
54	83361	16639	93313	06687	01427	98573	6
55	83547	16453	93462	06538	01550	98450	5
56	83732	16268	93609	06391	01673	98327	4
57	83916	16084	93756	06244	01796	98204	3
58	84100	15900	93903	06097	01918	98082	2
59	84282	15718	94049	05951	02040	97960	1
60	84464	15536	94195	05805	02162	97838	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	83°		85°		84°		

′	6°		7°		8°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.02162	10.97838	9.08914	10.91086	9.14780	10.85220	60
1	02283	97717	09019	90981	14872	85128	59
2	02404	97596	09123	90877	14963	85037	58
3	02525	97475	09227	90773	15054	84946	57
4	02645	97355	09330	90670	15145	84855	56
5	02766	97234	09434	90566	15236	84764	55
6	02885	97115	09537	90463	15327	84673	54
7	03005	96995	09640	90360	15417	84583	53
8	03124	96876	09742	90258	15508	84492	52
9	03242	96758	09845	90155	15598	84402	51
10	9.03361	10.96639	9.09947	10.90053	9.15688	10.84312	50
11	03479	96521	10049	89951	15777	84223	49
12	03597	96403	10150	89850	15867	84133	48
13	03714	96286	10252	89748	15956	84044	47
14	03832	96168	10353	89647	16046	83954	46
15	03948	96052	10454	89546	16135	83865	45
16	04065	95935	10555	89445	16224	83776	44
17	04181	95819	10656	89344	16312	83688	43
18	04297	95703	10756	89244	16401	83599	42
19	04413	95587	10856	89144	16489	83511	41
20	9.04528	10.95472	9.10956	10.89044	9.16577	10.83423	40
21	04643	95357	11056	88944	16665	83335	39
22	04758	95242	11155	88845	16753	83247	38
23	04873	95127	11254	88746	16841	83159	37
24	04987	95013	11353	88647	16928	83072	36
25	05101	94899	11452	88548	17016	82984	35
26	05214	94786	11551	88449	17103	82897	34
27	05328	94672	11649	88351	17190	82810	33
28	05441	94559	11747	88253	17277	82723	32
29	05553	94447	11845	88155	17363	82637	31
30	9.05666	10.94334	9.11943	10.88057	9.17450	10.82550	30
31	05778	94222	12040	87960	17536	82464	29
32	05890	94110	12138	87862	17622	82378	28
33	06002	93998	12235	87765	17708	82292	27
34	06113	93887	12332	87668	17794	82206	26
35	06224	93776	12428	87572	17880	82120	25
36	06335	93665	12525	87475	17965	82035	24
37	06445	93555	12621	87379	18051	81949	23
38	06556	93444	12717	87283	18136	81864	22
39	06666	93334	12813	87187	18221	81779	21
40	9.06775	10.93225	9.12909	10.87091	9.18306	10.81694	20
41	06885	93115	13004	86996	18391	81609	19
42	06994	93006	13099	86901	18475	81525	18
43	07103	92897	13194	86806	18560	81440	17
44	07211	92789	13289	86711	18644	81356	16
45	07320	92680	13384	86616	18728	81272	15
46	07428	92572	13478	86522	18812	81188	14
47	07536	92464	13573	86427	18896	81104	13
48	07643	92357	13667	86333	18979	81021	12
49	07751	92249	13761	86239	19063	80937	11
50	9.07858	10.92142	9.13854	10.86146	9.19146	10.80854	10
51	07964	92036	13948	86053	19229	80771	9
52	08071	91929	14041	85959	19312	80688	8
53	08177	91823	14134	85866	19395	80605	7
54	08283	91717	14227	85773	19478	80522	6
55	08389	91611	14320	85680	19561	80439	5
56	08495	91505	14412	85588	19643	80357	4
57	08600	91400	14504	85496	19725	80275	3
58	08705	91295	14597	85403	19807	80193	2
59	08810	91190	14688	85312	19889	80111	1
60	08914	91086	14780	85220	19971	80029	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	83°		82°		81°		

′	9°		10°		11°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.19971	10.80029	9.24632	10.75368	9.28865	10.71135	60
1	20053	79947	24706	75294	28933	71067	59
2	20134	79866	24779	75221	29000	71000	58
3	20216	79784	24853	75147	29067	70933	57
4	20297	79703	24926	75074	29134	70866	56
5	20378	79622	25000	75000	29201	70799	55
6	20459	79541	25073	74927	29268	70732	54
7	20540	79460	25146	74854	29335	70665	53
8	20621	79379	25219	74781	29402	70598	52
9	20701	79299	25292	74708	29468	70532	51
10	9.20782	10.79218	9.25365	10.74635	9.29535	10.70465	50
11	20862	79138	25437	74563	29601	70399	49
12	20942	79058	25510	74490	29668	70332	48
13	21022	78978	25582	74418	29734	70266	47
14	21102	78898	25655	74345	29800	70200	46
15	21182	78818	25727	74273	29866	70134	45
16	21261	78739	25799	74201	29932	70068	44
17	21341	78659	25871	74129	29998	70002	43
18	21420	78580	25943	74057	30064	69936	42
19	21499	78501	26015	73985	30130	69870	41
20	9.21578	10.78422	9.26086	10.73914	9.30195	10.69805	40
21	21657	78343	26158	73842	30261	69739	39
22	21736	78264	26229	73771	30326	69674	38
23	21814	78186	26301	73699	30391	69609	37
24	21893	78107	26372	73628	30457	69543	36
25	21971	78029	26443	73557	30522	69478	35
26	22049	77951	26514	73486	30587	69413	34
27	22127	77873	26585	73415	30652	69348	33
28	22205	77795	26655	73345	30717	69283	32
29	22283	77717	26726	73274	30782	69218	31
30	9.22361	10.77639	9.26797	10.73203	9.30846	10.69154	30
31	22438	77562	26867	73133	30911	69089	29
32	22516	77484	26937	73063	30975	69025	28
33	22593	77407	27008	72992	31040	68960	27
34	22670	77330	27078	72922	31104	68896	26
35	22747	77253	27148	72852	31168	68832	25
36	22824	77176	27218	72782	31233	68767	24
37	22901	77099	27288	72712	31297	68703	23
38	22977	77023	27357	72643	31361	68639	22
39	23054	76946	27427	72573	31425	68575	21
40	9.23130	10.76870	9.27496	10.72504	9.31489	10.68511	20
41	23206	76794	27566	72434	31552	68448	19
42	23283	76717	27635	72365	31616	68384	18
43	23359	76641	27704	72296	31679	68321	17
44	23435	76565	27773	72227	31743	68257	16
45	23510	76490	27842	72158	31806	68194	15
46	23586	76414	27911	72089	31870	68130	14
47	23661	76339	27980	72020	31933	68067	13
48	23737	76263	28049	71951	31996	68004	12
49	23812	76188	28117	71883	32059	67941	11
50	9.23887	10.76113	9.28186	10.71814	9.32122	10.67878	10
51	23962	76038	28254	71746	32185	67815	9
52	24037	75963	28323	71677	32248	67752	8
53	24112	75888	28391	71609	32311	67689	7
54	24186	75814	28459	71541	32373	67627	6
55	24261	75739	28527	71473	32436	67564	5
56	24335	75665	28595	71405	32498	67502	4
57	24410	75590	28662	71338	32561	67439	3
58	24484	75516	28730	71270	32623	67377	2
59	24558	75442	28798	71202	32685	67315	1
60	24632	75368	28865	71135	32747	67253	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	80°		79°		78°		

′	12°		13°		14°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.32747	10.67253	9.36336	10.63664	9.39677	10.60323	60
1	32810	67190	36394	63606	39731	60269	59
2	32872	67128	36452	63548	39785	60215	58
3	32933	67067	36509	63491	39838	60162	57
4	32995	67005	36566	63434	39892	60108	56
5	33057	66943	36624	63376	39945	60055	55
6	33119	66881	36681	63319	39999	60001	54
7	33180	66820	36738	63262	40052	59948	53
8	33242	66758	36795	63205	40106	59894	52
9	33303	66697	36852	63148	40159	59841	51
10	9.33365	10.66635	9.36909	10.63091	9.40212	10.59788	50
11	33426	66574	36966	63034	40266	59734	49
12	33487	66513	37023	62977	40319	59681	48
13	33548	66452	37080	62920	40372	59628	47
14	33609	66391	37137	62863	40425	59575	46
15	33670	66330	37193	62807	40478	59522	45
16	33731	66269	37250	62750	40531	59469	44
17	33792	66208	37306	62694	40584	59416	43
18	33853	66147	37363	62637	40636	59364	42
19	33913	66087	37419	62581	40689	59311	41
20	9.33974	10.66026	9.37476	10.62524	9.40742	10.59258	40
21	34034	65966	37532	62468	40795	59205	39
22	34095	65905	37588	62412	40847	59153	38
23	34155	65845	37644	62356	40900	59100	37
24	34215	65785	37700	62300	40952	59048	36
25	34276	65724	37756	62244	41005	58995	35
26	34336	65664	37812	62188	41057	58943	34
27	34396	65604	37868	62132	41109	58891	33
28	34456	65544	37924	62076	41161	58839	32
29	34516	65484	37980	62020	41214	58786	31
30	9.34576	10.65424	9.38035	10.61965	9.41266	10.58734	30
31	34635	65365	38091	61909	41318	58682	29
32	34695	65305	38147	61853	41370	58630	28
33	34755	65245	38202	61798	41422	58578	27
34	34814	65186	38257	61743	41474	58526	26
35	34874	65126	38313	61687	41526	58474	25
36	34933	65067	38368	61632	41578	58422	24
37	34992	65008	38423	61577	41629	58371	23
38	35051	64949	38479	61521	41681	58319	22
39	35111	64889	38534	61466	41733	58267	21
40	9.35170	10.64830	9.38589	10.61411	9.41784	10.58216	20
41	35229	64771	38644	61356	41836	58164	19
42	35288	64712	38699	61301	41887	58113	18
43	35347	64653	38754	61246	41939	58061	17
44	35405	64595	38808	61192	41990	58010	16
45	35464	64536	38863	61137	42041	57959	15
46	35523	64477	38918	61082	42093	57907	14
47	35581	64419	38972	61028	42144	57856	13
48	35640	64360	39027	60973	42195	57805	12
49	35698	64302	39082	60918	42246	57754	11
50	9.35757	10.64243	9.39136	10.60864	9.42297	10.57703	10
51	35815	64185	39190	60810	42348	57652	9
52	35873	64127	39245	60755	42399	57601	8
53	35931	64069	39299	60701	42450	57550	7
54	35989	64011	39353	60647	42501	57499	6
55	36047	63953	39407	60593	42552	57448	5
56	36105	63895	39461	60539	42603	57397	4
57	36163	63837	39515	60485	42653	57347	3
58	36221	63779	39569	60431	42704	57296	2
59	36279	63721	39623	60377	42755	57245	1
60	36336	63664	39677	60323	42805	57195	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	77°		76°		75°		

′	15°		16°		17°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.42805	10.57195	9.45750	10.54250	9.48584	10.51466	60
1	42856	57144	45797	54203	48579	51421	59
2	42906	57094	45845	54155	48624	51376	58
3	42957	57043	45892	54108	48669	51331	57
4	43007	56993	45940	54060	48714	51286	56
5	43057	56943	45987	54013	48759	51241	55
6	43108	56892	46035	53965	48804	51196	54
7	43158	56842	46082	53918	48849	51151	53
8	43208	56792	46130	53870	48894	51106	52
9	43258	56742	46177	53823	48939	51061	51
10	9.43308	10.56692	9.46224	10.53776	9.48984	10.51016	50
11	43358	56642	46271	53729	49029	50971	49
12	43408	56592	46319	53681	49073	50927	48
13	43458	56542	46366	53634	49118	50882	47
14	43508	56492	46413	53587	49163	50837	46
15	43558	56442	46460	53540	49207	50793	45
16	43607	56393	46507	53493	49252	50748	44
17	43657	56343	46554	53446	49296	50704	43
18	43707	56293	46601	53399	49341	50659	42
19	43756	56244	46648	53352	49385	50615	41
20	9.43806	10.56194	9.46694	10.53306	9.49430	10.50570	40
21	43855	56145	46741	53259	49474	50526	39
22	43905	56095	46788	53212	49519	50481	38
23	43954	56046	46835	53165	49563	50437	37
24	44004	55996	46881	53119	49607	50393	36
25	44053	55947	46928	53072	49652	50348	35
26	44102	55898	46975	53025	49696	50304	34
27	44151	55849	47021	52979	49740	50260	33
28	44201	55799	47068	52932	49784	50216	32
29	44250	55750	47114	52886	49828	50172	31
30	9.44299	10.55701	9.47160	10.52840	9.49872	10.50128	30
31	44348	55652	47207	52793	49916	50084	29
32	44397	55603	47253	52747	49960	50040	28
33	44446	55554	47299	52701	50004	49996	27
34	44495	55505	47346	52654	50048	49952	26
35	44544	55456	47392	52608	50092	49908	25
36	44592	55408	47438	52562	50136	49864	24
37	44641	55359	47484	52516	50180	49820	23
38	44690	55310	47530	52470	50223	49777	22
39	44738	55262	47576	52424	50267	49733	21
40	9.44787	10.55213	9.47622	10.52378	9.50311	10.49689	20
41	44836	55164	47668	52332	50355	49645	19
42	44884	55116	47714	52286	50398	49602	18
43	44933	55067	47760	52240	50442	49558	17
44	44981	55019	47806	52194	50485	49515	16
45	45029	54971	47852	52148	50529	49471	15
46	45078	54922	47897	52103	50572	49428	14
47	45126	54874	47943	52057	50616	49384	13
48	45174	54826	47989	52011	50659	49341	12
49	45222	54778	48035	51965	50703	49297	11
50	9.45271	10.54729	9.48080	10.51920	9.50746	10.49254	10
51	45319	54681	48126	51874	50789	49211	9
52	45367	54633	48171	51829	50833	49167	8
53	45415	54585	48217	51783	50876	49124	7
54	45463	54537	48262	51738	50919	49081	6
55	45511	54489	48307	51693	50962	49038	5
56	45559	54441	48353	51647	51005	48995	4
57	45606	54394	48398	51602	51048	48952	3
58	45654	54346	48443	51557	51092	48908	2
59	45702	54298	48489	51511	51135	48865	1
60	45750	54250	48534	51466	51178	48822	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	74°		73°		72°		

′	18°		19°		20°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.51178	10.48822	9.53697	10.46303	9.56107	10.43893	60
1	51221	48779	53738	46262	56146	43854	59
2	51264	48736	53779	46221	56185	438.5	58
3	51306	48694	53820	46180	56224	43776	57
4	51349	48651	53861	46139	56264	43736	56
5	51392	48608	53902	46098	56303	43697	55
6	51435	48565	53943	46057	56342	43658	54
7	51478	48522	53984	46016	56381	43619	53
8	51520	48480	54025	45975	56420	43580	52
9	51563	48437	54065	45935	56459	43541	51
10	9.51606	10.48394	9.54106	10.45894	9.56498	10.43502	50
11	51648	48352	54147	45853	56537	43463	49
12	51691	48309	54187	45813	56576	43424	48
13	51734	48266	54228	45772	56615	43385	47
14	51776	48224	54269	45731	56654	43346	46
15	51819	48181	54309	45691	56693	43307	45
16	51861	48139	54350	45650	56732	43268	44
17	51903	48097	54390	45610	56771	43229	43
18	51946	48054	54431	45569	56810	43190	42
19	51988	48012	54471	45529	56849	43151	41
20	9.52031	10.47969	9.54512	10.45488	9.56887	10.43113	40
21	52073	47927	54552	45448	56926	43074	39
22	52115	47885	54593	45407	56965	43035	38
23	52157	47843	54633	45367	57004	42996	37
24	52200	47800	54673	45327	57042	42958	36
25	52242	47758	54714	45286	57081	42919	35
26	52284	47716	54754	45246	57120	42880	34
27	52326	47674	54794	45206	57158	42842	33
28	52368	47632	54835	45165	57197	42803	32
29	52410	47590	54875	45125	57235	42765	31
30	9.52452	10.47548	9.54915	10.45085	9.57274	10.42726	30
31	52494	47506	54955	45045	57312	42688	29
32	52536	47464	54995	45005	57351	42649	28
33	52578	47422	55035	44965	57389	42611	27
34	52620	47380	55075	44925	57428	42572	26
35	52661	47339	55115	44885	57466	42534	25
36	52703	47297	55155	44845	57504	42496	24
37	52745	47255	55195	44805	57543	42457	23
38	52787	47213	55235	44765	57581	42419	22
39	52829	47171	55275	44725	57619	42381	21
40	9.52870	10.47130	9.55315	10.44685	9.57658	10.42342	20
41	52912	47088	55355	44645	57696	42304	19
42	52953	47047	55395	44605	57734	42266	18
43	52995	47005	55434	44566	57772	42228	17
44	53037	46963	55474	44526	57810	42190	16
45	53078	46922	55514	44486	57849	42151	15
46	53120	46880	55554	44446	57887	42113	14
47	53161	46839	55593	44407	57925	42075	13
48	53202	46798	55633	44367	57963	42037	12
49	53244	46756	55673	44327	58001	41999	11
50	9.53285	10.46715	9.55712	10.44288	9.58039	10.41961	10
51	53327	46673	55752	44248	58077	41723	9
52	53368	46632	55791	44209	58115	41885	8
53	53409	46591	55831	44169	58153	41847	7
54	53450	46550	55870	44130	58191	41809	6
55	53492	46508	55910	44090	58229	41771	5
56	53533	46467	55949	44051	58267	41733	4
57	53574	46426	55989	44011	58304	41696	3
58	53615	46385	56028	43972	58342	41658	2
59	53656	46344	56067	43933	58380	41620	1
60	53697	46303	56107	43893	58418	41582	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	71°		70°		69°		

′	21°		22°		23°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.58418	10.41582	9.60641	10.39359	9.62785	10.37215	60
1	58455	41545	60677	39323	62820	37180	59
2	58493	41507	60714	39286	62855	37145	58
3	58531	41469	60750	39250	62890	37110	57
4	58569	41431	60786	39214	62926	37074	56
5	58606	41394	60823	39177	62961	37039	55
6	58644	41356	60859	39141	62996	37004	54
7	58681	41319	60895	39105	63031	36969	53
8	58719	41281	60931	39069	63066	36934	52
9	58757	41243	60967	39033	63101	36899	51
10	9.58794	10.41206	9.61004	10.38996	9.63135	10.36865	50
11	58832	41168	61040	38960	63170	36830	49
12	58869	41131	61076	38924	63205	36795	48
13	58907	41093	61112	38888	63240	36760	47
14	58944	41056	61148	38852	63275	36725	46
15	58981	41019	61184	38816	63310	36690	45
16	59019	40981	61220	38780	63345	36655	44
17	59056	40944	61256	38744	63379	36621	43
18	59094	40906	61292	38708	63414	36586	42
19	59131	40869	61328	38672	63449	36551	41
20	59168	40832	61364	38636	9.63484	10.36516	40
21	59205	40795	61400	38600	63519	36481	39
22	59243	40757	61436	38564	63553	36447	38
23	59280	40720	61472	38528	63588	36412	37
24	59317	40683	61508	38492	63623	36377	36
25	59354	40646	61544	38456	63657	36343	35
26	59391	40609	61579	38421	63692	36308	34
27	59429	40571	61615	38385	63726	36274	33
28	59466	40534	61651	38349	63761	36239	32
29	59503	40497	61687	38313	63796	36204	31
30	9.59540	10.40460	9.61722	10.38278	9.63830	10.36170	30
31	59577	40423	61758	38242	63865	36135	29
32	59614	40386	61794	38206	63899	36101	28
33	59651	40349	61830	38170	63934	36066	27
34	59688	40312	61865	38135	63968	36032	26
35	59725	40275	61901	38099	64003	35997	25
36	59762	40238	61936	38064	64037	35963	24
37	59799	40201	61972	38028	64072	35928	23
38	59835	40165	62008	37992	64106	35894	22
39	59872	40128	62043	37957	64140	35860	21
40	9.59909	10.40091	9.62079	10.37921	9.64175	10.35825	20
41	59946	40054	62114	37886	64209	35791	19
42	59983	40017	62150	37850	64243	35757	18
43	60019	39981	62185	37815	64278	35722	17
44	60056	39944	62221	37779	64312	35688	16
45	60093	39907	62256	37744	64346	35654	15
46	60130	39870	62292	37708	64381	35619	14
47	60166	39834	62327	37673	64415	35585	13
48	60203	39797	62362	37638	64449	35551	12
49	60240	39760	62398	37602	64483	35517	11
50	9.60276	10.39724	9.62433	10.37567	9.64517	10.35483	10
51	60313	39687	62468	37532	64552	35448	9
52	60349	39651	62504	37496	64586	35414	8
53	60386	39614	62539	37461	64620	35380	7
54	60422	39578	62574	37426	64654	35346	6
55	60459	39541	62609	37391	64688	35312	5
56	60495	39505	62645	37355	64722	35278	4
57	60532	39468	62680	37320	64756	35244	3
58	60568	39432	62715	37285	64790	35210	2
59	60605	39395	62750	37250	64824	35176	1
60	60641	39359	62785	37215	64858	35142	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	68°		67°		66°		

′	24°		25°		26°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.64858	10.35142	9.66867	10.33133	9.68818	10.31182	60
1	64892	35108	66900	33100	68850	31150	59
2	64926	35074	66933	33067	68882	31118	58
3	64960	35040	66966	33034	68914	31086	57
4	64994	35006	66999	33001	68946	31054	56
5	65028	34972	67032	32968	68978	31022	55
6	65062	34938	67065	32935	69010	30990	54
7	65096	34904	67098	32902	69042	30958	53
8	65130	34870	67131	32869	69074	30926	52
9	65164	34836	67163	32837	69106	30894	51
10	9.65197	10.34803	9.67196	10.32804	9.69138	10.30862	50
11	65231	34769	67229	32771	69170	30830	49
12	65265	34735	67262	32738	69202	30798	48
13	65299	34701	67295	32705	69234	30766	47
14	65333	34667	67327	32673	69266	30734	46
15	65366	34634	67360	32640	69298	30702	45
16	65400	34600	67393	32607	69329	30671	44
17	65434	34566	67426	32574	69361	30639	43
18	65467	34533	67458	32542	69393	30607	42
19	65501	34499	67491	32509	69425	30575	41
20	9.65535	10.34465	9.67524	10.32476	9.69457	10.30543	40
21	65568	34432	67556	32444	69488	30512	39
22	65602	34398	67589	32411	69520	30480	38
23	65636	34364	67622	32378	69552	30448	37
24	65669	34331	67654	32346	69584	30416	36
25	65703	34297	67687	32313	69615	30385	35
26	65736	34264	67719	32281	69647	30353	34
27	65770	34230	67752	32248	69679	30321	33
28	65803	34197	67785	32215	69710	30290	32
29	65837	34163	67817	32183	69742	30258	31
30	9.65870	10.34130	9.67850	10.32150	9.69774	10.30226	30
31	65904	34096	67882	32118	69805	30195	29
32	65937	34063	67915	32085	69837	30163	28
33	65971	34029	67947	32053	69868	30132	27
34	66004	33996	67980	32020	69900	30100	26
35	66038	33962	68012	31988	69932	30068	25
36	66071	33929	68044	31956	69963	30037	24
37	66104	33896	68077	31923	69995	30005	23
38	66138	33862	68109	31891	70026	29974	22
39	66171	33829	68142	31858	70058	29942	21
40	9.66204	10.33796	9.68174	10.31826	9.70089	10.29911	20
41	66238	33762	68206	31794	70121	29879	19
42	66271	33729	68239	31761	70152	29848	18
43	66304	33696	68271	31729	70184	29816	17
44	66337	33663	68303	31697	70215	29785	16
45	66371	33629	68336	31664	70247	29753	15
46	66404	33596	68368	31632	70278	29722	14
47	66437	33563	68400	31600	70309	29691	13
48	66470	33530	68432	31568	70341	29659	12
49	66503	33497	68465	31535	70372	29628	11
50	9.66537	10.33463	9.68497	10.31503	9.70404	10.29596	10
51	66570	33430	68529	31471	70435	29565	9
52	66603	33397	68561	31439	70466	29534	8
53	66636	33364	68593	31407	70498	29502	7
54	66669	33331	68626	31374	70529	29471	6
55	66702	33298	68658	31342	70560	29440	5
56	66735	33265	68690	31310	70592	29408	4
57	66768	33232	68722	31278	70623	29377	3
58	66801	33199	68754	31246	70654	29346	2
59	66834	33166	68786	31214	70685	29315	1
60	66867	33133	68818	31182	70717	29283	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	65°		64°		63°		

′	27°		28°		29°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.70717	10.29283	9.72567	10.27433	9.74375	10.25625	60
1	70748	29252	72598	27402	74405	25595	59
2	70779	29221	72628	27372	74435	25565	58
3	70810	29190	72659	27341	74465	25535	57
4	70841	29159	72689	27311	74494	25506	56
5	70873	29127	72720	27280	74524	25476	55
6	70904	29096	72750	27250	74554	25446	54
7	70935	29065	72780	27220	74583	25417	53
8	70966	29034	72811	27189	74613	25387	52
9	70997	29003	72841	27159	74643	25357	51
10	9.71028	10.28972	9.72872	10.27128	9.74673	10.25327	50
11	71059	28941	72902	27098	74702	25298	49
12	71090	28910	72932	27068	74732	25268	48
13	71121	28879	72963	27037	74762	25238	47
14	71153	28847	72993	27007	74791	25209	46
15	71184	28816	73023	26977	74821	25179	45
16	71215	28785	73054	26946	74851	25149	44
17	71246	28754	73084	26916	74880	25120	43
18	71277	28723	73114	26886	74910	25090	42
19	71308	28692	73144	26856	74939	25061	41
20	9.71339	10.28661	9.73175	10.26825	9.74969	10.25031	40
21	71370	28630	73205	26795	74998	25002	39
22	71401	28599	73235	26765	75028	24972	38
23	71431	28569	73265	26735	75058	24942	37
24	71462	28538	73295	26705	75087	24913	36
25	71493	28507	73326	26674	75117	24883	35
26	71524	28476	73356	26644	75146	24854	34
27	71555	28445	73386	26614	75176	24824	33
28	71586	28414	73416	26584	75205	24795	32
29	71617	28383	73446	26554	75235	24765	31
30	9.71648	10.28352	9.73476	10.26524	9.75264	10.24736	30
31	71679	28321	73507	26493	75294	24706	29
32	71709	28291	73537	26463	75323	24677	28
33	71740	28260	73567	26433	75353	24647	27
34	71771	28229	73597	26403	75382	24618	26
35	71802	28198	73627	26373	75411	24589	25
36	71833	28167	73657	26343	75441	24559	24
37	71863	28137	73687	26313	75470	24530	23
38	71894	28106	73717	26283	75500	24500	22
39	71925	28075	73747	26253	75529	24471	21
40	9.71955	10.28045	9.73777	10.26223	9.75558	10.24442	20
41	71986	28014	73807	26193	75588	24412	19
42	72017	27983	73837	26163	75617	24383	18
43	72048	27952	73867	26133	75647	24353	17
44	72078	27922	73897	26103	75676	24324	16
45	72109	27891	73927	26073	75705	24295	15
46	72140	27860	73957	26043	75735	24265	14
47	72170	27830	73987	26013	75764	24236	13
48	72201	27799	74017	25983	75793	24207	12
49	72231	27769	74047	25953	75822	24178	11
50	9.72262	10.27738	9.74077	10.25923	9.75852	10.24148	10
51	72293	27707	74107	25893	75881	24119	9
52	72323	27677	74137	25863	75910	24090	8
53	72354	27646	74166	25834	75939	24061	7
54	72384	27616	74196	25804	75969	24031	6
55	72415	27585	74226	25774	75998	24002	5
56	72445	27555	74256	25744	76027	23973	4
57	72476	27524	74286	25714	76056	23944	3
58	72506	27494	74316	25684	76086	23914	2
59	72537	27463	74345	25655	76115	23885	1
60	72567	27433	74375	25625	76144	23856	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	62°		61°		60°		

′	30°		31°		32°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.76144	10.23856	9.77877	10.22123	9.79579	10.20421	60
1	76173	23827	77906	22094	79607	20393	59
2	76202	23798	77935	22065	79635	20365	58
3	76231	23769	77963	22037	79663	20337	57
4	76261	23739	77992	22008	79691	20309	56
5	76290	23710	78020	21980	79719	20281	55
6	76319	23681	78049	21951	79747	20253	54
7	76348	23652	78077	21923	79776	20224	53
8	76377	23623	78106	21894	79804	20196	52
9	76406	23594	78135	21865	79832	20168	51
10	9.76435	10.23565	9.78163	10.21837	9.79860	10.20140	50
11	76464	23536	78192	21808	79888	20112	49
12	76493	23507	78220	21780	79916	20084	48
13	76522	23478	78249	21751	79944	20056	47
14	76551	23449	78277	21723	79972	20028	46
15	76580	23420	78306	21694	80000	20000	45
16	76609	23391	78334	21666	80028	19972	44
17	76639	23361	78363	21637	80056	19944	43
18	76668	23332	78391	21609	80084	19916	42
19	76697	23303	78419	21581	80112	19888	41
20	9.76725	10.23275	9.78448	10.21552	9.80140	10.19860	40
21	76754	23246	78476	21524	80168	19832	39
22	76783	23217	78505	21495	80195	19805	38
23	76812	23188	78533	21467	80223	19777	37
24	76841	23159	78562	21438	80251	19749	36
25	76870	23130	78590	21410	80279	19721	35
26	76899	23101	78618	21382	80307	19693	34
27	76928	23072	78647	21353	80335	19665	33
28	76957	23043	78675	21325	80363	19637	32
29	76986	23014	78704	21296	80391	19609	31
30	9.77015	10.22985	9.78732	10.21268	9.80419	10.19581	30
31	77044	22956	78760	21240	80447	19553	29
32	77073	22927	78789	21211	80474	19526	28
33	77101	22899	78817	21183	80502	19498	27
34	77130	22870	78845	21155	80530	19470	26
35	77159	22841	78874	21126	80558	19442	25
36	77188	22812	78902	21098	80586	19414	24
37	77217	22783	78930	21070	80614	19386	23
38	77246	22754	78959	21041	80642	19358	22
39	77274	22726	78987	21013	80669	19331	21
40	9.77303	10.22697	9.79015	10.20985	9.80697	10.19303	20
41	77332	22668	79043	20957	80725	19275	19
42	77361	22639	79072	20928	80753	19247	18
43	77390	22610	79100	20900	80781	19219	17
44	77418	22582	79128	20872	80808	19192	16
45	77447	22553	79156	20844	80836	19164	15
46	77476	22524	79185	20815	80864	19136	14
47	77505	22495	79213	20787	80892	19108	13
48	77533	22467	79241	20759	80919	19081	12
49	77562	22438	79269	20731	80947	19053	11
50	9.77591	10.22409	9.79297	10.20703	9.80975	10.19025	10
51	77619	22381	79326	20674	81003	18997	9
52	77648	22352	79354	20646	81030	18970	8
53	77677	22323	79382	20618	81058	18942	7
54	77706	22294	79410	20590	81086	18914	6
55	77734	22266	79438	20562	81113	18887	5
56	77763	22237	79466	20534	81141	18859	4
57	77791	22209	79495	20505	81169	18831	3
58	77820	22180	79523	20477	81196	18804	2
59	77849	22151	79551	20449	81224	18776	1
60	77877	22123	79579	20421	81252	18748	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	59°		58°		57°		

′	33°		34°		35°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.81252	10.18748	9.82899	10.17101	9.84523	10.15477	60
1	81279	18721	82926	17074	84550	15450	59
2	81307	18693	82953	17047	84576	15424	58
3	81335	18665	82980	17020	84603	15397	57
4	81362	18638	83008	16992	84630	15370	56
5	81390	18610	83035	16965	84657	15343	55
6	81418	18582	83062	16938	84684	15316	54
7	81445	18555	83089	16911	84711	15289	53
8	81473	18527	83117	16883	84738	15262	52
9	81500	18500	83144	16856	84764	15236	51
10	9.81528	10.18472	9.83171	10.16829	9.84791	10.15209	50
11	81556	18444	83198	16802	84818	15182	49
12	81583	18417	83225	16775	84845	15155	48
13	81611	18389	83252	16748	84872	15128	47
14	81638	18362	83280	16720	84899	15101	46
15	81666	18334	83307	16693	84925	15075	45
16	81693	18307	83334	16666	84952	15048	44
17	81721	18279	83361	16639	84979	15021	43
18	81748	18252	83388	16612	85006	14994	42
19	81776	18224	83415	16585	85033	14967	41
20	9.81803	10.18197	9.83442	10.16558	9.85059	10.14941	40
21	81831	18169	83470	16530	85086	14914	39
22	81858	18142	83497	16503	85113	14887	38
23	81886	18114	83524	16476	85140	14860	37
24	81913	18087	83551	16449	85166	14834	36
25	81941	18059	83578	16422	85193	14807	35
26	81968	18032	83605	16395	85220	14780	34
27	81996	18004	83632	16368	85247	14753	33
28	82023	17977	83659	16341	85273	14727	32
29	82051	17949	83686	16314	85300	14700	31
30	9.82078	10.17922	9.83713	10.16287	9.85327	10.14673	30
31	82106	17894	83740	16260	85354	14646	29
32	82133	17867	83768	16232	85380	14620	28
33	82161	17839	83795	16205	85407	14593	27
34	82188	17812	83822	16178	85434	14566	26
35	82215	17785	83849	16151	85460	14540	25
36	82243	17757	83876	16124	85487	14513	24
37	82270	17730	83903	16097	85514	14486	23
38	82298	17702	83930	16070	85540	14460	22
39	82325	17675	83957	16043	85567	14433	21
40	9.82352	10.17648	9.83984	10.16016	9.85594	10.14406	20
41	82380	17620	84011	15989	85620	14380	19
42	82407	17593	84038	15962	85647	14353	18
43	82435	17565	84065	15935	85674	14326	17
44	82462	17538	84092	15908	85700	14300	16
45	82489	17511	84119	15881	85727	14273	15
46	82517	17483	84146	15854	85754	14246	14
47	82544	17456	84173	15827	85780	14220	13
48	82571	17429	84200	15800	85807	14193	12
49	82599	17401	84227	15773	85834	14166	11
50	9.82626	10.17374	9.84254	10.15746	9.85860	10.14140	10
51	82653	17347	84280	15720	85887	14113	9
52	82681	17319	84307	15693	85913	14087	8
53	82708	17292	84334	15666	85940	14060	7
54	82735	17265	84361	15639	85967	14033	6
55	82762	17238	84388	15612	85993	14007	5
56	82790	17210	84415	15585	86020	13980	4
57	82817	17183	84442	15558	86046	13954	3
58	82844	17156	84469	15531	86073	13927	2
59	82871	17129	84496	15504	86100	13900	1
60	82899	17101	84523	15477	86126	13874	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	56°		55°		54°		

′	36°		37°		38°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.86126	10.13874	9.87711	10.12289	9.89281	10.10719	60
1	86153	13847	87738	12262	89307	10693	59
2	86179	13821	87764	12236	89333	10667	58
3	86206	13794	87790	12210	89359	10641	57
4	86232	13768	87817	12183	89385	10615	56
5	86259	13741	87843	12157	89411	10589	55
6	86285	13715	87869	12131	89437	10563	54
7	86312	13688	87895	12105	89463	10537	53
8	86338	13662	87922	12078	89489	10511	52
9	86365	13635	87948	12052	89515	10485	51
10	9.86392	10.13608	9.87974	10.12026	9.89541	10.10459	50
11	86418	13582	88000	12000	89567	10433	49
12	86445	13555	88027	11973	89593	10407	48
13	86471	13529	88053	11947	89619	10381	47
14	86498	13502	88079	11921	89645	10355	46
15	86524	13476	88105	11895	89671	10329	45
16	86551	13449	88131	11869	89697	10303	44
17	86577	13423	88158	11842	89723	10277	43
18	86603	13397	88184	11816	89749	10251	42
19	86630	13370	88210	11790	89775	10225	41
20	9.86656	10.13344	9.88236	10.11764	9.89801	10.10199	40
21	86683	13317	88262	11738	89827	10173	39
22	86709	13291	88289	11711	89853	10147	38
23	86736	13264	88315	11685	89879	10121	37
24	86762	13238	88341	11659	89905	10095	36
25	86789	13211	88367	11633	89931	10069	35
26	86815	13185	88393	11607	89957	10043	34
27	86842	13158	88420	11580	89983	10017	33
28	86868	13132	88446	11554	90009	09991	32
29	86894	13106	88472	11528	90035	09965	31
30	9.86921	10.13079	9.88498	10.11502	9.90061	10.09939	30
31	86947	13053	88524	11476	90086	09914	29
32	86974	13026	88550	11450	90112	09888	28
33	87000	13000	88577	11423	90138	09862	27
34	87027	12973	88603	11397	90164	09836	26
35	87053	12947	88629	11371	90190	09810	25
36	87079	12921	88655	11345	90216	09784	24
37	87106	12894	88681	11319	90242	09758	23
38	87132	12868	88707	11293	90268	09732	22
39	87158	12842	88733	11267	90294	09706	21
40	9.87185	10.12815	9.88759	10.11241	9.90320	10.09680	20
41	87211	12789	88786	11214	90346	09654	19
42	87238	12762	88812	11188	90371	09629	18
43	87264	12736	88838	11162	90397	09603	17
44	87290	12710	88864	11136	90423	09577	16
45	87317	12683	88890	11110	90449	09551	15
46	87343	12657	88916	11084	90475	09525	14
47	87369	12631	88942	11058	90501	09499	13
48	87396	12604	88968	11032	90527	09473	12
49	87422	12578	88994	11006	90553	09447	11
50	9.87448	10.12552	9.89020	10.10980	9.90578	10.09422	10
51	87475	12525	89046	10954	90604	09396	9
52	87501	12499	89073	10927	90630	09370	8
53	87527	12473	89099	10901	90656	09344	7
54	87554	12446	89125	10875	90682	09318	6
55	87580	12420	89151	10849	90708	09292	5
56	87606	12394	89177	10823	90734	09266	4
57	87633	12367	89203	10797	90759	09241	3
58	87659	12341	89229	10771	90785	09215	2
59	87685	12315	89255	10745	90811	09189	1
60	87711	12289	89281	10719	90837	09163	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	53°		52°		51°		

′	39°		40°		41°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.90837	10.09163	9.92381	10.07619	9.93916	10.06084	60
1	90863	09137	92407	07593	93942	06058	59
2	90889	09111	92433	07567	93967	06033	58
3	90914	09086	92458	07542	93993	06007	57
4	90940	09060	92484	07516	94018	05982	56
5	90966	09034	92510	07490	94044	05956	55
6	90992	09008	92535	07465	94069	05931	54
7	91018	08982	92561	07439	94095	05905	53
8	91043	08957	92587	07413	94120	05880	52
9	91069	08931	92612	07388	94146	05854	51
10	9.91095	10.08905	9.92638	10.07362	9.94171	10.05829	50
11	91121	08879	92663	07337	94197	05803	49
12	91147	08853	92689	07311	94222	05778	48
13	91172	08828	92715	07285	94248	05752	47
14	91198	08802	92740	07260	94273	05727	46
15	91224	08776	92766	07234	94299	05701	45
16	91250	08750	92792	07208	94324	05676	44
17	91276	08724	92817	07183	94350	05650	43
18	91301	08699	92843	07157	94375	05625	42
19	91327	08673	92868	07132	94401	05599	41
20	9.91353	10.08647	9.92894	10.07106	9.94426	10.05574	40
21	91379	08621	92920	07080	94452	05548	39
22	91404	08596	92945	07055	94477	05523	38
23	91430	08570	92971	07029	94503	05497	37
24	91456	08544	92996	07004	94528	05472	36
25	91482	08518	93022	06978	94554	05446	35
26	91507	08493	93048	06952	94579	05421	34
27	91533	08467	93073	06927	94604	05396	33
28	91559	08441	93099	06901	94630	05370	32
29	91585	08415	93124	06876	94655	05345	31
30	9.91610	10.08390	9.93150	10.06850	9.94681	10.05319	30
31	91636	08364	93175	06825	94706	05294	29
32	91662	08338	93201	06799	94732	05268	28
33	91688	08312	93227	06773	94757	05243	27
34	91713	08287	93252	06748	94783	05217	26
35	91739	08261	93278	06722	94808	05192	25
36	91765	08235	93303	06697	94834	05166	24
37	91791	08209	93329	06671	94859	05141	23
38	91816	08184	93354	06646	94884	05116	22
39	91842	08158	93380	06620	94910	05090	21
40	9.91868	10.08132	9.93406	10.06594	9.94935	10.05065	20
41	91893	08107	93431	06569	94961	05039	19
42	91919	08081	93457	06543	94986	05014	18
43	91945	08055	93482	06518	95012	04988	17
44	91971	08029	93508	06492	95037	04963	16
45	91996	08004	93533	06467	95062	04938	15
46	92022	07978	93559	06441	95088	04912	14
47	92048	07952	93584	06416	95113	04887	13
48	92073	07927	93610	06390	95139	04861	12
49	92099	07901	93636	06364	95164	04836	11
50	9.92125	10.07875	9.93661	10.06389	9.95190	10.04810	10
51	92150	07850	93687	06313	95215	04785	9
52	92176	07824	93712	06288	95240	04760	8
53	92202	07798	93738	06262	95266	04734	7
54	92227	07773	93763	06237	95291	04709	6
55	92253	07747	93789	06211	95317	04683	5
56	92279	07721	93814	06186	95342	04658	4
57	92304	07696	93840	06160	95368	04632	3
58	92330	07670	93865	06135	95393	04607	2
59	92356	07644	93891	06109	95418	04582	1
60	92381	07619	93916	06084	95444	04556	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	50°		49°		48°		

′	42° Tan	42° Cotan	43° Tan	43° Cotan	44° Tan	44° Cotan	′
0	9.95444	10.04556	9.96966	10.03034	9.98484	10.01516	60
1	95469	04531	96991	03009	98509	01491	59
2	95495	04505	97016	02984	98534	01466	58
3	95520	04480	97042	02958	98560	01440	57
4	95545	04455	97067	02933	98585	01415	56
5	95571	04429	97092	02908	98610	01390	55
6	95596	04404	97118	02882	98635	01365	54
7	95622	04378	97143	02857	98661	01339	53
8	95647	04353	97168	02832	98686	01314	52
9	95672	04328	97193	02807	98711	01289	51
10	9.95698	10.04302	9.97219	10.02781	9.98737	10.01263	50
11	95723	04277	97244	02756	98762	01238	49
12	95748	04252	97269	02731	98787	01213	48
13	95774	04226	97295	02705	98812	01188	47
14	95799	04201	97320	02680	98838	01162	46
15	95825	04175	97345	02655	98863	01137	45
16	95850	04150	97371	02629	98888	01112	44
17	95875	04125	97396	02604	98913	01087	43
18	95901	04099	97421	02579	98939	01061	42
19	95926	04074	97447	02553	98964	01036	41
20	9.95952	10.04048	9.97472	10.02528	9.98989	10.01011	40
21	95977	04023	97497	02503	99015	00985	39
22	96002	03998	97523	02477	99040	00960	38
23	96028	03972	97548	02452	99065	00935	37
24	96053	03947	97573	02427	99090	00910	36
25	96078	03922	97598	02402	99116	00884	35
26	96104	03896	97624	02376	99141	00859	34
27	96129	03871	97649	02351	99166	00834	33
28	96155	03845	97674	02326	99191	00809	32
29	96180	03820	97700	02300	99217	00783	31
30	9.96205	10.03795	9.97725	10.02275	9.99242	10.00758	30
31	96231	03769	97750	02250	99267	00733	29
32	96256	03744	97776	02224	99293	00707	28
33	96281	03719	97801	02199	99318	00682	27
34	96307	03693	97826	02174	99343	00657	26
35	96332	03668	97851	02149	99368	00632	25
36	96357	03643	97877	02123	99394	00606	24
37	96383	03617	97902	02098	99419	00581	23
38	96408	03592	97927	02073	99444	00556	22
39	96433	03567	97953	02047	99469	00531	21
40	9.96459	10.03541	9.97978	10.02022	9.99495	10.00505	20
41	96484	03516	98003	01997	99520	00480	19
42	96510	03490	98029	01971	99545	00455	18
43	96535	03465	98054	01946	99570	00430	17
44	96560	03440	98079	01921	99596	00404	16
45	96586	03414	98104	01896	99621	00379	15
46	96611	03389	98130	01870	99646	00354	14
47	96636	03364	98155	01845	99672	00328	13
48	96662	03338	98180	01820	99697	00303	12
49	96687	03313	98206	01794	99722	00278	11
50	9.96712	10.03288	9.98231	10.01769	9.99747	10.00253	10
51	96738	03262	98256	01744	99773	00227	9
52	96763	03237	98281	01719	99798	00202	8
53	96788	03212	98307	01693	99823	00177	7
54	96814	03186	98332	01668	99848	00152	6
55	96839	03161	98357	01643	99874	00126	5
56	96864	03136	98383	01617	99899	00101	4
57	96890	03110	98408	01592	99924	00076	3
58	96915	03085	98433	01567	99949	00051	2
59	96940	03060	98458	01542	99975	00025	1
60	96966	03034	98484	01516	10.00000	00000	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	47°		46°		45°		

TABLE 17.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENT

	0°				1°				
	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.00000	One	.00000	Infinite	.01745	.99985	.01746	57.2900	6
1	.00029	One	.00029	3437.75	.01774	.99984	.01775	56.3506	5
2	.00058	One	.00058	1718.87	.01803	.99984	.01804	55.4415	5
3	.00087	One	.00087	1145.92	.01832	.99983	.01833	54.5613	5
4	.00116	One	.00116	859.436	.01862	.99983	.01862	53.7086	5
5	.00145	One	.00145	687.549	.01891	.99982	.01891	52.8821	5
6	.00175	One	.00175	572.957	.01920	.99982	.01920	52.0807	5
7	.00204	One	.00204	491.106	.01949	.99981	.01949	51.3032	5
8	.00233	One	.00233	429.718	.01978	.99980	.01978	50.5485	5
9	.00262	One	.00262	381.971	.02007	.99980	.02007	49.8157	5
10	.00291	One	.00291	343.774	.02036	.99979	.02036	49.1039	5
11	.00320	.99999	.00320	312.521	.02065	.99979	.02066	48.4121	5
12	.00349	.99999	.00349	286.478	.02094	.99978	.02095	47.7395	5
13	.00378	.99999	.00378	264.441	.02123	.99977	.02124	47.0853	5
14	.00407	.99999	.00407	245.552	.02152	.99977	.02153	46.4489	5
15	.00436	.99999	.00436	229.182	.02181	.99976	.02182	45.8294	5
16	.00465	.99999	.00465	214.858	.02211	.99976	.02211	45.2261	5
17	.00495	.99999	.00495	202.219	.02240	.99975	.02240	44.6386	5
18	.00524	.99999	.00524	190.984	.02269	.99974	.02269	44.0661	5
19	.00553	.99998	.00553	180.932	.02298	.99974	.02298	43.5081	5
20	.00582	.99998	.00582	171.885	.02327	.99973	.02328	42.9641	4
21	.00611	.99998	.00611	163.700	.02356	.99972	.02357	42.4335	4
22	.00640	.99998	.00640	156.259	.02385	.99972	.02386	41.9158	4
23	.00669	.99998	.00669	149.465	.02414	.99971	.02415	41.4106	4
24	.00698	.99998	.00698	143.237	.02443	.99970	.02444	40.9174	4
25	.00727	.99997	.00727	137.507	.02472	.99969	.02473	40.4358	4
26	.00756	.99997	.00756	132.219	.02501	.99969	.02502	39.9655	4
27	.00785	.99997	.00785	127.321	.02530	.99968	.02531	39.5059	4
28	.00814	.99997	.00815	122.774	.02560	.99967	.02560	39.0568	4
29	.00844	.99996	.00844	118.540	.02589	.99966	.02589	38.6177	4
30	.00873	.99996	.00873	114.589	.02618	.99966	.02619	38.1885	3
31	.00902	.99996	.00902	110.892	.02647	.99965	.02648	37.7686	3
32	.00931	.99996	.00931	107.426	.02676	.99964	.02677	37.3579	3
33	.00960	.99995	.00960	104.171	.02705	.99963	.02706	36.9560	3
34	.00989	.99995	.00989	101.107	.02734	.99963	.02735	36.5627	3
35	.01018	.99995	.01018	98.2179	.02763	.99962	.02764	36.1776	3
36	.01047	.99995	.01047	95.4895	.02792	.99961	.02793	35.8006	3
37	.01076	.99994	.01076	92.9085	.02821	.99960	.02822	35.4313	3
38	.01105	.99994	.01105	90.4633	.02850	.99959	.02851	35.0695	3
39	.01134	.99994	.01135	88.1436	.02879	.99959	.02881	34.7151	3
40	.01164	.99993	.01164	85.9398	.02908	.99958	.02910	34.3678	2
41	.01193	.99993	.01193	83.8435	.02938	.99957	.02939	34.0273	2
42	.01222	.99993	.01222	81.8470	.02967	.99956	.02968	33.6935	2
43	.01251	.99992	.01251	79.9434	.02996	.99955	.02997	33.3662	2
44	.01280	.99992	.01280	78.1263	.03025	.99954	.03026	33.0452	2
45	.01309	.99991	.01309	76.3900	.03054	.99953	.03055	32.7303	2
46	.01338	.99991	.01338	74.7292	.03083	.99952	.03084	32.4213	2
47	.01367	.99991	.01367	73.1390	.03112	.99952	.03114	32.1181	2
48	.01396	.99990	.01396	71.6151	.03141	.99951	.03143	31.8205	2
49	.01425	.99990	.01425	70.1533	.03170	.99950	.03172	31.5284	2
50	.01454	.99989	.01455	68.7501	.03199	.99949	.03201	31.2416	1
51	.01483	.99989	.01484	67.4019	.03228	.99948	.03230	30.9599	1
52	.01513	.99989	.01513	66.1055	.03257	.99947	.03259	30.6833	1
53	.01542	.99988	.01542	64.8580	.03286	.99946	.03288	30.4116	1
54	.01571	.99988	.01571	63.6567	.03316	.99945	.03317	30.1446	1
55	.01600	.99987	.01600	62.4992	.03345	.99944	.03346	29.8823	1
56	.01629	.99987	.01629	61.3829	.03374	.99943	.03376	29.6245	1
57	.01658	.99986	.01658	60.3058	.03403	.99942	.03405	29.3711	1
58	.01687	.99986	.01687	59.2659	.03432	.99941	.03434	29.1220	1
59	.01716	.99985	.01716	58.2612	.03461	.99940	.03463	28.8771	1
60	.01745	.99985	.01746	57.2900	.03490	.99939	.03492	28.6363	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	

Inserted by permission from Webb's Railroad Construction.

	2°			3°				
	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
	.03490	.99939	.03492	28.6363	.05234	.99863	.05241	19.0811	60
	.03519	.99938	.03521	28.3994	.05263	.99861	.05270	18.9755	59
	.03548	.99937	.03550	28.1664	.05292	.99860	.05299	18.8711	58
	.03577	.99936	.03579	27.9372	.05321	.99858	.05328	18.7678	57
	.03606	.99935	.03609	27.7117	.05350	.99857	.05357	18.6656	56
	.03635	.99934	.03638	27.4899	.05379	.99855	.05387	18.5645	55
	.03664	.99933	.03667	27.2715	.05408	.99854	.05416	18.4645	54
	.03693	.99932	.03696	27.0566	.05437	.99852	.05445	18.3655	53
	.03723	.99931	.03725	26.8450	.05466	.99851	.05474	18.2677	52
	.03752	.99930	.03754	26.6367	.05495	.99849	.05503	18.1708	51
	.03781	.99929	.03783	26.4316	.05524	.99847	.05533	18.0750	50
	.03810	.99927	.03812	26.2296	.05553	.99846	.05562	17.9802	49
	.03839	.99926	.03842	26.0307	.05582	.99844	.05591	17.8863	48
	.03868	.99925	.03871	25.8348	.05611	.99842	.05620	17.7934	47
	.03897	.99924	.03900	25.6418	.05640	.99841	.05649	17.7015	46
	.03926	.99923	.03929	25.4517	.05669	.99839	.05678	17.6106	45
	.03955	.99922	.03958	25.2644	.05698	.99838	.05708	17.5205	44
	.03984	.99921	.03987	25.0798	.05727	.99836	.05737	17.4314	43
	.04013	.99919	.04016	24.8978	.05756	.99834	.05768	17.3432	42
	.04042	.99918	.04046	24.7185	.05785	.99833	.05795	17.2558	41
	.04071	.99917	.04075	24.5418	.05814	.99831	.05824	17.1693	40
	.04100	.99916	.04104	24.3675	.05844	.99829	.05854	17.0837	39
	.04129	.99915	.04133	24.1957	.05873	.99827	.05883	16.9990	38
	.04159	.99913	.04162	24.0263	.05902	.99826	.05912	16.9150	37
	.04188	.99912	.04191	23.8593	.05931	.99824	.05941	16.8319	36
	.04217	.99911	.04220	23.6945	.05960	.99822	.05970	16.7496	35
	.04246	.99910	.04250	23.5321	.05989	.99821	.05999	16.6681	34
	.04275	.99909	.04279	23.3718	.06018	.99819	.06029	16.5874	33
	.04304	.99907	.04308	23.2137	.06047	.99817	.06058	16.5075	32
	.04333	.99906	.04337	23.0577	.06076	.99815	.06087	16.4283	31
	.04362	.99905	.04366	22.9038	.06105	.99813	.06116	16.3499	30
	.04391	.99904	.04395	22.7519	.06134	.99812	.06145	16.2722	29
	.04420	.99902	.04424	22.6020	.06163	.99810	.06175	16.1952	28
	.04449	.99901	.04454	22.4541	.06192	.99808	.06204	16.1190	27
	.04478	.99900	.04483	22.3081	.06221	.99806	.06233	16.0435	26
	.04507	.99898	.04512	22.1640	.06250	.99804	.06262	15.9687	25
	.04536	.99897	.04541	22.0217	.06279	.99803	.06291	15.8945	24
	.04565	.99896	.04570	21.8813	.06308	.99801	.06321	15.8211	23
	.04594	.99894	.04599	21.7426	.06337	.99799	.06350	15.7483	22
	.04623	.99893	.04628	21.6056	.06366	.99797	.06379	15.6762	21
	.04653	.99892	.04658	21.4704	.06395	.99795	.06408	15.6048	20
	.04682	.99890	.04687	21.3369	.06424	.99793	.06437	15.5340	19
	.04711	.99889	.04716	21.2049	.06453	.99792	.06467	15.4638	18
	.04740	.99888	.04745	21.0747	.06482	.99790	.06496	15.3943	17
	.04769	.99886	.04774	20.9460	.06511	.99788	.06525	15.3254	16
	.04798	.99885	.04803	20.8188	.06540	.99786	.06554	15.2571	15
	.04827	.99883	.04833	20.6932	.06569	.99784	.06584	15.1893	14
	.04856	.99882	.04862	20.5691	.06598	.99782	.06613	15.1222	13
	.04885	.99881	.04891	20.4465	.06627	.99780	.06642	15.0557	12
	.04914	.99879	.04920	20.3253	.06656	.99778	.06671	14.9898	11
	.04943	.99878	.04949	20.2056	.06685	.99776	.06700	14.9244	10
	.04972	.99876	.04978	20.0872	.06714	.99774	.06730	14.8596	9
	.05001	.99875	.05007	19.9702	.06743	.99772	.06759	14.7954	8
	.05030	.99873	.05037	19.8546	.06773	.99770	.06788	14.7317	7
	.05059	.99872	.05066	19.7403	.06802	.99768	.06817	14.6685	6
	.05088	.99870	.05095	19.6273	.06831	.99766	.06847	14.6059	5
	.05117	.99869	.05124	19.5156	.06860	.99764	.06876	14.5438	4
	.05146	.99867	.05153	19.4051	.06889	.99762	.06905	14.4823	3
	.05175	.99866	.05182	19.2959	.06918	.99760	.06934	14.4212	2
	.05205	.99864	.05212	19.1879	.06947	.99758	.06963	14.3607	1
	.05234	.99863	.05241	19.0811	.06976	.99756	.06993	14.3007	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	87°						86°		

	4°				5°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	.06976	.99756	.06993	14.3007	.08716	.99619	.08749	11.4301
1	.07005	.99754	.07022	14.2411	.08745	.99617	.08778	11.3919
2	.07034	.99752	.07051	14.1821	.08774	.99614	.08807	11.3540
3	.07063	.99750	.07080	14.1235	.08803	.99612	.08837	11.3163
4	.07092	.99748	.07110	14.0655	.08831	.99609	.08866	11.2789
5	.07121	.99746	.07139	14.0079	.08860	.99607	.08895	11.2417
6	.07150	.99744	.07168	13.9507	.08889	.99604	.08925	11.2048
7	.07179	.99742	.07197	13.8940	.08918	.99602	.08954	11.1681
8	.07208	.99740	.07227	13.8378	.08947	.99599	.08983	11.1316
9	.07237	.99738	.07256	13.7821	.08976	.99596	.09013	11.0954
10	.07266	.99736	.07285	13.7267	.09005	.99594	.09042	11.0594
11	.07295	.99734	.07314	13.6719	.09034	.99591	.09071	11.0237
12	.07324	.99731	.07344	13.6174	.09063	.99588	.09101	10.9882
13	.07353	.99729	.07373	13.5634	.09092	.99586	.09130	10.9529
14	.07382	.99727	.07402	13.5098	.09121	.99583	.09159	10.9178
15	.07411	.99725	.07431	13.4566	.09150	.99580	.09189	10.8829
16	.07440	.99723	.07461	13.4039	.09179	.99578	.09218	10.8483
17	.07469	.99721	.07490	13.3515	.09208	.99575	.09247	10.8139
18	.07498	.99719	.07519	13.2996	.09237	.99572	.09277	10.7797
19	.07527	.99716	.07548	13.2480	.09266	.99570	.09306	10.7457
20	.07556	.99714	.07578	13.1969	.09295	.99567	.09335	10.7119
21	.07585	.99712	.07607	13.1461	.09324	.99564	.09365	10.6783
22	.07614	.99710	.07636	13.0958	.09353	.99562	.09394	10.6450
23	.07643	.99708	.07665	13.0458	.09382	.99559	.09423	10.6118
24	.07672	.99705	.07695	12.9962	.09411	.99556	.09453	10.5789
25	.07701	.99703	.07724	12.9469	.09440	.99553	.09482	10.5462
26	.07730	.99701	.07753	12.8981	.09469	.99551	.09511	10.5136
27	.07759	.99699	.07782	12.8496	.09498	.99548	.09541	10.4813
28	.07788	.99696	.07812	12.8014	.09527	.99545	.09570	10.4491
29	.07817	.99694	.07841	12.7536	.09556	.99542	.09600	10.4172
30	.07846	.99692	.07870	12.7062	.09585	.99540	.09629	10.3854
31	.07875	.99689	.07899	12.6591	.09614	.99537	.09658	10.3538
32	.07904	.99687	.07929	12.6124	.09642	.99534	.09688	10.3224
33	.07933	.99685	.07958	12.5660	.09671	.99531	.09717	10.2913
34	.07962	.99683	.07987	12.5199	.09700	.99528	.09746	10.2602
35	.07991	.99680	.08017	12.4742	.09729	.99526	.09776	10.2294
36	.08020	.99678	.08046	12.4288	.09758	.99523	.09805	10.1988
37	.08049	.99676	.08075	12.3838	.09787	.99520	.09834	10.1683
38	.08078	.99673	.08104	12.3390	.09816	.99517	.09864	10.1381
39	.08107	.99671	.08134	12.2946	.09845	.99514	.09893	10.1080
40	.08136	.99668	.08163	12.2505	.09874	.99511	.09923	10.0780
41	.08165	.99666	.08192	12.2067	.09903	.99508	.09952	10.0483
42	.08194	.99664	.08221	12.1632	.09932	.99506	.09981	10.0187
43	.08223	.99661	.08251	12.1201	.09961	.99503	.10011	9.98931
44	.08252	.99659	.08280	12.0772	.09990	.99500	.10040	9.96007
45	.08281	.99657	.08309	12.0346	.10019	.99497	.10069	9.93101
46	.08310	.99654	.08339	11.9923	.10048	.99494	.10099	9.90211
47	.08339	.99652	.08368	11.9504	.10077	.99491	.10128	9.87338
48	.08368	.99649	.08397	11.9087	.10106	.99488	.10158	9.84482
49	.08397	.99647	.08427	11.8673	.10135	.99485	.10187	9.81641
50	.08426	.99644	.08456	11.8262	.10164	.99482	.10216	9.78817
51	.08455	.99642	.08485	11.7853	.10192	.99479	.10246	9.76009
52	.08484	.99639	.08514	11.7448	.10221	.99476	.10275	9.73217
53	.08513	.99637	.08544	11.7045	.10250	.99473	.10305	9.70441
54	.08542	.99635	.08573	11.6645	.10279	.99470	.10334	9.67680
55	.08571	.99632	.08602	11.6248	.10308	.99467	.10363	9.64935
56	.08600	.99630	.08632	11.5853	.10337	.99464	.10393	9.62205
57	.08629	.99627	.08661	11.5461	.10366	.99461	.10422	9.59490
58	.08658	.99625	.08690	11.5072	.10395	.99458	.10452	9.56791
59	.08687	.99622	.08720	11.4685	.10424	.99455	.10481	9.54106
60	.08716	.99619	.08749	11.4301	.10453	.99452	.10510	9.51436
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.

	6°			7°				
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
.10453	.99452	.10510	9.51436	.12187	.99255	.12278	8.14435	60
.10482	.99449	.10540	9.48781	.12216	.99251	.12308	8.12481	59
.10511	.99446	.10569	9.46141	.12245	.99248	.12338	8.10536	58
.10540	.99443	.10599	9.43515	.12274	.99244	.12367	8.08600	57
.10569	.99440	.10628	9.40904	.12302	.99240	.12397	8.06674	56
.10597	.99437	.10657	9.38307	.12331	.99237	.12426	8.04756	55
.10626	.99434	.10687	9.35724	.12360	.99233	.12456	8.02848	54
.10655	.99431	.10716	9.33155	.12389	.99230	.12485	8.00948	53
.10684	.99428	.10746	9.30599	.12418	.99226	.12515	7.99058	52
.10713	.99424	.10775	9.28058	.12447	.99222	.12544	7.97176	51
.10742	.99421	.10805	9.25530	.12476	.99219	.12574	7.95302	50
.10771	.99418	.10834	9.23016	.12504	.99215	.12603	7.93438	49
.10800	.99415	.10863	9.20516	.12533	.99211	.12633	7.91582	48
.10829	.99412	.10893	9.18028	.12562	.99208	.12662	7.89734	47
.10858	.99409	.10922	9.15554	.12591	.99204	.12692	7.87895	46
.10887	.99406	.10952	9.13093	.12620	.99200	.12722	7.86064	45
.10916	.99402	.10981	9.10646	.12649	.99197	.12751	7.84242	44
.10945	.99399	.11011	9.08211	.12678	.99193	.12781	7.82428	43
.10973	.99396	.11040	9.05789	.12706	.99189	.12810	7.80622	42
.11002	.99393	.11070	9.03379	.12735	.99186	.12840	7.78825	41
.11031	.99390	.11099	9.00983	.12764	.99182	.12869	7.77035	40
.11060	.99386	.11128	8.98598	.12793	.99178	.12899	7.75254	39
.11089	.99383	.11158	8.96227	.12822	.99175	.12929	7.73480	38
.11118	.99380	.11187	8.93867	.12851	.99171	.12958	7.71715	37
.11147	.99377	.11217	8.91520	.12880	.99167	.12988	7.69957	36
.11176	.99374	.11246	8.89185	.12908	.99163	.13017	7.68208	35
.11205	.99370	.11276	8.86862	.12937	.99160	.13047	7.66466	34
.11234	.99367	.11305	8.84551	.12966	.99156	.13076	7.64732	33
.11263	.99364	.11335	8.82252	.12995	.99152	.13106	7.63005	32
.11291	.99360	.11364	8.79964	.13024	.99148	.13136	7.61287	31
.11320	.99357	.11394	8.77689	.13053	.99144	.13165	7.59575	30
.11349	.99354	.11423	8.75425	.13081	.99141	.13195	7.57872	29
.11378	.99351	.11452	8.73172	.13110	.99137	.13224	7.56176	28
.11407	.99347	.11482	8.70931	.13139	.99133	.13254	7.54487	27
.11436	.99344	.11511	8.68701	.13168	.99129	.13284	7.52806	26
.11465	.99341	.11541	8.66482	.13197	.99125	.13313	7.51132	25
.11494	.99337	.11570	8.64275	.13226	.99122	.13343	7.49465	24
.11523	.99334	.11600	8.62078	.13254	.99118	.13372	7.47806	23
.11552	.99331	.11629	8.59893	.13283	.99114	.13402	7.46154	22
.11580	.99327	.11659	8.57718	.13312	.99110	.13432	7.44509	21
.11609	.99324	.11688	8.55555	.13341	.99106	.13461	7.42871	20
.11638	.99320	.11718	8.53402	.13370	.99102	.13491	7.41240	19
.11667	.99317	.11747	8.51259	.13399	.99098	.13521	7.39616	18
.11696	.99314	.11777	8.49128	.13427	.99094	.13550	7.37999	17
.11725	.99310	.11806	8.47007	.13456	.99091	.13580	7.36389	16
.11754	.99307	.11836	8.44896	.13485	.99087	.13609	7.34786	15
.11783	.99303	.11865	8.42795	.13514	.99083	.13639	7.33190	14
.11812	.99300	.11895	8.40705	.13543	.99079	.13669	7.31600	13
.11840	.99297	.11924	8.38625	.13572	.99075	.13698	7.30018	12
.11869	.99293	.11954	8.36555	.13600	.99071	.13728	7.28442	11
.11898	.99290	.11983	8.34496	.13629	.99067	.13758	7.26873	10
.11927	.99286	.12013	8.32446	.13658	.99063	.13787	7.25310	9
.11956	.99283	.12042	8.30406	.13687	.99059	.13817	7.23754	8
.11985	.99279	.12072	8.28376	.13716	.99055	.13846	7.22204	7
.12014	.99276	.12101	8.26355	.13744	.99051	.13876	7.20661	6
.12043	.99272	.12131	8.24345	.13773	.99047	.13906	7.19125	5
.12071	.99269	.12160	8.22344	.13802	.99043	.13935	7.17594	4
.12100	.99265	.12190	8.20352	.13831	.99039	.13965	7.16071	3
.12129	.99262	.12219	8.18370	.13860	.99035	.13995	7.14553	2
.12158	.99258	.12249	8.16398	.13889	.99031	.14024	7.13042	1
.12187	.99255	.12278	8.14435	.13917	.99027	.14054	7.11537	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	8°				9°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	·13917	·99027	·14054	7·11537	·15643	·98769	·15838	6·31375
1	·13946	·99023	·14084	7·10038	·15672	·98764	·15868	6·30189
2	·13975	·99019	·14113	7·08546	·15701	·98760	·15898	6·29007
3	·14004	·99015	·14143	7·07059	·15730	·98755	·15928	6·27829
4	·14033	·99011	·14173	7·05579	·15758	·98751	·15958	6·26655
5	·14061	·99006	·14202	7·04105	·15787	·98746	·15988	6·25486
6	·14090	·99002	·14232	7·02637	·15816	·98741	·16017	6·24321
7	·14119	·98998	·14262	7·01174	·15845	·98737	·16047	6·23160
8	·14148	·98994	·14291	6·99718	·15873	·98732	·16077	6·22003
9	·14177	·98990	·14321	6·98268	·15902	·98728	·16107	6·20851
10	·14205	·98986	·14351	6·96823	·15931	·98723	·16137	6·19703
11	·14234	·98982	·14381	6·95385	·15959	·98718	·16167	6·18559
12	·14263	·98978	·14410	6·93952	·15938	·98714	·16196	6·17419
13	·14292	·98973	·14440	6·92525	·16017	·98709	·16226	6·16283
14	·14320	·98969	·14470	6·91104	·16046	·98704	·16256	6·15151
15	·14349	·98965	·14499	6·89688	·16074	·98700	·16286	6·14023
16	·14378	·98961	·14529	6·88278	·16103	·98695	·16316	6·12899
17	·14407	·98957	·14559	6·86874	·16132	·98690	·16346	6·11779
18	·14436	·98953	·14588	6·85475	·16160	·98686	·16376	6·10664
19	·14464	·98948	·14618	6·84082	·16189	·98681	·16405	6·09552
20	·14493	·98944	·14648	6·82694	·16218	·98676	·16435	6·08444
21	·14522	·98940	·14678	6·81312	·16246	·98671	·16465	6·07340
22	·14551	·98936	·14707	6·79936	·16275	·98667	·16495	6·06240
23	·14580	·98931	·14737	6·78564	·16304	·98662	·16525	6·05143
24	·14608	·98927	·14767	6·77199	·16333	·98657	·16555	6·04051
25	·14637	·98923	·14796	6·75838	·16361	·98652	·16585	6·02962
26	·14666	·98919	·14826	6·74483	·16390	·98648	·16615	6·01878
27	·14695	·98914	·14856	6·73133	·13419	·98643	·16645	6·00797
28	·14723	·98910	·14886	6·71789	·16447	·98638	·16674	5·99720
29	·14752	·98906	·14915	6·70450	·16476	·98633	·16704	5·98646
30	·14781	·98902	·14945	6·69116	·16505	·98629	·16734	5·97576
31	·14810	·98897	·14975	6·67787	·16533	·98624	·16764	5·96510
32	·14838	·98893	·15005	6·66463	·16562	·98619	·16794	5·95448
33	·14867	·98889	·15034	6·65144	·16591	·98614	·16824	5·94390
34	·14896	·98884	·15064	6·63831	·16620	·98609	·16854	5·93335
35	·14925	·98880	·15094	6·62523	·16648	·98604	·16884	5·92283
36	·14954	·98876	·15124	6·61219	·16677	·98600	·16914	5·91236
37	·14982	·98871	·15153	6·59921	·16706	·98595	·16944	5·90191
38	·15011	·98867	·15183	6·58627	·16734	·98590	·16974	5·89151
39	·15040	·98863	·15213	6·57339	·16763	·98585	·17004	5·88114
40	·15069	·98858	·15243	6·56055	·16792	·98580	·17033	5·87080
41	·15097	·98854	·15272	6·54777	·16820	·98575	·17063	5·86051
42	·15126	·98849	·15302	6·53503	·16849	·98570	·17093	5·85024
43	·15155	·98845	·15332	6·52234	·16878	·98565	·17123	5·84001
44	·15184	·98841	·15362	6·50970	·16906	·98561	·17153	5·82982
45	·15212	·98836	·15391	6·49710	·16935	·98556	·17183	5·81966
46	·15241	·98832	·15421	6·48456	·16964	·98551	·17213	5·80953
47	·15270	·98827	·15451	6·47206	·16992	·98546	·17243	5·79944
48	·15299	·98823	·15481	6·45961	·17021	·98541	·17273	5·78938
49	·15327	·98818	·15511	6·44720	·17050	·98536	·17303	5·77936
50	·15356	·98814	·15540	6·43484	·17078	·98531	·17333	5·76937
51	·15385	·98809	·15570	6·42253	·17107	·98526	·17363	5·75941
52	·15414	·98805	·15600	6·41026	·17136	·98521	·17393	5·74949
53	·15442	·98800	·15630	6·39804	·17164	·98516	·17423	5·73960
54	·15471	·98796	·15660	6·38587	·17193	·98511	·17453	5·72974
55	·15500	·98791	·15689	6·37374	·17222	·98506	·17483	5·71992
56	·15529	·98787	·15719	6·36165	·17250	·98501	·17513	5·71013
57	·15557	·98782	·15749	6·34961	·17279	·98496	·17543	5·70037
58	·15586	·98778	·15779	6·33761	·17308	·98491	·17573	5·69064
59	·15615	·98773	·15809	6·32566	·17336	·98486	·17603	5·68094
60	·15643	·98769	·15838	6·31375	·17365	·98481	·17633	5·67128
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.

	10°			11°				
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
.17365	.98481	.17633	5.67128	.19081	.98163	.19438	5.14455	60
.17393	.98476	.17663	5.66165	.19109	.98157	.19468	5.13658	59
.17422	.98471	.17693	5.65205	.19138	.98152	.19498	5.12862	58
.17451	.98466	.17723	5.64248	.19167	.98146	.19529	5.12069	57
.17479	.98461	.17753	5.63295	.19195	.98140	.19559	5.11279	56
.17508	.98455	.17783	5.62344	.19224	.98135	.19589	5.10490	55
.17537	.98450	.17813	5.61397	.19252	.98129	.19619	5.09704	54
.17565	.98445	.17843	5.60452	.19281	.98124	.19649	5.08921	53
.17594	.98440	.17873	5.59511	.19309	.98118	.19680	5.08139	52
.17623	.98435	.17903	5.58573	.19338	.98112	.19710	5.07360	51
.17651	.98430	.17933	5.57638	.19366	.98107	.19740	5.06584	50
.17680	.98425	.17963	5.56706	.19395	.98101	.19770	5.05809	49
.17708	.98420	.17993	5.55777	.19423	.98096	.19801	5.05037	48
.17737	.98414	.18023	5.54851	.19452	.98090	.19831	5.04267	47
.17766	.98409	.18053	5.53927	.19481	.98084	.19861	5.03499	46
.17794	.98404	.18083	5.53007	.19509	.98079	.19891	5.02734	45
.17823	.98399	.18113	5.52090	.19538	.98073	.19921	5.01971	44
.17852	.98394	.18143	5.51176	.19566	.98067	.19952	5.01210	43
.17880	.98389	.18173	5.50264	.19595	.98061	.19982	5.00451	42
.17909	.98383	.18203	5.49356	.19623	.98056	.20012	4.99695	41
.17937	.98378	.18233	5.48451	.19652	.98050	.20042	4.98940	40
.17966	.98373	.18263	5.47548	.19680	.98044	.20073	4.98188	39
.17995	.98368	.18293	5.46648	.19709	.98039	.20103	4.97438	38
.18023	.98362	.18323	5.45751	.19737	.98033	.20133	4.96690	37
.18052	.98357	.18353	5.44857	.19766	.98027	.20164	4.95945	36
.18081	.98352	.18384	5.43966	.19794	.98021	.20194	4.95201	35
.18109	.98347	.18414	5.43077	.19823	.98016	.20224	4.94460	34
.18138	.98341	.18444	5.42192	.19851	.98010	.20254	4.93721	33
.18166	.98336	.18474	5.41309	.19880	.98004	.20285	4.92984	32
.18195	.98331	.18504	5.40429	.19908	.97998	.20315	4.92249	31
.18224	.98325	.18534	5.39552	.19937	.97992	.20345	4.91516	30
.18252	.98320	.18564	5.38677	.19965	.97987	.20376	4.90785	29
.18281	.98315	.18594	5.37805	.19994	.97981	.20406	4.90056	28
.18309	.98310	.18624	5.36936	.20022	.97975	.20438	4.89330	27
.18338	.98304	.18654	5.36070	.20051	.97969	.20466	4.88605	26
.18367	.98299	.18684	5.35206	.20079	.97963	.20497	4.87882	25
.18395	.98294	.18714	5.34345	.20108	.97958	.20527	4.87162	24
.18424	.98288	.18745	5.33487	.20136	.97952	.20557	4.86444	23
.18452	.98283	.18775	5.32631	.20165	.97946	.20588	4.85727	22
.18481	.98277	.18805	5.31778	.20193	.97940	.20618	4.85013	21
.18509	.98272	.18835	5.30928	.20222	.97934	.20648	4.84300	20
.18538	.98267	.18865	5.30080	.20250	.97928	.20679	4.83590	19
.18567	.98261	.18895	5.29235	.20279	.97922	.20709	4.82882	18
.18595	.98256	.18925	5.28393	.20307	.97916	.20739	4.82175	17
.18624	.98250	.18955	5.27553	.20336	.97910	.20770	4.81471	16
.18652	.98245	.18986	5.26715	.20364	.97905	.20800	4.80769	15
.18681	.98240	.19016	5.25880	.20393	.97899	.20830	4.80068	14
.18710	.98234	.19046	5.25048	.20421	.97893	.20861	4.79370	13
.18738	.98229	.19076	5.24218	.20450	.97887	.20891	4.78673	12
.18767	.98223	.19106	5.23391	.20478	.97881	.20921	4.77978	11
.18795	.98218	.19136	5.22566	.20507	.97875	.20952	4.77286	10
.18824	.98212	.19166	5.21744	.20535	.97869	.20982	4.76595	9
.18852	.98207	.19197	5.20925	.20563	.97863	.21013	4.75906	8
.18881	.98201	.19227	5.20107	.20592	.97857	.21043	4.75219	7
.18910	.98196	.19257	5.19293	.20620	.97851	.21073	4.74534	6
.18938	.98190	.19287	5.18480	.20649	.97845	.21104	4.73851	5
.18967	.98185	.19317	5.17671	.20677	.97839	.21134	4.73170	4
.18995	.98179	.19347	5.16863	.20706	.97833	.21164	4.72490	3
.19024	.98174	.19378	5.16058	.20734	.97827	.21195	4.71813	2
.19052	.98168	.19408	5.15256	.20763	.97821	.21225	4.71137	1
.19081	.98163	.19438	5.14455	.20791	.97815	.21256	4.70463	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

TABLE 17.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

	12°				13°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	.20791	.97815	.21256	4.70463	.22495	.97437	.23087	4.33148
1	.20820	.97809	.21286	4.69791	.22523	.97430	.23117	4.32573
2	.20848	.97803	.21316	4.69121	.22552	.97424	.23148	4.32001
3	.20877	.97797	.21347	4.68452	.22580	.97417	.23179	4.31430
4	.20905	.97791	.21377	4.67786	.22608	.97411	.23209	4.30860
5	.20933	.97784	.21408	4.67121	.22637	.97404	.23240	4.30291
6	.20962	.97778	.21438	4.66458	.22665	.97398	.23271	4.29724
7	.20990	.97772	.21469	4.65797	.22693	.97391	.23301	4.29159
8	.21019	.97766	.21499	4.65138	.22722	.97384	.23332	4.28595
9	.21047	.97760	.21529	4.64480	.22750	.97378	.23363	4.28032
10	.21076	.97754	.21560	4.63825	.22778	.97371	.23393	4.27471
11	.21104	.97748	.21590	4.63171	.22807	.97365	.23424	4.26911
12	.21132	.97742	.21621	4.62518	.22835	.97358	.23455	4.26352
13	.21161	.97735	.21651	4.61868	.22863	.97351	.23485	4.25795
14	.21189	.97729	.21682	4.61219	.22892	.97345	.23516	4.25239
15	.21218	.97723	.21712	4.60572	.22920	.97338	.23547	4.24685
16	.21246	.97717	.21743	4.59927	.22948	.97331	.23578	4.24132
17	.21275	.97711	.21773	4.59283	.22977	.97325	.23608	4.23580
18	.21303	.97705	.21804	4.58641	.23005	.97318	.23639	4.23030
19	.21331	.97698	.21834	4.58001	.23033	.97311	.23670	4.22481
20	.21360	.97692	.21864	4.57363	.23062	.97304	.23700	4.21933
21	.21388	.97686	.21895	4.56726	.23090	.97298	.23731	4.21387
22	.21417	.97680	.21925	4.56091	.23118	.97291	.23762	4.20842
23	.21445	.97673	.21956	4.55458	.23146	.97284	.23793	4.20298
24	.21474	.97667	.21986	4.54826	.23175	.97278	.23823	4.19756
25	.21502	.97661	.22017	4.54196	.23203	.97271	.23854	4.19215
26	.21530	.97655	.22047	4.53568	.23231	.97264	.23885	4.18675
27	.21559	.97648	.22078	4.52941	.23260	.97257	.23916	4.18137
28	.21587	.97642	.22108	4.52316	.23288	.97251	.23946	4.17600
29	.21616	.97636	.22139	4.51693	.23316	.97244	.23977	4.17064
30	.21644	.97630	.22169	4.51071	.23345	.97237	.24008	4.16530
31	.21672	.97623	.22200	4.50451	.23373	.97230	.24039	4.15997
32	.21701	.97617	.22231	4.49832	.23401	.97223	.24069	4.15465
33	.21729	.97611	.22261	4.49215	.23429	.97217	.24100	4.14934
34	.21758	.97604	.22292	4.48600	.23458	.97210	.24131	4.14405
35	.21786	.97598	.22322	4.47986	.23486	.97203	.24162	4.13877
36	.21814	.97592	.22353	4.47374	.23514	.97196	.24193	4.13350
37	.21843	.97585	.22383	4.46764	.23542	.97189	.24223	4.12825
38	.21871	.97579	.22414	4.46155	.23571	.97182	.24254	4.12301
39	.21899	.97573	.22444	4.45548	.23599	.97176	.24285	4.11778
40	.21928	.97566	.22475	4.44942	.23627	.97169	.24316	4.11256
41	.21956	.97560	.22505	4.44338	.23656	.97162	.24347	4.10736
42	.21985	.97553	.22536	4.43735	.23684	.97155	.24377	4.10216
43	.22013	.97547	.22567	4.43134	.23712	.97148	.24408	4.09699
44	.22041	.97541	.22597	4.42534	.23740	.97141	.24439	4.09182
45	.22070	.97534	.22628	4.41936	.23769	.97134	.24470	4.08666
46	.22098	.97528	.22658	4.41340	.23797	.97127	.24501	4.08152
47	.22126	.97521	.22689	4.40745	.23825	.97120	.24532	4.07639
48	.22155	.97515	.22719	4.40152	.23853	.97113	.24562	4.07127
49	.22183	.97508	.22750	4.39560	.23882	.97106	.24593	4.06616
50	.22212	.97502	.22781	4.38969	.23910	.97100	.24624	4.06107
51	.22240	.97496	.22811	4.38381	.23938	.97093	.24655	4.05599
52	.22268	.97489	.22842	4.37793	.23966	.97086	.24686	4.05092
53	.22297	.97483	.22872	4.37207	.23995	.97079	.24717	4.04586
54	.22325	.97476	.22903	4.36623	.24023	.97072	.24747	4.04081
55	.22353	.97470	.22934	4.36040	.24051	.97065	.24778	4.03578
56	.22382	.97463	.22964	4.35459	.24079	.97058	.24809	4.03076
57	.22410	.97457	.22995	4.34879	.24108	.97051	.24840	4.02574
58	.22438	.97450	.23026	4.34300	.24136	.97044	.24871	4.02074
59	.22467	.97444	.23056	4.33723	.24164	.97037	.24902	4.01576
60	.22495	.97437	.23087	4.33148	.24192	.97030	.24933	4.01078
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.

	14°			15°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
60	.24192	.97030	.24933	4.01078	.25882	.96593	.26795	3.73205
59	.24220	.97023	.24964	4.00582	.25910	.96585	.26826	3.72771
58	.24249	.97015	.24995	4.00086	.25938	.96578	.26857	3.72338
57	.24277	.97008	.25026	3.99592	.25966	.96570	.26888	3.71907
56	.24305	.97001	.25056	3.99099	.25994	.96562	.26920	3.71476
55	.24333	.96994	.25087	3.98607	.26022	.96555	.26951	3.71046
54	.24362	.96987	.25118	3.98117	.26050	.96547	.26982	3.70616
53	.24390	.96980	.25149	3.97627	.26079	.96540	.27013	3.70188
52	.24418	.96973	.25180	3.97139	.26107	.96532	.27044	3.69761
51	.24446	.96966	.25211	3.96651	.26135	.96524	.27076	3.69335
50	.24474	.96959	.25242	3.96165	.26163	.96517	.27107	3.68909
49	.24503	.96952	.25273	3.95680	.26191	.96509	.27138	3.68485
48	.24531	.96945	.25304	3.95196	.26219	.96502	.27169	3.68061
47	.24559	.96937	.25335	3.94713	.26247	.96494	.27201	3.67638
46	.24587	.96930	.25366	3.94232	.26275	.96486	.27232	3.67217
45	.24615	.96923	.25397	3.93751	.26303	.96479	.27263	3.66796
44	.24644	.96916	.25428	3.93271	.26331	.96471	.27294	3.66376
43	.24672	.96909	.25459	3.92793	.26359	.96463	.27326	3.65957
42	.24700	.96902	.25490	3.92316	.26387	.96456	.27357	3.65538
41	.24728	.96894	.25521	3.91839	.26415	.96448	.27388	3.65121
40	.24756	.96887	.25552	3.91364	.26443	.96440	.27419	3.64705
39	.24784	.96880	.25583	3.90890	.26471	.96433	.27451	3.64289
38	.24813	.96873	.25614	3.90417	.26500	.96425	.27482	3.63874
37	.24841	.96866	.25645	3.89945	.26528	.96417	.27513	3.63461
36	.24869	.96858	.25678	3.89474	.26556	.96410	.27545	3.63048
35	.24897	.96851	.25707	3.89004	.26584	.96402	.27576	3.62636
34	.24925	.96844	.25738	3.88536	.26612	.96394	.27607	3.62224
33	.24954	.96837	.25769	3.88068	.26640	.96386	.27638	3.61814
32	.24982	.96829	.25800	3.87601	.26668	.96379	.27670	3.61405
31	.25010	.96822	.25831	3.87136	.26696	.96371	.27701	3.60996
30	.25038	.96815	.25862	3.86671	.26724	.96363	.27732	3.60588
29	.25066	.96807	.25893	3.86208	.26752	.96355	.27764	3.60181
28	.25094	.96800	.25924	3.85745	.26780	.96347	.27795	3.59775
27	.25122	.96793	.25955	3.85284	.26808	.96340	.27826	3.59370
26	.25151	.96786	.25986	3.84824	.26836	.96332	.27858	3.58966
25	.25179	.96778	.26017	3.84364	.26864	.96324	.27889	3.58562
24	.25207	.96771	.26048	3.83906	.26892	.96316	.27921	3.58160
23	.25235	.96764	.26079	3.83449	.26920	.96308	.27952	3.57758
22	.25263	.96756	.26110	3.82992	.26948	.96301	.27983	3.57357
21	.25291	.96749	.26141	3.82537	.26976	.96293	.28015	3.56957
20	.25320	.96742	.26172	3.82083	.27004	.96285	.28046	3.56557
19	.25348	.96734	.26203	3.81630	.27032	.96277	.28077	3.56159
18	.25376	.96727	.26235	3.81177	.27060	.96269	.28109	3.55761
17	.25404	.96719	.26266	3.80726	.27088	.96261	.28140	3.55364
16	.25432	.96712	.26297	3.80276	.27116	.96253	.28172	3.54968
15	.25460	.96705	.26328	3.79827	.27144	.96246	.28203	3.54573
14	.25488	.96697	.26359	3.79378	.27172	.96238	.28234	3.54179
13	.25516	.96690	.26390	3.78931	.27200	.96230	.28266	3.53785
12	.25545	.96682	.26421	3.78485	.27228	.96222	.28297	3.53393
11	.25573	.96675	.26452	3.78040	.27256	.96214	.28329	3.53001
10	.25601	.96667	.26483	3.77595	.27284	.96206	.28360	3.52609
9	.25629	.96660	.26515	3.77152	.27312	.96198	.28391	3.52219
8	.25657	.96653	.26546	3.76709	.27340	.96190	.28423	3.51829
7	.25685	.96645	.26577	3.76268	.27368	.96182	.28454	3.51441
6	.25713	.96638	.26608	3.75828	.27396	.96174	.28486	3.51053
5	.25741	.96630	.26639	3.75388	.27424	.96166	.28517	3.50666
4	.25769	.96623	.26670	3.74950	.27452	.96158	.28549	3.50279
3	.25798	.96615	.26701	3.74512	.27480	.96150	.28580	3.49894
2	.25826	.96608	.26733	3.74075	.27508	.96142	.28612	3.49509
1	.25854	.96600	.26764	3.73640	.27536	.96134	.28643	3.49125
0	.25882	.96593	.26795	3.73205	.27564	.96126	.28675	3.48741
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.

TABLE 17.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

	16°				17°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	.27564	.96126	.28675	3.48741	.29237	.95630	.30573	3.27085
1	.27592	.96118	.28706	3.48359	.29265	.95622	.30605	3.26745
2	.27620	.96110	.28738	3.47977	.29293	.95613	.30637	3.26406
3	.27648	.96102	.28769	3.47596	.29321	.95605	.30669	3.26067
4	.27676	.96094	.28800	3.47216	.29348	.95596	.30700	3.25729
5	.27704	.96086	.28832	3.46837	.29376	.95588	.30732	3.25392
6	.27731	.96078	.28864	3.46458	.29404	.95579	.30764	3.25055
7	.27759	.96070	.28895	3.46080	.29432	.95571	.30796	3.24719
8	.27787	.96062	.28927	3.45703	.29460	.95562	.30828	3.24383
9	.27815	.96054	.28958	3.45327	.29487	.95554	.30860	3.24049
10	.27843	.96046	.28990	3.44951	.29515	.95545	.30891	3.23714
11	.27871	.96037	.29021	3.44576	.29543	.95536	.30923	3.23381
12	.27899	.96029	.29053	3.44202	.29571	.95528	.30955	3.23048
13	.27927	.96021	.29084	3.43829	.29599	.95519	.30987	3.22715
14	.27955	.96013	.29116	3.43456	.29626	.95511	.31019	3.22384
15	.27983	.96005	.29147	3.43084	.29654	.95502	.31051	3.22053
16	.28011	.95997	.29179	3.42713	.29682	.95493	.31083	3.21722
17	.28039	.95989	.29210	3.42343	.29710	.95485	.31115	3.21392
18	.28067	.95981	.29242	3.41973	.29737	.95476	.31147	3.21063
19	.28095	.95972	.29274	3.41604	.29765	.95467	.31178	3.20734
20	.28123	.95964	.29305	3.41236	.29793	.95459	.31210	3.20406
21	.28150	.95956	.29337	3.40869	.29821	.95450	.31242	3.20079
22	.28178	.95948	.29368	3.40502	.29849	.95441	.31274	3.19752
23	.28206	.95940	.29400	3.40136	.29876	.95433	.31306	3.19426
24	.28234	.95931	.29432	3.39771	.29904	.95424	.31338	3.19100
25	.28262	.95923	.29463	3.39406	.29932	.95415	.31370	3.18775
26	.28290	.95915	.29495	3.39042	.29960	.95407	.31402	3.18451
27	.28318	.95907	.29526	3.38679	.29987	.95398	.31434	3.18127
28	.28346	.95898	.29558	3.38317	.30015	.95389	.31466	3.17804
29	.28374	.95890	.29590	3.37955	.30043	.95380	.31498	3.17481
30	.28402	.95882	.29621	3.37594	.30071	.95372	.31530	3.17159
31	.28429	.95874	.29653	3.37234	.30098	.95363	.31562	3.16838
32	.28457	.95865	.29685	3.36875	.30126	.95354	.31594	3.16517
33	.28485	.95857	.29716	3.36516	.30154	.95345	.31626	3.16197
34	.28513	.95849	.29748	3.36158	.30182	.95337	.31658	3.15877
35	.28541	.95841	.29780	3.35800	.30209	.95328	.31690	3.15558
36	.28569	.95832	.29811	3.35443	.30237	.95319	.31722	3.15240
37	.28597	.95824	.29843	3.35087	.30265	.95310	.31754	3.14922
38	.28625	.95816	.29875	3.34732	.30292	.95301	.31786	3.14605
39	.28652	.95807	.29906	3.34377	.30320	.95293	.31818	3.14288
40	.28680	.95799	.29938	3.34023	.30348	.95284	.31850	3.13972
41	.28708	.95791	.29970	3.33670	.30376	.95275	.31882	3.13656
42	.28736	.95782	.30001	3.33317	.30403	.95266	.31914	3.13341
43	.28764	.95774	.30033	3.32965	.30431	.95257	.31946	3.13027
44	.28792	.95766	.30065	3.32614	.30459	.95248	.31978	3.12713
45	.28820	.95757	.30097	3.32264	.30486	.95240	.32010	3.12400
46	.28847	.95749	.30128	3.31914	.30514	.95231	.32042	3.12087
47	.28875	.95740	.30160	3.31565	.30542	.95222	.32074	3.11775
48	.28903	.95732	.30192	3.31216	.30570	.95213	.32106	3.11464
49	.28931	.95724	.30224	3.30868	.30597	.95204	.32139	3.11153
50	.28959	.95715	.30255	3.30521	.30625	.95195	.32171	3.10842
51	.28987	.95707	.30287	3.30174	.30653	.95186	.32203	3.10532
52	.29015	.95698	.30319	3.29829	.30680	.95177	.32235	3.10223
53	.29042	.95690	.30351	3.29483	.30708	.95168	.32267	3.09914
54	.29070	.95681	.30382	3.29139	.30736	.95159	.32299	3.09606
55	.29098	.95673	.30414	3.28795	.30763	.95150	.32331	3.09298
56	.29126	.95664	.30446	3.28452	.30791	.95142	.32363	3.08991
57	.29154	.95656	.30478	3.28109	.30819	.95133	.32396	3.08685
58	.29182	.95647	.30509	3.27767	.30846	.95124	.32428	3.08379
59	.29209	.95639	.30541	3.27426	.30874	.95115	.32460	3.08073
60	.29237	.95630	.30573	3.27085	.30902	.95106	.32492	3.07768
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.

73°	72°

	18°			19°				
	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
	·30902	·95106	·32492	3·07768	·32557	·94552	·34433	2·90421	60
	·30929	·95097	·32524	3·07464	·32584	·94542	·34465	2·90147	59
	·30957	·95088	·32556	3·07160	·32612	·94533	·34498	2·89873	58
	·30985	·95079	·32588	3·06857	·32639	·94523	·34530	2·89600	57
	·31012	·95070	·32621	3·06554	·32667	·94514	·34563	2·89327	56
	·31040	·95061	·32653	3·06252	·32694	·94504	·34596	2·89055	55
	·31068	·95052	·32685	3·05950	·32722	·94495	·34628	2·88783	54
	·31095	·95043	·32717	3·05649	·32749	·94485	·34661	2·88511	53
	·31123	·95033	·32749	3·05349	·32777	·94476	·34693	2·88240	52
	·31151	·95024	·32782	3·05049	·32804	·94466	·34726	2·87970	51
	·31178	·95015	·32814	3·04749	·32832	·94457	·34758	2·87700	50
	·31206	·95006	·32846	3·04450	·32859	·94447	·34791	2·87430	49
	·31233	·94997	·32878	3·04152	·32887	·94438	·34824	2·87161	48
	·31261	·94988	·32911	3·03854	·32914	·94428	·34856	2·86892	47
	·31289	·94979	·32943	3·03556	·32942	·94418	·34889	2·86624	46
	·31316	·94970	·32975	3·03260	·32969	·94409	·34922	2·86356	45
	·31344	·94961	·33007	3·02963	·32997	·94399	·34954	2·86089	44
	·31372	·94952	·33040	3·02667	·33024	·94390	·34987	2·85822	43
	·31399	·94943	·33072	3·02372	·33051	·94380	·35020	2·85555	42
	·31427	·94933	·33104	3·02077	·33079	·94370	·35052	2·85289	41
	·31454	·94924	·33136	3·01783	·33106	·94361	·35085	2·85023	40
	·31482	·94915	·33169	3·01489	·33134	·94351	·35118	2·84758	39
	·31510	·94906	·33201	3·01196	·33161	·94342	·35150	2·84494	38
	·31537	·94897	·33233	3·00903	·33189	·94332	·35183	2·84229	37
	·31565	·94888	·33266	3·00611	·33216	·94322	·35216	2·83965	36
	·31593	·94878	·33298	3·00319	·33244	·94313	·35248	2·83702	35
	·31620	·94869	·33330	3·00028	·33271	·94303	·35281	2·83439	34
	·31648	·94860	·33363	2·99738	·33298	·94293	·35314	2·83176	33
	·31675	·94851	·33395	2·99447	·33326	·94284	·35346	2·82914	32
	·31703	·94842	·33427	2·99158	·33353	·94274	·35379	2·82653	31
	·31730	·94832	·33460	2·98868	·33381	·94264	·35412	2·82391	30
	·31758	·94823	·33492	2·98580	·33408	·94254	·35445	2·82130	29
	·31786	·94814	·33524	2·98292	·33436	·94245	·35477	2·81870	28
	·31813	·94805	·33557	2·98004	·33463	·94235	·35510	2·81610	27
	·31841	·94795	·33589	2·97717	·33490	·94225	·35543	2·81350	26
	·31868	·94786	·33621	2·97430	·33518	·94215	·35576	2·81091	25
	·31896	·94777	·33654	2·97144	·33545	·94206	·35608	2·80833	24
	·31923	·94768	·33686	2·96858	·33573	·94196	·35641	2·80574	23
	·31951	·94758	·33718	2·96573	·33600	·94186	·35674	2·80316	22
	·31979	·94749	·33751	2·96288	·33627	·94176	·35707	2·80059	21
	·32006	·94740	·33783	2·96004	·33655	·94167	·35740	2·79802	20
	·32034	·94730	·33816	2·95721	·33682	·94157	·35772	2·79545	19
	·32061	·94721	·33848	2·95437	·33710	·94147	·35805	2·79289	18
	·32089	·94712	·33881	2·95155	·33737	·94137	·35838	2·79033	17
	·32116	·94702	·33913	2·94872	·33764	·94127	·35871	2·78778	16
	·32144	·94693	·33945	2·94591	·33792	·94118	·35904	2·78523	15
	·32171	·94684	·33978	2·94309	·33819	·94108	·35937	2·78269	14
	·32199	·94674	·34010	2·94028	·33846	·94098	·35969	2·78014	13
	·32227	·94665	·34043	2·93748	·33874	·94088	·36002	2·77761	12
	·32254	·94656	·34075	2·93468	·33901	·94078	·36035	2·77507	11
	·32282	·94646	·34108	2·93189	·33929	·94068	·36068	2·77254	10
	·32309	·94637	·34140	2·92910	·33956	·94058	·36101	2·77002	9
	·32337	·94627	·34173	2·92632	·33983	·94049	·36134	2·76750	8
	·32364	·94618	·34205	2·92354	·34011	·94039	·36167	2·76498	7
	·32392	·94609	·34238	2·92076	·34038	·94029	·36199	2·76247	6
	·32419	·94599	·34270	2·91799	·34065	·94019	·36232	2·75996	5
	·32447	·94590	·34303	2·91523	·34093	·94009	·36265	2·75746	4
	·32474	·94580	·34335	2·91246	·34120	·93999	·36298	2·75496	3
	·32502	·94571	·34368	2·90971	·34147	·93989	·36331	2·75246	2
	·32529	·94561	·34400	2·90696	·34175	·93979	·36364	2·74997	1
	·32557	·94552	·34433	2·90421	·34202	·93969	·36397	2·74748	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

′	20° Sin.	20° Cos.	20° Tan.	20° Cot.	21° Sin.	21° Cos.	21° Tan.	21° Cot.	′
0	.34202	.93969	.36397	2.74748	.35837	.93358	.38386	2.60509	60
1	.34229	.93959	.36430	2.74499	.35864	.93348	.38420	2.60283	59
2	.34257	.93949	.36463	2.74251	.35891	.93337	.38453	2.60057	58
3	.34284	.93939	.36496	2.74004	.35918	.93327	.38487	2.59831	57
4	.34311	.93929	.36529	2.73756	.35945	.93316	.38520	2.59606	56
5	.34339	.93919	.36562	2.73509	.35973	.93306	.38553	2.59381	55
6	.34366	.93909	.36595	2.73263	.36000	.93295	.38587	2.59156	54
7	.34393	.93899	.36628	2.73017	.36027	.93285	.38620	2.58932	53
8	.34421	.93889	.36661	2.72771	.36054	.93274	.38654	2.58708	52
9	.34448	.93879	.36694	2.72526	.36081	.93264	.38687	2.58484	51
10	.34475	.93869	.36727	2.72281	.36108	.93253	.38721	2.58261	50
11	.34503	.93859	.36760	2.72036	.36135	.93243	.38754	2.58038	49
12	.34530	.93849	.36793	2.71792	.36162	.93232	.38787	2.57815	48
13	.34557	.93839	.36826	2.71548	.36190	.93222	.38821	2.57593	47
14	.34584	.93829	.36859	2.71305	.36217	.93211	.38854	2.57371	46
15	.34612	.93819	.36892	2.71062	.36244	.93201	.38888	2.57150	45
16	.34639	.93809	.36925	2.70819	.36271	.93190	.38921	2.56928	44
17	.34666	.93799	.36958	2.70577	.36298	.93180	.38955	2.56707	43
18	.34694	.93789	.36991	2.70335	.36325	.93169	.38988	2.56487	42
19	.34721	.93779	.37024	2.70094	.36352	.93159	.39022	2.56266	41
20	.34748	.93769	.37057	2.69853	.36379	.93148	.39055	2.56046	40
21	.34775	.93759	.37090	2.69612	.36406	.93137	.39089	2.55827	39
22	.34803	.93748	.37123	2.69371	.36434	.93127	.39122	2.55608	38
23	.34830	.93738	.37157	2.69131	.36461	.93116	.39156	2.55389	37
24	.34857	.93728	.37190	2.68892	.36488	.93106	.39190	2.55170	36
25	.34884	.93718	.37223	2.68653	.36515	.93095	.39223	2.54952	35
26	.34912	.93708	.37256	2.68414	.36542	.93084	.39257	2.54734	34
27	.34939	.93698	.37289	2.68175	.36569	.93074	.39290	2.54516	33
28	.34966	.93688	.37322	2.67937	.36596	.93063	.39324	2.54299	32
29	.34993	.93677	.37355	2.67700	.36623	.93052	.39357	2.54082	31
30	.35021	.93667	.37388	2.67462	.36650	.93042	.39391	2.53865	30
31	.35048	.93657	.37422	2.67225	.36677	.93031	.39425	2.53648	29
32	.35075	.93647	.37455	2.66989	.36704	.93020	.39458	2.53432	28
33	.35102	.93637	.37488	2.66752	.36731	.93010	.39492	2.53217	27
34	.35130	.93626	.37521	2.66516	.36758	.92999	.39526	2.53001	26
35	.35157	.93616	.37554	2.66281	.36785	.92988	.39559	2.52786	25
36	.35184	.93606	.37588	2.66046	.36812	.92978	.39593	2.52571	24
37	.35211	.93596	.37621	2.65811	.36839	.92967	.39626	2.52357	23
38	.35239	.93585	.37654	2.65576	.36867	.92956	.39660	2.52142	22
39	.35266	.93575	.37687	2.65342	.36894	.92945	.39694	2.51929	21
40	.35293	.93565	.37720	2.65109	.36921	.92935	.39727	2.51715	20
41	.35320	.93555	.37754	2.64875	.36948	.92924	.39761	2.51502	19
42	.35347	.93544	.37787	2.64642	.36975	.92913	.39795	2.51289	18
43	.35375	.93534	.37820	2.64410	.37002	.92902	.39829	2.51076	17
44	.35402	.93524	.37853	2.64177	.37029	.92892	.39862	2.50864	16
45	.35429	.93514	.37887	2.63945	.37056	.92881	.39896	2.50652	15
46	.35456	.93503	.37920	2.63714	.37083	.92870	.39930	2.50440	14
47	.35484	.93493	.37953	2.63483	.37110	.92859	.39963	2.50229	13
48	.35511	.93483	.37986	2.63252	.37137	.92849	.39997	2.50018	12
49	.35538	.93472	.38020	2.63021	.37164	.92838	.40031	2.49807	11
50	.35565	.93462	.38053	2.62791	.37191	.92827	.40065	2.49597	10
51	.35592	.93452	.38086	2.62561	.37218	.92816	.40098	2.49388	9
52	.35619	.93441	.38120	2.62332	.37245	.92805	.40132	2.49177	8
53	.35647	.93431	.38153	2.62103	.37272	.92794	.40166	2.48967	7
54	.35674	.93420	.38186	2.61874	.37299	.92784	.40200	2.48758	6
55	.35701	.93410	.38220	2.61646	.37326	.92773	.40234	2.48549	5
56	.35728	.93400	.38253	2.61418	.37353	.92762	.40267	2.48340	4
57	.35755	.93389	.38286	2.61190	.37380	.92751	.40301	2.48132	3
58	.35782	.93379	.38320	2.60963	.37407	.92740	.40335	2.47924	2
59	.35810	.93368	.38353	2.60736	.37434	.92729	.40369	2.47716	1
60	.35837	.93358	.38386	2.60509	.37461	.92718	.40403	2.47509	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	22°			23°				
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
.37461	.92718	.40403	2.47509	.39073	.92050	.42447	2.35585	60
.37488	.92707	.40436	2.47302	.39100	.92039	.42482	2.35395	59
.37515	.92697	.40470	2.47095	.39127	.92028	.42516	2.35205	58
.37542	.92686	.40504	2.46888	.39153	.92018	.42551	2.35015	57
.37569	.92675	.40538	2.46682	.39180	.92005	.42585	2.34825	56
.37595	.92664	.40572	2.46476	.39207	.91994	.42619	2.34636	55
.37622	.92653	.40606	2.46270	.39234	.91982	.42654	2.34447	54
.37649	.92642	.40640	2.46065	.39260	.91971	.42688	2.34258	53
.37676	.92631	.40674	2.45860	.39287	.91959	.42722	2.34069	52
.37703	.92620	.40707	2.45655	.39314	.91948	.42757	2.33881	51
.37730	.92609	.40741	2.45451	.39341	.91936	.42791	2.33693	50
.37757	.92598	.40775	2.45246	.39367	.91925	.42826	2.33505	49
.37784	.92587	.40809	2.45043	.39394	.91914	.42860	2.33317	48
.37811	.92576	.40843	2.44839	.39421	.91902	.42894	2.33130	47
.37838	.92565	.40877	2.44636	.39448	.91891	.42929	2.32943	46
.37865	.92554	.40911	2.44433	.39474	.91879	.42963	2.32756	45
.37892	.92543	.40945	2.44230	.39501	.91868	.42998	2.32570	44
.37919	.92532	.40979	2.44027	.39528	.91856	.43032	2.32383	43
.37946	.92521	.41013	2.43825	.39555	.91845	.43067	2.32197	42
.37973	.92510	.41047	2.43623	.39581	.91833	.43101	2.32012	41
.37999	.92499	.41081	2.43422	.39608	.91822	.43136	2.31826	40
.38026	.92488	.41115	2.43220	.39635	.91810	.43170	2.31641	39
.38053	.92477	.41149	2.43019	.39661	.91799	.43205	2.31456	38
.38080	.92466	.41183	2.42819	.39688	.91787	.43239	2.31271	37
.38107	.92455	.41217	2.42618	.39715	.91775	.43274	2.31086	36
.38134	.92444	.41251	2.42418	.39741	.91764	.43308	2.30902	35
.38161	.92432	.41285	2.42218	.39768	.91752	.43343	2.30718	34
.38188	.92421	.41319	2.42019	.39795	.91741	.43378	2.30534	33
.38215	.92410	.41353	2.41819	.39822	.91729	.43412	2.30351	32
.38241	.92399	.41387	2.41620	.39848	.91718	.43447	2.30167	31
.38268	.92388	.41421	2.41421	.39875	.91706	.43481	2.29984	30
.38295	.92377	.41455	2.41223	.39902	.91694	.43516	2.29801	29
.38322	.92366	.41490	2.41025	.39928	.91683	.43550	2.29619	28
.38349	.92355	.41524	2.40827	.39955	.91671	.43585	2.29437	27
.38376	.92343	.41558	2.40629	.39982	.91660	.43620	2.29254	26
.38403	.92332	.41592	2.40432	.40008	.91648	.43654	2.29073	25
.38430	.92321	.41626	2.40235	.40035	.91636	.43689	2.28891	24
.38456	.92310	.41660	2.40038	.40062	.91625	.43724	2.28710	23
.38483	.92299	.41694	2.39841	.40088	.91613	.43758	2.28528	22
.38510	.92287	.41728	2.39645	.40115	.91601	.43793	2.28348	21
.38537	.92276	.41763	2.39449	.40141	.91590	.43828	2.28167	20
.38564	.92265	.41797	2.39253	.40168	.91578	.43862	2.27987	19
.38591	.92254	.41831	2.39058	.40195	.91566	.43897	2.27806	18
.38617	.92243	.41865	2.38863	.40221	.91555	.43932	2.27626	17
.38644	.92231	.41899	2.38668	.40248	.91543	.43966	2.27447	16
.38671	.92220	.41933	2.38473	.40275	.91531	.44001	2.27267	15
.38698	.92209	.41968	2.38279	.40301	.91519	.44036	2.27088	14
.38725	.92198	.42002	2.38084	.40328	.91508	.44071	2.26909	13
.38752	.92186	.42036	2.37891	.40355	.91496	.44105	2.26730	12
.38778	.92175	.42070	2.37697	.40381	.91484	.44140	2.26552	11
.38805	.92164	.42105	2.37504	.40408	.91472	.44175	2.26374	10
.38832	.92152	.42139	2.37311	.40434	.91461	.44210	2.26196	9
.38859	.92141	.42173	2.37118	.40461	.91449	.44244	2.26018	8
.38886	.92130	.42207	2.36925	.40488	.91437	.44279	2.25840	7
.38912	.92119	.42242	2.36733	.40514	.91425	.44314	2.25663	6
.38939	.92107	.42276	2.36541	.40541	.91414	.44349	2.25486	5
.38966	.92096	.42310	2.36349	.40567	.91402	.44384	2.25309	4
.38993	.92085	.42345	2.36158	.40594	.91390	.44418	2.25132	3
.39020	.92073	.42379	2.35967	.40621	.91378	.44453	2.24956	2
.39046	.92062	.42413	2.35776	.40647	.91366	.44488	2.24780	1
.39073	.92050	.42447	2.35585	.40674	.91355	.44523	2.24604	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	67°					66°		

	24°				25°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.40674	.91355	.44523	2.24604	.42262	.90631	.46631	2.14451	60
1	.40700	.91343	.44558	2.24428	.42288	.90618	.46666	2.14288	59
2	.40727	.91331	.44593	2.24252	.42315	.90606	.46702	2.14125	58
3	.40753	.91319	.44627	2.24077	.42341	.90594	.46737	2.13963	57
4	.40780	.91307	.44662	2.23902	.42367	.90582	.46772	2.13801	56
5	.40806	.91295	.44697	2.23727	.42394	.90569	.46808	2.13639	55
6	.40833	.91283	.44732	2.23553	.42420	.90557	.46843	2.13477	54
7	.40860	.91272	.44767	2.23378	.42446	.90545	.46879	2.13316	53
8	.40886	.91260	.44802	2.23204	.42473	.90532	.46914	2.13154	52
9	.40913	.91248	.44837	2.23030	.42499	.90520	.46950	2.12993	51
10	.40939	.91236	.44872	2.22857	.42525	.90507	.46985	2.12832	50
11	.40966	.91224	.44907	2.22683	.42552	.90495	.47021	2.12671	49
12	.40992	.91212	.44942	2.22510	.42578	.90483	.47056	2.12511	48
13	.41019	.91200	.44977	2.22337	.42604	.90470	.47092	2.12350	47
14	.41045	.91188	.45012	2.22164	.42631	.90458	.47128	2.12190	46
15	.41072	.91176	.45047	2.21992	.42657	.90446	.47163	2.12030	45
16	.41098	.91164	.45082	2.21819	.42683	.90433	.47199	2.11871	44
17	.41125	.91152	.45117	2.21647	.42709	.90421	.47234	2.11711	43
18	.41151	.91140	.45152	2.21475	.42736	.90408	.47270	2.11552	42
19	.41178	.91128	.45187	2.21304	.42762	.90396	.47305	2.11392	41
20	.41204	.91116	.45222	2.21132	.42788	.90383	.47341	2.11233	40
21	.41231	.91104	.45257	2.20961	.42815	.90371	.47377	2.11075	39
22	.41257	.91092	.45292	2.20790	.42841	.90358	.47412	2.10916	38
23	.41284	.91080	.45327	2.20619	.42867	.90346	.47448	2.10758	37
24	.41310	.91068	.45362	2.20449	.42894	.90334	.47483	2.10600	36
25	.41337	.91056	.45397	2.20278	.42920	.90321	.47519	2.10442	35
26	.41363	.91044	.45432	2.20108	.42946	.90309	.47555	2.10284	34
27	.41390	.91032	.45467	2.19938	.42972	.90296	.47590	2.10126	33
28	.41416	.91020	.45502	2.19769	.42999	.90284	.47626	2.09969	32
29	.41443	.91008	.45538	2.19599	.43025	.90271	.47662	2.09811	31
30	.41469	.90996	.45573	2.19430	.43051	.90259	.47698	2.09654	30
31	.41496	.90984	.45608	2.19261	.43077	.90246	.7733	2.09498	29
32	.41522	.90972	.45643	2.19092	.43104	.90233	.47769	2.09341	28
33	.41549	.90960	.45678	2.18923	.43130	.90221	.47805	2.09184	27
34	.41575	.90948	.45713	2.18755	.43156	.90208	.47840	2.09028	26
35	.41602	.90936	.45748	2.18587	.43182	.90196	.47876	2.08872	25
36	.41628	.90924	.45784	2.18419	.43209	.90183	.47912	2.08716	24
37	.41655	.90911	.45819	2.18251	.43235	.90171	.47948	2.08560	23
38	.41681	.90899	.45854	2.18084	.43261	.90158	.47984	2.08405	22
39	.41707	.90887	.45889	2.17916	.43287	.90146	.48019	2.08250	21
40	.41734	.90875	.45924	2.17749	.43313	.90133	.48055	2.08094	20
41	.41760	.90863	.45960	2.17582	.43340	.90120	.48091	2.07939	19
42	.41787	.90851	.45995	2.17416	.43366	.90108	.48127	2.07785	18
43	.41813	.90839	.46030	2.17249	.43392	.90095	.48163	2.07630	17
44	.41840	.90826	.46065	2.17083	.43418	.90082	.48198	2.07476	16
45	.41866	.90814	.46101	2.16917	.43445	.90070	.48234	2.07321	15
46	.41892	.90802	.46136	2.16751	.43471	.90057	.48270	2.07167	14
47	.41919	.90790	.46171	2.16585	.43497	.90045	.48306	2.07014	13
48	.41945	.90778	.46206	2.16420	.43523	.90032	.48342	2.06860	12
49	.41972	.90766	.46242	2.16255	.43549	.90019	.48378	2.06706	11
50	.41998	.90753	.46277	2.16090	.43575	.90007	.48414	2.06553	10
51	.42024	.90741	.46312	2.15925	.43602	.89994	.48450	2.06400	9
52	.42051	.90729	.46348	2.15760	.43628	.89981	.48486	2.06247	8
53	.42077	.90717	.46383	2.15596	.43654	.89968	.48521	2.06094	7
54	.42104	.90704	.46418	2.15432	.43680	.89956	.48557	2.05942	6
55	.42130	.90692	.46454	2.15268	.43706	.89943	.48593	2.05790	5
56	.42156	.90680	.46489	2.15104	.43733	.89930	.48629	2.05637	4
57	.42183	.90668	.46525	2.14940	.43759	.89918	.48665	2.05485	3
58	.42209	.90655	.46560	2.14777	.43785	.89905	.48701	2.05333	2
59	.42235	.90643	.46595	2.14614	.43811	.89892	.48737	2.05182	1
60	.42262	.90631	.46631	2.14451	.43837	.89879	.48773	2.05030	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	65°							64°	

	26°			27°				
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
.43837	.89879	.48773	2.05030	.45399	.89101	.50953	1.96261	60
.43863	.89867	.48809	2.04879	.45425	.89087	.50989	1.96120	59
.43889	.89854	.48845	2.04728	.45451	.89074	.51026	1.95979	58
.43916	.89841	.48881	2.04577	.45477	.89061	.51063	1.95838	57
.43942	.89828	.48917	2.04426	.45503	.89048	.51099	1.95698	56
.43968	.89816	.48953	2.04276	.45529	.89035	.51136	1.95557	55
.43994	.89803	.48989	2.04125	.45554	.89021	.51173	1.95417	54
.44020	.89790	.49026	2.03975	.45580	.89008	.51209	1.95277	53
.44046	.89777	.49062	2.03825	.45606	.88995	.51246	1.95137	52
.44072	.89764	.49098	2.03675	.45632	.88981	.51283	1.94997	51
.44098	.89752	.49134	2.03526	.45658	.88968	.51319	1.94858	50
.44124	.89739	.49170	2.03376	.45684	.88955	.51356	1.94718	49
.44151	.89726	.49206	2.03227	.45710	.88942	.51393	1.94579	48
.44177	.89713	.49242	2.03078	.45736	.88928	.51430	1.94440	47
.44203	.89700	.49278	2.02929	.45762	.88915	.51467	1.94301	46
.44229	.89687	.49315	2.02780	.45787	.88902	.51503	1.94162	45
.44255	.89674	.49351	2.02631	.45813	.88888	.51540	1.94023	44
.44281	.89662	.49387	2.02483	.45839	.88875	.51577	1.93885	43
.44307	.89649	.49423	2.02335	.45865	.88862	.51614	1.93746	42
.44333	.89636	.49459	2.02187	.45891	.88848	.51651	1.93608	41
.44359	.89623	.49495	2.02039	.45917	.88835	.51688	1.93470	40
.44385	.89610	.49532	2.01891	.45942	.88822	.51724	1.93332	39
.44411	.89597	.49568	2.01743	.45968	.88808	.51761	1.93195	38
.44437	.89584	.49604	2.01596	.45994	.88795	.51798	1.93057	37
.44464	.89571	.49640	2.01449	.46020	.88782	.51835	1.92920	36
.44490	.89558	.49677	2.01302	.46046	.88768	.51872	1.92782	35
.44516	.89545	.49713	2.01155	.46072	.88755	.51909	1.92645	34
.44542	.89532	.49749	2.01008	.46097	.88741	.51948	1.92508	33
.44568	.89519	.49786	2.00862	.46123	.88728	.51983	1.92371	32
.44594	.89506	.49822	2.00715	.46149	.88715	.52020	1.92235	31
.44620	.89493	.49858	2.00569	.46175	.88701	.52057	1.92098	30
.44646	.89480	.49894	2.00423	.46201	.88688	.52094	1.91962	29
.44672	.89467	.49931	2.00277	.46226	.88674	.52131	1.91826	28
.44698	.89454	.49967	2.00131	.46252	.88661	.52168	1.91690	27
.44724	.89441	.50004	1.99986	.46278	.88647	.52205	1.91554	26
.44750	.89428	.50040	1.99841	.46304	.88634	.52242	1.91418	25
.44776	.89415	.50076	1.99695	.46330	.88620	.52279	1.91282	24
.44802	.89402	.50113	1.99550	.46355	.88607	.52316	1.91147	23
.44828	.89389	.50149	1.99406	.46381	.88593	.52353	1.91012	22
.44854	.89376	.50185	1.99261	.46407	.88580	.52390	1.90876	21
.44880	.89363	.50222	1.99116	.46433	.88566	.52427	1.90741	20
.44906	.89350	.50258	1.98972	.46458	.88553	.52464	1.90607	19
.44932	.89337	.50295	1.98828	.46484	.88539	.52501	1.90472	18
.44958	.89324	.50331	1.98684	.46510	.88526	.52538	1.90337	17
.44984	.89311	.50368	1.98540	.46536	.88512	.52575	1.90203	16
.45010	.89298	.50404	1.98396	.46561	.88499	.52613	1.90069	15
.45036	.89285	.50441	1.98253	.46587	.88485	.52650	1.89935	14
.45062	.89272	.50477	1.98110	.46613	.88472	.52687	1.89801	13
.45088	.89259	.50514	1.97966	.46639	.88458	.52724	1.89667	12
.45114	.89245	.50550	1.97823	.46664	.88445	.52761	1.89533	11
.45140	.89232	.50587	1.97681	.46690	.88431	.52798	1.89400	10
.45166	.89219	.50623	1.97538	.46716	.88417	.52836	1.89266	9
.45192	.89206	.50660	1.97395	.46742	.88404	.52873	1.89133	8
.45218	.89193	.50696	1.97253	.46767	.88390	.52910	1.89000	7
.45243	.89180	.50733	1.97111	.46793	.88377	.52947	1.88867	6
.45269	.89167	.50769	1.96969	.46819	.88363	.52985	1.88734	5
.45295	.89153	.50806	1.96827	.46844	.88349	.53022	1.88602	4
.45321	.89140	.50843	1.96685	.46870	.88336	.53059	1.88469	3
.45347	.89127	.50879	1.96544	.46896	.88322	.53098	1.88337	2
.45373	.89114	.50916	1.96402	.46921	.88308	.53134	1.88205	1
.45399	.89101	.50953	1.96261	.46947	.88295	.53171	1.88073	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	28°			29°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.46947	.88295	.53171	1.88073	.48481	.87462	.55431	1.80405	6
1	.46973	.88281	.53208	1.87941	.48506	.87448	.55469	1.80281	5
2	.46999	.88267	.53246	1.87809	.48532	.87434	.55507	1.80158	5
3	.47024	.88254	.53283	1.87677	.48557	.87420	.55545	1.80034	5
4	.47050	.88240	.53320	1.87546	.48583	.87406	.55583	1.79911	5
5	.47076	.88226	.53358	1.87415	.48608	.87391	.55621	1 79788	5
6	.47101	.88213	.53395	1.87283	.48634	.87377	.55659	1 79665	5
7	.47127	.88199	.53432	1.87152	.48659	.87363	.55697	1.79542	5
8	.47153	.88185	.53470	1.87021	.48684	.87349	.55736	1.79419	5
9	.47178	.88172	.53507	1.86891	.48710	.87335	.55774	1.79296	5
10	.47204	.88158	.53545	1.86760	.48735	.87321	.55812	1.79174	5
11	.47229	.88144	.53582	1.86630	.48761	.87306	.55850	1.79051	4
12	.47255	.88130	.53620	1.86499	.48786	.87292	.55888	1.78929	4
13	.47281	.88117	.53657	1.86369	.48811	.87278	.55926	1.78807	4
14	.47306	.88103	.53694	1.86239	.48837	.87264	.55964	1.78685	4
15	.47332	.88089	.53732	1.86109	.48862	.87250	.56003	1.78563	4
16	.47358	.88075	.53769	1.85979	.48888	.87235	.56041	1.78441	4
17	.47383	.88062	.53807	1.85850	.48913	.87221	.56079	1.78319	4
18	.47409	.88048	.53844	1.85720	.48938	.87207	.56117	1.78198	4
19	.47434	.88034	.53882	1.85591	.48964	.87193	.56156	1.78077	4
20	.47460	.88020	.53920	1.85462	.48989	.87178	.56194	1.77955	4
21	.47486	.88006	.53957	1.85333	.49014	.87164	.56232	1.77834	3
22	.47511	.87993	.53995	1.85204	.49040	.87150	.56270	1.77713	3
23	.47537	.87979	.54032	1.85075	.49065	.87136	.56309	1.77592	3
24	.47562	.87965	.54070	1.84946	.49090	.87121	.56347	1.77471	3
25	.47588	.87951	.54107	1.84818	.49116	.87107	.56385	1.77351	3
26	.47614	.87937	.54145	1.84689	.49141	.87093	.56424	1.77230	3
27	.47639	.87923	.54183	1.84561	.49166	.87079	.56462	1.77110	3
28	.47665	.87909	.54220	1.84433	.49192	.87064	.56501	1.76990	3
29	.47690	.87896	.54258	1.84305	.49217	.87050	.56539	1.76869	3
30	.47716	.87882	.54296	1.84177	.49242	.87036	.56577	1.76749	3
31	.47741	.87868	.54333	1.84049	.49268	.87021	.56616	1.76629	2
32	.47767	.87854	.54371	1.83922	.49293	.87007	.56654	1.76510	2
33	.47793	.87840	.54409	1.83794	.49318	.86993	.56693	1.76390	2
34	.47818	.87826	.54446	1.83667	.49344	.86978	.56731	1.76271	2
35	.47844	.87812	.54484	1.83540	.49369	.86964	.56769	1.76151	2
36	.47869	.87798	.54522	1.83413	.49394	.86949	.56808	1.76032	2
37	.47895	.87784	.54560	1.83286	.49419	.86935	.56848	1.75913	2
38	.47920	.87770	.54597	1.83159	.49445	.86921	.56885	1.75794	2
39	.47946	.87758	.54635	1.83033	.49470	.86906	.56923	1.75675	2
40	.47971	.87743	.54673	1.82906	.49495	.86892	.56962	1.75556	2
41	.47997	.87729	.54711	1.82780	.49521	.86878	.57000	1.75437	1
42	.48022	.87715	.54748	1.82654	.49546	.86863	.57039	1.75319	1
43	.48048	.87701	.54786	1.82528	.49571	.86849	.57078	1.75200	1
44	.48073	.87687	.54824	1.82402	.49596	.86834	.57116	1.75082	1
45	.48099	.87673	.54862	1.82276	.49622	.86820	.57155	1.74964	1
46	.48124	.87659	.54900	1.82150	.49647	.86805	.57193	1.74846	1
47	.48150	.87645	.54938	1.82025	.49672	.86791	.57232	1.74728	1
48	.48175	.87631	.54975	1.81899	.49697	.86777	.57271	1.74610	1
49	.48201	.87617	.55013	1.81774	.49723	.86762	.57309	1.74492	1
50	.48226	.87603	.55051	1.81649	.49748	.86748	.57348	1.74375	1
51	.48252	.87589	.55089	1.81524	.49773	.86733	.57386	1.74257	
52	.48277	.87575	.55127	1.81399	.49798	.86719	.57425	1.74140	
53	.48303	.87561	.55165	1.81274	.49824	.86704	.57464	1.74022	
54	.48328	.87546	.55203	1.81150	.49849	.86690	.57503	1.73905	
55	.48354	.87532	.55241	1.81025	.49874	.86675	.57541	1.73788	
56	.48379	.87518	.55279	1.80901	.49899	.86661	.57580	1.73671	
57	.48405	.87504	.55317	1.80777	.49924	.86646	.57619	1.73555	
58	.48430	.87490	.55355	1.80653	.49950	.86632	.57657	1.73438	
59	.48456	.87476	.55393	1.80529	.49975	.86617	.57696	1.73321	
60	.48481	.87462	.55431	1.80405	.50000	.86603	.57735	1.73205	
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	30°				31°			
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
.50000	.86603	.57735	1.73205	.51504	.85717	.60086	1.66428	60
.50025	.86588	.57774	1.73089	.51529	.85702	.60126	1.66318	59
.50050	.86573	.57813	1.72973	.51554	.85687	.60165	1.66209	58
.50076	.86559	.57851	1.72857	.51579	.85672	.60205	1.66099	57
.50101	.86544	.57890	1.72741	.51604	.85657	.60245	1.65990	56
.50126	.86530	.57929	1.72625	.51628	.85642	.60284	1.65881	55
.50151	.86515	.57968	1.72509	.51653	.85627	.60324	1.65772	54
.50176	.86501	.58007	1.72393	.51678	.85612	.60364	1.65663	53
.50201	.86486	.58046	1.72278	.51703	.85597	.60403	1.65554	52
.50227	.86471	.58085	1.72163	.51728	.85582	.60443	1.65445	51
.50252	.86457	.58124	1.72047	.51753	.85567	.60483	1.65337	50
.50277	.86442	.58162	1.71932	.51778	.85551	.60522	1.65228	49
.50302	.86427	.58201	1.71817	.51803	.85536	.60562	1.65120	48
.50327	.86413	.58240	1.71702	.51828	.85521	.60602	1.65011	47
.50352	.86398	.58279	1.71588	.51852	.85506	.60642	1.64903	46
.50377	.86384	.58318	1.71473	.51877	.85491	.60681	1.64795	45
.50403	.86369	.58357	1.71358	.51902	.85476	.60721	1.64687	44
.50428	.86354	.58396	1.71244	.51927	.85461	.60761	1.64579	43
.50453	.86340	.58435	1.71129	.51952	.85446	.60801	1.64471	42
.50478	.86325	.58474	1.71015	.51977	.85431	.60841	1.64363	41
.50503	.86310	.58513	1.70901	.52002	.85416	.60881	1.64256	40
.50528	.86295	.58552	1.70787	.52026	.85401	.60921	1.64148	39
.50553	.86281	.58591	1.70673	.52051	.85385	.60960	1.64041	38
.50578	.86266	.58631	1.70560	.52076	.85370	.61000	1.63934	37
.50603	.86251	.58670	1.70446	.52101	.85355	.61040	1.63826	36
.50628	.86237	.58709	1.70332	.52126	.85340	.61080	1.63719	35
.50654	.86222	.58748	1.70219	.52151	.85325	.61120	1.63612	34
.50679	.86207	.58787	1.70106	.52175	.85310	.61160	1.63505	33
.50704	.86192	.58826	1.69992	.52200	.85294	.61200	1.63398	32
.50729	.86178	.58865	1.69879	.52225	.85279	.61240	1.63292	31
.50754	.86163	.58905	1.69766	.52250	.85264	.61280	1.63185	30
.50779	.86148	.58944	1.69653	.52275	.85249	.61320	1.63079	29
.50804	.86133	.58983	1.69541	.52299	.85234	.61360	1.62972	28
.50829	.86119	.59022	1.69428	.52324	.85218	.61400	1.62866	27
.50854	.86104	.59061	1.69316	.52349	.85203	.61440	1.62760	26
.50878	.86089	.59101	1.69203	.52374	.85188	.61480	1.62654	25
.50904	.86074	.59140	1.69091	.52399	.85173	.61520	1.62548	24
.50929	.86059	.59179	1.68979	.52423	.85157	.61561	1.62442	23
.50954	.86045	.59218	1.68866	.52448	.85142	.61601	1.62336	22
.50979	.86030	.59258	1.68754	.52473	.85127	.61641	1.62230	21
.51004	.86015	.59297	1.68643	.52498	.85112	.61681	1.62125	20
.51029	.86000	.59336	1.68531	.52522	.85096	.61721	1.62019	19
.51054	.85985	.59376	1.68419	.52547	.85081	.61761	1.61914	18
.51079	.85970	.59415	1.68308	.52572	.85066	.61801	1.61808	17
.51104	.85956	.59454	1.68196	.52597	.85051	.61842	1.61703	16
.51129	.85941	.59494	1.68085	.52621	.85035	.61882	1.61598	15
.51154	.85926	.59533	1.67974	.52646	.85020	.61922	1.61493	14
.51179	.85911	.59573	1.67863	.52671	.85005	.61962	1.61388	13
.51204	.85896	.59612	1.67752	.52696	.84989	.62003	1.61283	12
.51229	.85881	.59651	1.67641	.52720	.84974	.62043	1.61179	11
.51254	.85866	.59691	1.67530	.52745	.84959	.62083	1.61074	10
.51279	.85851	.59730	1.67419	.52770	.84943	.62124	1.60970	9
.51304	.85836	.59770	1.67309	.52794	.84928	.62164	1.60865	8
.51329	.85821	.59809	1.67198	.52819	.84913	.62204	1.60761	7
.51354	.85806	.59849	1.67088	.52844	.84897	.62245	1.60657	6
.51379	.85792	.59888	1.66978	.52869	.84882	.62285	1.60553	5
.51404	.85777	.59928	1.66867	.52893	.84866	.62325	1.60449	4
.51429	.85762	.59967	1.66757	.52918	.84851	.62366	1.60345	3
.51454	.85747	.60007	1.66647	.52943	.84836	.62406	1.60241	2
.51479	.85732	.60046	1.66538	.52967	.84820	.62446	1.60137	1
.51504	.85717	.60086	1.66428	.52992	.84805	.62487	1.60033	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

TABLE 17.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENT

′	32° Sin.	Cos	Tan.	Cot	33° Sin.	Cos.	Tan.	Cot.	′
0	.52992	.84805	.62487	1.60033	.54464	.83867	.64941	1.53986	6
1	.53017	.84789	.62527	1.59930	.54488	.83851	.64982	1.53888	5
2	.53041	.84774	.62568	1.59826	.54513	.83835	.65024	1.53791	5
3	.53066	.84759	.62608	1.59723	.54537	.83819	.65065	1.53693	5
4	.53091	.84743	.62649	1.59620	.54561	.83804	.65106	1.53595	5
5	.53115	.84728	.62689	1.59517	.54586	.83788	.65148	1.53497	5
6	.53140	.84712	.62730	1.59414	.54610	.83772	.65189	1.53400	5
7	.53164	.84697	.62770	1.59311	.54635	.83756	.65231	1.53302	5
8	.53189	.84681	.62811	1.59208	.54659	.83740	.65272	1.53205	5
9	.53214	.84666	.62852	1.59105	.54683	.83724	.65314	1.53107	5
10	.53238	.84650	.62892	1.59002	.54708	.83708	.65355	1.53010	5
11	.53263	.84635	.62933	1.58900	.54732	.83692	.65397	1.52913	4
12	.53288	.84619	.62973	1.58797	.54756	.83676	.65438	1.52816	4
13	.53312	.84604	.63014	1.58695	.54781	.83660	.65480	1.52719	4
14	.53337	.84588	.63055	1.58593	.54805	.83645	.65521	1.52622	4
15	.53361	.84573	.63095	1.58490	.54829	.83629	.65563	1.52525	4
16	.53386	.84557	.63136	1.58388	.54854	.83613	.65604	1.52429	4
17	.53411	.84542	.63177	1.58286	.54878	.83597	.65646	1.52332	4
18	.53435	.84526	.63217	1.58184	.54902	.83581	.65688	1.52235	4
19	.53460	.84511	.63258	1.58083	.54927	.83565	.65729	1.52139	4
20	.53484	.84495	.63299	1.57981	.54951	.83549	.65771	1.52043	4
21	.53509	.84480	.63340	1.57879	.54975	.83533	.65813	1.51948	3
22	.53534	.84464	.63380	1.57778	.54999	.83517	.65854	1.51850	3
23	.53558	.84448	.63421	1.57676	.55024	.83501	.65896	1.51754	3
24	.53583	.84433	.63462	1.57575	.55048	.83485	.65938	1.51658	3
25	.53607	.84417	.63503	1.57474	.55072	.83469	.65980	1.51562	3
26	.53632	.84402	.63544	1.57372	.55097	.83453	.66021	1.51466	3
27	.53656	.84386	.63584	1.57271	.55121	.83437	.66063	1.51370	3
28	.53681	.84370	.63625	1.57170	.55145	.83421	.66105	1.51275	3
29	.53705	.84355	.63666	1.57069	.55169	.83405	.66147	1.51179	3
30	.53730	.84339	.63707	1.56969	.55194	.83389	.66189	1.51084	3
31	.53754	.84324	.63748	1.56868	.55218	.83373	.66230	1.50988	2
32	.53779	.84308	.63789	1.56767	.55242	.83356	.66272	1.50893	2
33	.53804	.84292	.63830	1.56667	.55266	.83340	.66314	1.50797	2
34	.53828	.84277	.63871	1.56566	.55291	.83324	.66356	1.50702	2
35	.53853	.84261	.63912	1.56466	.55315	.83308	.66398	1.50607	2
36	.53877	.84245	.63953	1.56366	.55339	.83292	.66440	1.50512	2
37	.53902	.84230	.63994	1.56265	.55363	.83276	.66482	1.50417	2
38	.53926	.84214	.64035	1.56165	.55388	.83260	.66524	1.50322	2
39	.53951	.84198	.64076	1.56065	.55412	.83244	.66566	1.50228	2
40	.53975	.84182	.64117	1.55966	.55436	.83228	.66608	1.50133	2
41	.54000	.84167	.64158	1.55866	.55460	.83212	.66650	1.50038	1
42	.54024	.84151	.64199	1.55766	.55484	.83195	.66692	1.49944	1
43	.54049	.84135	.64240	1.55666	.55509	.83179	.66734	1.49849	1
44	.54073	.84120	.64281	1.55567	.55533	.83163	.66776	1.49755	1
45	.54097	.84104	.64322	1.55467	.55557	.83147	.66818	1.49661	1
46	.54122	.84088	.64363	1.55368	.55581	.83131	.66860	1.49566	1
47	.54146	.84072	.64404	1.55269	.55605	.83115	.66902	1.49472	1
48	.54171	.84057	.64446	1.55170	.55630	.83098	.66944	1.49378	1
49	.54195	.84041	.64487	1.55071	.55654	.83082	.66986	1.49284	1
50	.54220	.84025	.64528	1.54972	.55678	.83066	.67028	1.49190	1
51	.54244	.84009	.64569	1.54873	.55702	.83050	.67071	1.49097	
52	.54269	.83994	.64610	1.54774	.55726	.83034	.67113	1.49003	
53	.54293	.83978	.64652	1.54675	.55750	.83017	.67155	1.48909	
54	.54317	.83962	.64693	1.54576	.55775	.83001	.67197	1.48816	
55	.54342	.83946	.64734	1.54478	.55799	.82985	.67239	1.48722	
56	.54366	.83930	.64775	1.54379	.55823	.82969	.67282	1.48629	
57	.54391	.83915	.64817	1.54281	.55847	.82953	.67324	1.48536	
58	.54415	.83899	.64858	1.54183	.55871	.82936	.67366	1.48442	
59	.54440	.83883	.64899	1.54085	.55895	.82920	.67409	1.48349	
60	.54464	.83867	.64941	1.53986	.55919	.82904	.67451	1.48256	
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	34°			35°				
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
.55919	.82904	.67451	1.48256	.57358	.81915	.70021	1.42815	60
.55943	.82887	.67493	1.48163	.57381	.81899	.70064	1.42726	59
.55968	.82871	.67536	1.48070	.57405	.81882	.70107	1.42638	58
.55992	.82855	.67578	1.47977	.57429	.81865	.70151	1.42550	57
.56016	.82839	.67620	1.47885	.57453	.81848	.70194	1.42462	56
.56040	.82822	.67663	1.47792	.57477	.81832	.70238	1.42374	55
.56064	.82806	.67705	1.47699	.57501	.81815	.70281	1.42286	54
.56088	.82790	.67748	1.47607	.57524	.81798	.70325	1.42198	53
.56112	.82773	.67790	1.47514	.57548	.81782	.70368	1.42110	52
.56136	.82757	.67832	1.47422	.57572	.81765	.70412	1.42022	51
.56160	.82741	.67875	1.47330	.57596	.81748	.70455	1.41934	50
.56184	.82724	.67917	1.47238	.57619	.81731	.70499	1.41847	49
.56208	.82708	.67960	1.47146	.57643	.81714	.70542	1.41759	48
.56232	.82692	.68002	1.47053	.57667	.81698	.70586	1.41672	47
.56256	.82675	.68045	1.46962	.57691	.81681	.70629	1.41584	46
.56280	.82659	.68088	1.46870	.57715	.81664	.70673	1.41497	45
.56305	.82643	.68130	1.46778	.57738	.81647	.70717	1.41409	44
.56329	.82626	.68173	1.46686	.57762	.81631	.70760	1.41322	43
.56353	.82610	.68215	1.46595	.57786	.81614	.70804	1.41235	42
.56377	.82593	.68258	1.46503	.57810	.81597	.70848	1.41148	41
.56401	.82577	.68301	1.46411	.57833	.81580	.70891	1.41061	40
.56425	.82561	.68343	1.46320	.57857	.81563	.70935	1.40974	39
.56449	.82544	.68386	1.46229	.57881	.81546	.70979	1.40887	38
.56473	.82528	.68429	1.46137	.57904	.81530	.71023	1.40800	37
.56497	.82511	.68471	1.46046	.57928	.81513	.71066	1.40714	36
.56521	.82495	.68514	1.45955	.57952	.81496	.71110	1.40627	35
.56545	.82478	.68557	1.45864	.57976	.81479	.71154	1.40540	34
.56569	.82462	.68600	1.45773	.57999	.81462	.71198	1.40454	33
.56593	.82446	.68642	1.45682	.58023	.81445	.71242	1.40367	32
.56617	.82429	.68685	1.45592	.58047	.81428	.71285	1.40281	31
.56641	.82413	.68728	1.45501	.58070	.81412	.71329	1.40195	30
.56665	.82396	.68771	1.45410	.58094	.81395	.71373	1.40109	29
.56689	.82380	.68814	1.45320	.58118	.81378	.71417	1.40022	28
.56713	.82363	.68857	1.45229	.58141	.81361	.71461	1.39936	27
.56736	.82347	.68900	1.45139	.58165	.81344	.71505	1.39850	26
.56760	.82330	.68942	1.45049	.58189	.81327	.71549	1.39764	25
.56784	.82314	.68985	1.44958	.58212	.81310	.71593	1.39679	24
.56808	.82297	.69028	1.44868	.58236	.81293	.71637	1.39593	23
.56832	.82281	.69071	1.44778	.58260	.81276	.71681	1.39507	22
.56856	.82264	.69114	1.44688	.58283	.81259	.71725	1.39421	21
.56880	.82248	.69157	1.44598	.58307	.81242	.71769	1.39336	20
.56904	.82231	.69200	1.44508	.58330	.81225	.71813	1.39250	19
.56928	.82214	.69243	1.44418	.58354	.81208	.71857	1.39165	18
.56952	.82198	.69286	1.44329	.58378	.81191	.71901	1.39079	17
.56976	.82181	.69329	1.44239	.58401	.81174	.71946	1.38994	16
.57000	.82165	.69372	1.44149	.58425	.81157	.71990	1.38909	15
.57024	.82148	.69416	1.44060	.58449	.81140	.72034	1.38824	14
.57047	.82132	.69459	1.43970	.58472	.81123	.72078	1.38738	13
.57071	.82115	.69502	1.43881	.58496	.81106	.72122	1.38653	12
.57095	.82098	.69545	1.43792	.58519	.81089	.72167	1.38568	11
.57119	.82082	.69588	1.43703	.58543	.81072	.72211	1.38484	10
.57143	.82065	.69631	1.43614	.58567	.81055	.72255	1.38399	9
.57167	.82048	.69675	1.43525	.58590	.81038	.72299	1.38314	8
.57191	.82032	.69718	1.43436	.58614	.81021	.72344	1.38229	7
.57215	.82015	.69761	1.43347	.58637	.81004	.72388	1.38145	6
.57238	.81999	.69804	1.43258	.58661	.80987	.72432	1.38060	5
.57262	.81982	.69847	1.43169	.58684	.80970	.72477	1.37976	4
.57286	.81965	.69891	1.43080	.58708	.80953	.72521	1.37891	3
.57310	.81949	.69934	1.42992	.58731	.80936	.72565	1.37807	2
.57334	.81932	.69977	1.42903	.58755	.80919	.72610	1.37722	1
.57358	.81915	.70021	1.42815	.58779	.80902	.72654	1.37638	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

′	36° Sin.	36° Cos.	36° Tan.	36° Cot.	37° Sin.	37° Cos.	37° Tan.	37° Cot.	′
0	.58779	.80902	.72654	1.37638	.60182	.79864	.75355	1.32704	60
1	.58802	.80885	.72699	1.37554	.60205	.79846	.75401	1.32624	59
2	.58826	.80867	.72743	1.37470	.60228	.79829	.75447	1.32544	58
3	.58849	.80850	.72788	1.37386	.60251	.79811	.75492	1.32464	57
4	.58873	.80833	.72832	1.37302	.60274	.79793	.75538	1.32384	56
5	.58896	.80816	.72877	1.37218	.60298	.79776	.75584	1.32304	55
6	.58920	.80799	.72921	1.37134	.60321	.79758	.75629	1.32224	54
7	.58943	.80782	.72966	1.37050	.60344	.79741	.75675	1.32144	53
8	.58967	.80765	.73010	1.36967	.60367	.79723	.75721	1.32064	52
9	.58990	.80748	.73055	1.36883	.60390	.79706	.75767	1.31984	51
10	.59014	.80730	.73100	1.36800	.60414	.79688	.75812	1.31904	50
11	.59037	.80713	.73144	1.36716	.60437	.79671	.75858	1.31825	49
12	.59061	.80696	.73189	1.36633	.60460	.79653	.75904	1.31745	48
13	.59084	.80679	.73234	1.36549	.60483	.79635	.75950	1.31666	47
14	.59108	.80662	.73278	1.36466	.60506	.79618	.75996	1.31586	46
15	.59131	.80644	.73323	1.36383	.60529	.79600	.76042	1.31507	45
16	.59154	.80627	.73368	1.36300	.60553	.79583	.76088	1.31427	44
17	.59178	.80610	.73413	1.36217	.60576	.79565	.76134	1.31348	43
18	.59201	.80593	.73457	1.36134	.60599	.79547	.76180	1.31269	42
19	.59225	.80576	.73502	1.36051	.60622	.79530	.76226	1.31190	41
20	.59248	.80558	.73547	1.35968	.60645	.79512	.76272	1.31110	40
21	.59272	.80541	.73592	1.35885	.60668	.79494	.76318	1.31031	39
22	.59295	.80524	.73637	1.35802	.60691	.79477	.76364	1.30952	38
23	.59318	.80507	.73681	1.35719	.60714	.79459	.76410	1.30873	37
24	.59342	.80489	.73726	1.35637	.60738	.79441	.76456	1.30795	36
25	.59365	.80472	.73771	1.35554	.60761	.79424	.76502	1.30716	35
26	.59389	.80455	.73816	1.35472	.60784	.79406	.76548	1.30637	34
27	.59412	.80438	.73861	1.35389	.60807	.79388	.76594	1.30558	33
28	.59436	.80420	.73906	1.35307	.60830	.79371	.76640	1.30480	32
29	.59459	.80403	.73951	1.35224	.60853	.79353	.76688	1.30401	31
30	.59482	.80386	.73996	1.35142	.60876	.79335	.76733	1.30323	30
31	.59506	.80368	.74041	1.35060	.60899	.79318	.76779	1.30244	29
32	.59529	.80351	.74086	1.34978	.60922	.79300	.76825	1.30166	28
33	.59552	.80334	.74131	1.34896	.60945	.79282	.76871	1.30087	27
34	.59576	.80316	.74176	1.34814	.60968	.79264	.76918	1.30009	26
35	.59599	.80299	.74221	1.34732	.60991	.79247	.76964	1.29931	25
36	.59622	.80282	.74267	1.34650	.61015	.79229	.77010	1.29853	24
37	.59646	.80264	.74312	1.34568	.61038	.79211	.77057	1.29775	23
38	.59669	.80247	.74357	1.34487	.61061	.79193	.77103	1.29698	22
39	.59693	.80230	.74402	1.34405	.61084	.79176	.77149	1.29618	21
40	.59716	.80212	.74447	1.34323	.61107	.79158	.77196	1.29541	20
41	.59739	.80195	.74492	1.34242	.61130	.79140	.77242	1.29463	19
42	.59763	.80178	.74538	1.34160	.61153	.79122	.77289	1.29385	18
43	.59786	.80160	.74583	1.34079	.61176	.79105	.77335	1.29307	17
44	.59809	.80143	.74628	1.33998	.61199	.79087	.77382	1.29229	16
45	.59832	.80125	.74674	1.33916	.61222	.79069	.77428	1.29152	15
46	.59856	.80108	.74719	1.33835	.61245	.79051	.77475	1.29074	14
47	.59879	.80091	.74764	1.33754	.61268	.79033	.77521	1.28997	13
48	.59902	.80073	.74810	1.33673	.61291	.79016	.77568	1.28919	12
49	.59926	.80056	.74855	1.33592	.61314	.78998	.77615	1.28842	11
50	.59949	.80038	.74900	1.33511	.61337	.78980	.77661	1.28764	10
51	.59972	.80021	.74946	1.33430	.61360	.78962	.77708	1.28687	9
52	.59995	.80003	.74991	1.33349	.61383	.78944	.77754	1.28610	8
53	.60019	.79986	.75037	1.33268	.61406	.78926	.77801	1.28533	7
54	.60042	.79968	.75082	1.33187	.61429	.78908	.77848	1.28456	6
55	.60065	.79951	.75128	1.33107	.61451	.78891	.77895	1.28379	5
56	.60089	.79934	.75173	1.33026	.61474	.78873	.77941	1.28302	4
57	.60112	.79916	.75219	1.32946	.61497	.78855	.77988	1.28225	3
58	.60135	.79899	.75264	1.32865	.61520	.78837	.78035	1.28148	2
59	.60158	.79881	.75310	1.32785	.61543	.78819	.78082	1.28071	1
60	.60182	.79864	.75355	1.32704	.61566	.78801	.78129	1.27994	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	53°						52°		

	38°				39°			
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
·61566	·78801	·78129	1·27994	·62932	·77715	·80978	1·23490	60
·61589	·78783	·78175	1·27917	·62955	·77698	·81027	1·23416	59
·61612	·78765	·78222	1·27841	·62977	·77678	·81075	1·23343	58
·61635	·78747	·78269	1·27764	·63000	·77660	·81123	1·23270	57
·61658	·78729	·78316	1·27688	·63022	·77641	·81171	1·23196	56
·61681	·78711	·78363	1·27611	·63045	·77623	·81220	1·23123	55
·61704	·78694	·78410	1·27535	·63068	·77605	·81268	1·23050	54
·61726	·78676	·78457	1·27458	·63090	·77586	·81316	1·22977	53
·61749	·78658	·78504	1·27382	·63113	·77568	·81364	1·22904	52
·61772	·78640	·78551	1·27306	·63135	·77550	·81413	1·22831	51
·61795	·78622	·78598	1·27230	·63158	·77531	·81461	1·22758	50
·61818	·78604	·78645	1·27153	·63180	·77513	·81510	1·22685	49
·61841	·78586	·78692	1·27077	·63203	·77494	·81558	1·22612	48
·61864	·78568	·78739	1·27001	·63225	·77476	·81606	1·22539	47
·61887	·78550	·78786	1·26925	·63248	·77458	·81655	1·22467	46
·61909	·78532	·78834	1·26849	·63271	·77439	·81703	1·22394	45
·61932	·78514	·78881	1·26774	·63293	·77421	·81752	1·22321	44
·61955	·78496	·78928	1·26698	·63316	·77402	·81800	1·22249	43
·61978	·78478	·78975	1·26622	·63338	·77384	·81849	1·22176	42
·62001	·78460	·79022	1·26546	·63361	·77366	·81898	1·22104	41
·62024	·78442	·79070	1·26471	·63383	·77347	·81946	1·22031	40
·62046	·78424	·79117	1·26395	·63406	·77329	·81995	1·21959	39
·62069	·78405	·79164	1·26319	·63428	·77310	·82044	1·21886	38
·62092	·78387	·79212	1·26244	·63451	·77292	·82092	1·21814	37
·62115	·78369	·79259	1·26169	·63473	·77273	·82141	1·21742	36
·62138	·78351	·79306	1·26093	·63496	·77255	·82190	1·21670	35
·62160	·78333	·79354	1·26018	·63518	·77236	·82238	1·21598	34
·62183	·78315	·79401	1·25943	·63540	·77218	·82287	1·21526	33
·62206	·78297	·79449	1·25867	·63563	·77199	·82336	1·21454	32
·62229	·78279	·79496	1·25792	·63585	·77181	·82385	1·21382	31
·62251	·78261	·79544	1·25717	·63608	·77162	·82434	1·21310	30
·62274	·78243	·79591	1·25642	·63630	·77144	·82483	1·21238	29
·62297	·78225	·79639	1·25567	·63653	·77125	·82531	1·21166	28
·62320	·78206	·79686	1·25492	·63675	·77107	·82580	1·21094	27
·62342	·78188	·79734	1·25417	·63698	·77088	·82629	1·21023	26
·62365	·78170	·79781	1·25343	·63720	·77070	·82678	1·20951	25
·62388	·78152	·79829	1·25268	·63742	·77051	·82727	1·20879	24
·62411	·78134	·79877	1·25193	·63765	·77033	·82776	1·20808	23
·62433	·78116	·79924	1·25118	·63787	·77014	·82825	1·20736	22
·62456	·78098	·79972	1·25044	·63810	·76996	·82874	1·20665	21
·62479	·78079	·80020	1·24969	·63832	·76977	·82923	1·20593	20
·62502	·78061	·80067	1·24895	·63854	·76959	·82972	1·20522	19
·62524	·78043	·80115	1·24820	·63877	·76940	·83022	1·20451	18
·62547	·78025	·80163	1·24746	·63899	·76921	·83071	1·20379	17
·62570	·78007	·80211	1·24672	·63922	·76903	·83120	1·20308	16
·62592	·77988	·80258	1·24597	·63944	·76884	·83169	1·20237	15
·62615	·77970	·80306	1·24523	·63966	·76866	·83218	1·20166	14
·62638	·77952	·80354	1·24449	·63989	·76847	·83268	1·20095	13
·62660	·77934	·80402	1·24375	·64011	·76828	·83317	1·20024	12
·62683	·77916	·80450	1·24301	·64033	·76810	·83366	1·19953	11
·62706	·77897	·80498	1·24227	·64056	·76791	·83415	1·19882	10
·62728	·77879	·80546	1·24153	·64078	·76772	·83465	1·19811	9
·62751	·77861	·80594	1·24079	·64100	·76754	·83514	1·19740	8
·62774	·77843	·80642	1·24005	·64123	·76735	·83564	1·19669	7
·62796	·77824	·80690	1·23931	·64145	·76717	·83613	1·19599	6
·62819	·77806	·80738	1·23858	·64167	·76698	·83662	1·19528	5
·62842	·77788	·80786	1·23784	·64190	·76679	·83712	1·19457	4
·62864	·77769	·80834	1·23710	·64212	·76661	·83761	1·19387	3
·62887	·77751	·80882	1·23637	·64234	·76642	·83811	1·19316	2
·62909	·77733	·80930	1·23563	·64256	·76623	·83860	1·19246	1
·62932	·77715	·80978	1·23490	·64279	·76604	·83910	1·19175	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	51°				50°			

′	40° Sin.	40° Cos.	40° Tan.	40° Cot.	41° Sin.	41° Cos.	41° Tan.	41° Cot.
0	.64279	.76604	.83910	1.19175	.65606	.75471	.86929	1.15037
1	.64301	.76586	.83960	1.19105	.65628	.75452	.86980	1.14969
2	.64323	.76567	.84009	1.19035	.65650	.75433	.87031	1.14902
3	.64346	.76548	.84059	1.18964	.65672	.75414	.87082	1.14834
4	.64368	.76530	.84108	1.18894	.65694	.75395	.87133	1.14767
5	.64390	.76511	.84158	1.18824	.65716	.75375	.87184	1.14699
6	.64412	.76492	.84208	1.18754	.65738	.75356	.87236	1.14632
7	.64435	.76473	.84258	1.18684	.65759	.75337	.87287	1.14565
8	.64457	.76455	.84307	1.18614	.65781	.75318	.87338	1.14498
9	.64479	.76436	.84357	1.18544	.65803	.75299	.87389	1.14430
10	.64501	.76417	.84407	1.18474	.65825	.75280	.87441	1.14363
11	.64524	.76398	.84457	1.18404	.65847	.75261	.87492	1.14296
12	.64546	.76380	.84507	1.18334	.65869	.75241	.87543	1.14229
13	.64568	.76361	.84556	1.18264	.65891	.75222	.87595	1.14162
14	.64590	.76342	.84606	1.18194	.65913	.75203	.87646	1.14095
15	.64612	.76323	.84656	1.18125	.65935	.75184	.87698	1.14028
16	.64635	.76304	.84706	1.18055	.65956	.75165	.87749	1.13961
17	.64657	.76286	.84756	1.17986	.65978	.75146	.87801	1.13894
18	.64679	.76267	.84806	1.17916	.66000	.75126	.87852	1.13828
19	.64701	.76248	.84856	1.17846	.66022	.75107	.87904	1.13761
20	.64723	.76229	.84906	1.17777	.66044	.75088	.87955	1.13694
21	.64746	.76210	.84956	1.17708	.66066	.75069	.88007	1.13627
22	.64768	.76192	.85006	1.17638	.66088	.75050	.88059	1.13561
23	.64790	.76173	.85057	1.17569	.66109	.75030	.88110	1.13494
24	.64812	.76154	.85107	1.17500	.66131	.75011	.88162	1.13428
25	.64834	.76135	.85157	1.17430	.66153	.74992	.88204	1.13361
26	.64856	.76116	.85207	1.17361	.66175	.74973	.88265	1.13295
27	.64878	.76097	.85257	1.17292	.66197	.74953	.88317	1.13228
28	.64901	.76078	.85308	1.17223	.66218	.74934	.88369	1.13162
29	.64923	.76059	.85358	1.17154	.66240	.74915	.88421	1.13096
30	.64945	.76041	.85408	1.17085	.66262	.74896	.88473	1.13029
31	.64967	.76022	.85458	1.17016	.66284	.74876	.88524	1.12963
32	.64989	.76003	.85509	1.16947	.66306	.74857	.88576	1.12897
33	.65011	.75984	.85559	1.16878	.66327	.74838	.88628	1.12831
34	.65033	.75965	.85609	1.16809	.66349	.74818	.88680	1.12765
35	.65055	.75946	.85660	1.16741	.66371	.74799	.88732	1.12699
36	.65077	.75927	.85710	1.16672	.66393	.74780	.88784	1.12633
37	.65100	.75908	.85761	1.16603	.66414	.74760	.88836	1.12567
38	.65122	.75889	.85811	1.16535	.66436	.74741	.88888	1.12501
39	.65144	.75870	.85862	1.16466	.66458	.74722	.88940	1.12435
40	.65166	.75851	.85912	1.16398	.66480	.74703	.88992	1.12369
41	.65188	.75832	.85963	1.16329	.66501	.74683	.89045	1.12303
42	.65210	.75813	.86014	1.16261	.66523	.74664	.89097	1.12238
43	.65232	.75794	.86064	1.16192	.66545	.74644	.89149	1.12172
44	.65254	.75775	.86115	1.16124	.66566	.74625	.89201	1.12106
45	.65276	.75756	.86166	1.16056	.66588	.74606	.89253	1.12041
46	.65298	.75738	.86216	1.15987	.66610	.74586	.89306	1.11975
47	.65320	.75719	.86267	1.15919	.66632	.74567	.89358	1.11909
48	.65342	.75700	.86318	1.15851	.66653	.74548	.89410	1.11844
49	.65364	.75680	.86368	1.15783	.66675	.74528	.89463	1.11778
50	.65386	.75661	.86419	1.15715	.66697	.74509	.89515	1.11713
51	.65408	.75642	.86470	1.15647	.66718	.74489	.89567	1.11648
52	.65430	.75623	.86521	1.15579	.66740	.74470	.89620	1.11582
53	.65452	.75604	.86572	1.15511	.66762	.74451	.89672	1.11517
54	.65474	.75585	.86623	1.15443	.66783	.74431	.89725	1.11452
55	.65496	.75566	.86674	1.15375	.66805	.74412	.89777	1.11387
56	.65518	.75547	.86725	1.15308	.66827	.74392	.89830	1.11321
57	.65540	.75528	.86776	1.15240	.66848	.74373	.89883	1.11256
58	.65562	.75509	.86827	1.15172	.66870	.74353	.89935	1.11191
59	.65584	.75490	.86878	1.15104	.66891	.74334	.89988	1.11126
60	.65606	.75471	.86929	1.15037	.66913	.74314	.90040	1.11061
′	49° Cos.	49° Sin.	49° Cot.	49° Tan.	48° Cos.	48° Sin.	48° Cot.	48° Tan.

	42°			43°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
60	.66913	.74314	.90040	1.11061	.68200	.73135	.93252	1.07237
59	.66935	.74295	.90093	1 10996	.68221	.73116	.93306	1.07174
58	.66956	.74278	.90146	1.10931	.68242	.73098	.93360	1.07112
57	.66978	.74256	.90199	1.10867	.68264	.73078	.93415	1.07049
56	.66999	.74237	.90251	1.10802	.68285	.73056	.93469	1.06987
55	.67021	.74217	.90304	1.10737	.68306	.73036	.93524	1.06925
54	.67043	.74198	.90357	1.10672	.68327	.73018	.93578	1.06862
53	.67064	.74178	.90410	1.10607	.68349	.72998	.93633	1.06800
52	.67086	.74159	.90463	1.10543	.68370	.72978	.93688	1.06738
51	.67107	.74139	.90516	1.10478	.68391	.72957	.93742	1.06676
50	.67129	.74120	.90569	1.10414	.68412	.72937	.93797	1.06613
49	.67151	.74100	.90621	1.10349	.68434	.72917	.93852	1.06551
48	.67172	.74080	.90674	1.10285	.68455	.72897	.93906	1.06489
47	.67194	.74061	.90727	1.10220	.68476	.72877	.93961	1.06427
46	.67215	.74041	.90781	1.10156	.68497	.72857	.94016	1.06365
45	.67237	.74022	.90834	1.10091	.68518	.72837	.94071	1.06303
44	.67258	.74002	.90887	1.10027	.68539	.72817	.94125	1.06241
43	.67280	.73983	.90940	1.09963	.68561	.72797	.94180	1.06179
42	.67301	.73963	.90993	1.09899	.68582	.72777	.94235	1.06117
41	.67323	.73944	.91046	1.09834	.68603	.72757	.94290	1.06056
40	.67344	.73924	.91099	1.09770	.68624	.72737	.94345	1.05994
39	.67366	.73904	.91153	1.09706	.68645	.72717	.94400	1.05932
38	.67387	.73885	.91206	1.09642	.68666	.72697	.94455	1.05870
37	.67409	.73865	.91259	1.09578	.68688	.72677	.94510	1.05809
36	.67430	.73846	.91313	1.09514	.68709	.72657	.94565	1.05747
35	.67452	.73826	.91366	1.09450	.68730	.72637	.94620	1.05685
34	.67473	.73806	.91419	1.09386	.68751	.72617	.94676	1.05624
33	.67495	.73787	.91473	1.09322	.68772	.72597	.94731	1.05562
32	.67516	.73767	.91526	1.09258	.68793	.72577	.94786	1.05501
31	.67538	.73747	.91580	1.09195	.68814	.72557	.94841	1.05439
30	.67559	.73728	.91633	1.09131	.68835	.72537	.94896	1.05378
29	.67580	.73703	.91687	1.09067	.68857	.72517	.94952	1.05317
28	.67602	.73688	.91740	1.09003	.68878	.72497	.95007	1.05255
27	.67623	.73669	.91794	1.08940	.68899	.72477	.95062	1.05194
26	.67645	.73649	.91847	1.08876	.68920	.72457	.95118	1.05133
25	.67666	.73629	.91901	1.08813	.68941	.72437	.95173	1.05072
24	.67688	.73610	.91955	1.08749	.68962	.72417	.95229	1.05010
23	.67709	.73590	.92008	1.08686	.68983	.72397	.95284	1.04949
22	.67730	.73570	.92062	1.08622	.69004	.72377	.95340	1.04888
21	.67752	.73551	.92116	1.08559	.69025	.72357	.95395	1.04827
20	.67773	.73531	.92170	1.08496	.69046	.72337	.95451	1.04766
19	.67795	.73511	.92224	1.08432	.69067	.72317	.95506	1.04705
18	.67816	.73491	.92277	1.08369	.69088	.72297	.95562	1.04644
17	.67837	.73472	.92331	1.08306	.69109	.72277	.95618	1.04583
16	.67859	.73452	.92385	1.08243	.69130	.72257	.95673	1.04522
15	.67880	.73432	.92439	1.08179	.69151	.72236	.95729	1.04461
14	.67901	.73413	.92493	1.08116	.69172	.72216	.95785	1.04401
13	.67923	.73393	.92547	1.08053	.69193	.72196	.95841	1.04340
12	.67944	.73373	.92601	1.07990	.69214	.72176	.95897	1.04279
11	.67965	.73353	.92655	1.07927	.69235	.72156	.95952	1.04218
10	.67987	.73333	.92709	1.07864	.69256	.72136	.96008	1.04158
9	.68008	.73314	.92763	1.07801	.69277	.72116	.96064	1.04097
8	.68029	.73294	.92817	1.07738	.69298	.72095	.96120	1.04036
7	.68051	.73274	.92872	1.07676	.69319	.72075	.96176	1.03976
6	.68072	.73254	.92926	1.07613	.69340	.72055	.96232	1.03915
5	.68093	.73234	.92980	1.07550	.69361	.72035	.96288	1.03855
4	.68115	.73215	.93034	1.07487	.69382	.72015	.96344	1.03794
3	.68136	.73195	.93088	1.07425	.69403	.71995	.96400	1.03734
2	.68157	.73175	.93143	1.07362	.69424	.71974	.96457	1.03674
1	.68179	.73155	.93197	1.07299	.69445	.71954	.96513	1.03613
0	.68200	.73135	.93252	1.07237	.69466	.71934	.96569	1.03553
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.

44°

′	Sin.	Cos.	Tan.	Cot.	′
0	.69466	.71934	.96569	1.03553	60
1	.69487	.71914	.96625	1.03493	59
2	.69508	.71894	.96681	1.03433	58
3	.69529	.71873	.96738	1.03372	57
4	.69549	.71853	.96794	1.03312	56
5	.69570	.71833	.96850	1.03252	55
6	.69591	.71813	.96907	1.03192	54
7	.69612	.71792	.96963	1.03132	53
8	.69633	.71772	.97020	1.03072	52
9	.69654	.71752	.97076	1.03012	51
10	.69675	.71732	.97133	1.02952	50
11	.69696	.71711	.97189	1.02892	49
12	.69717	.71691	.97246	1.02832	48
13	.69737	.71671	.97302	1.02772	47
14	.69758	.71650	.97359	1.02713	46
15	.69779	.71630	.97416	1.02653	45
16	.69800	.71610	.97472	1.02593	44
17	.69821	.71590	.97529	1.02533	43
18	.69842	.71569	.97586	1.02474	42
19	.69862	.71549	.97643	1.02414	41
20	.69883	.71529	.97700	1.02355	40
21	.69904	.71508	.97756	1.02295	39
22	.69925	.71488	.97813	1.02236	38
23	.69946	.71468	.97870	1.02176	37
24	.69966	.71447	.97927	1.02117	36
25	.69987	.71427	.97984	1.02057	35
26	.70008	.71407	.98041	1.01998	34
27	.70029	.71386	.98098	1.01939	33
28	.70049	.71366	.98155	1.01879	32
29	.70070	.71345	.98213	1.01820	31
30	.70091	.71325	.98270	1.01761	30
′	Cos.	Sin.	Cot.	Tan.	′

45°

44°

′	Sin.	Cos.	Tan.	Cot.	′
30	.70091	.71325	.98270	1.01761	30
31	.70112	.71305	.98327	1.01702	29
32	.70132	.71284	.98384	1.01642	28
33	.70153	.71264	.98441	1.01583	27
34	.70174	.71243	.98499	1.01524	26
35	.70195	.71223	.98556	1.01465	25
36	.70215	.71203	.98613	1.01406	24
37	.70236	.71182	.98671	1.01347	23
38	.70257	.71162	.98728	1.01288	22
39	.70277	.71141	.98786	1.01229	21
40	.70298	.71121	.98843	1.01170	20
41	.70319	.71100	.98901	1.01112	19
42	.70339	.71080	.98958	1.01053	18
43	.70360	.71059	.99016	1.00994	17
44	.70381	.71039	.99073	1.00935	16
45	.70401	.71019	.99131	1.00876	15
46	.70422	.70998	.99189	1.00818	14
47	.70443	.70978	.99247	1.00759	13
48	.70463	.70957	.99304	1.00701	12
49	.70484	.70937	.99362	1.00642	11
50	.70505	.70916	.99420	1.00583	10
51	.70525	.70896	.99478	1.00525	9
52	.70546	.70875	.99536	1.00467	8
53	.70567	.70855	.99594	1.00408	7
54	.70587	.70834	.99652	1.00350	6
55	.70608	.70813	.99710	1.00291	5
56	.70628	.70793	.99768	1.00233	4
57	.70649	.70772	.99826	1.00175	3
58	.70670	.70752	.99884	1.00116	2
59	.70690	.70731	.99942	1.00058	1
60	.70711	.70711	1.00000	1.00000	0
′	Cos.	Sin.	Cot.	Tan.	′

45°

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

0°		1°		2°		3°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
.00000	.00000	.00015	.00015	.00061	.00061	.00137	.00137	0
.00000	.00000	.00016	.00016	.00062	.00062	.00139	.00139	1
.00000	.00000	.00016	.00016	.00063	.00063	.00140	.00140	2
.00000	.00000	.00017	.00017	.00064	.00064	.00142	.00142	3
.00000	.00000	.00017	.00017	.00065	.00065	.00143	.00143	4
.00000	.00000	.00018	.00018	.00066	.00066	.00145	.00145	5
.00000	.00000	.00018	.00018	.00067	.00067	.00146	.00147	6
.00000	.00000	.00019	.00019	.00068	.00068	.00148	.00148	7
.00000	.00000	.00020	.00020	.00069	.00069	.00150	.00150	8
.00000	.00000	.00020	.00020	.00070	.00070	.00151	.00151	9
.00000	.00000	.00021	.00021	.00071	.00072	.00153	.00153	10
.00001	.00001	.00021	.00021	.00073	.00073	.00154	.00155	11
.00001	.00001	.00022	.00022	.00074	.00074	.00156	.00156	12
.00001	.00001	.00023	.00023	.00075	.00075	.00158	.00158	13
.00001	.00001	.00023	.00023	.00076	.00076	.00159	.00159	14
.00001	.00001	.00024	.00024	.00077	.00077	.00161	.00161	15
.00001	.00001	.00024	.00024	.00078	.00078	.00162	.00163	16
.00001	.00001	.00025	.00025	.00079	.00079	.00164	.00164	17
.00001	.00001	.00026	.00026	.00081	.00081	.00166	.00166	18
.00002	.00002	.00026	.00026	.00082	.00082	.00168	.00168	19
.00002	.00002	.00027	.00027	.00083	.00083	.00169	.00169	20
.00002	.00002	.00028	.00028	.00084	.00084	.00171	.00171	21
.00002	.00002	.00028	.00028	.00085	.00085	.00173	.00173	22
.00002	.00002	.00029	.00029	.00087	.00087	.00174	.00175	23
.00002	.00002	.00030	.00030	.00088	.00088	.00176	.00176	24
.00003	.00003	.00031	.00031	.00089	.00089	.00178	.00178	25
.00003	.00003	.00031	.00031	.00090	.00090	.00179	.00180	26
.00003	.00003	.00032	.00032	.00091	.00091	.00181	.00182	27
.00003	.00003	.00033	.00033	.00093	.00093	.00183	.00183	28
.00004	.00004	.00034	.00034	.00094	.00094	.00185	.00185	29
.00004	.00004	.00034	.00034	.00095	.00095	.00187	.00187	30
.00004	.00004	.00035	.00035	.00096	.00097	.00188	.00189	31
.00004	.00004	.00036	.00036	.00098	.00098	.00190	.00190	32
.00005	.00005	.00037	.00037	.00099	.00099	.00192	.00192	33
.00005	.00005	.00037	.00037	.00100	.00100	.00194	.00194	34
.00005	.00005	.00038	.00038	.00102	.00102	.00196	.00196	35
.00005	.00005	.00039	.00039	.00103	.00103	.00197	.00198	36
.00006	.00006	.00040	.00040	.00104	.00104	.00199	.00200	37
.00006	.00006	.00041	.00041	.00106	.00106	.00201	.00201	38
.00006	.00006	.00041	.00041	.00107	.00107	.00203	.00203	39
.00007	.00007	.00042	.00042	.00108	.00108	.00205	.00205	40
.00007	.00007	.00043	.00043	.00110	.00110	.00207	.00207	41
.00007	.00007	.00044	.00044	.00111	.00111	.00208	.00209	42
.00008	.00008	.00045	.00045	.00112	.00113	.00210	.00211	43
.00008	.00008	.00046	.00046	.00114	.00114	.00212	.00213	44
.00009	.00009	.00047	.00047	.00115	.00115	.00214	.00215	45
.00009	.00009	.00048	.00048	.00117	.00117	.00216	.00216	46
.00009	.00009	.00048	.00048	.00118	.00118	.00218	.00218	47
.00010	.00010	.00049	.00049	.00119	.00120	.00220	.00220	48
.00010	.00010	.00050	.00050	.00121	.00121	.00222	.00222	49
.00011	.00011	.00051	.00051	.00122	.00122	.00224	.00224	50
.00011	.00011	.00052	.00052	.00124	.00124	.00226	.00226	51
.00011	.00011	.00053	.00053	.00125	.00125	.00228	.00228	52
.00012	.00012	.00054	.00054	.00127	.00127	.00230	.00230	53
.00012	.00012	.00055	.00055	.00128	.00128	.00232	.00232	54
.00013	.00013	.00056	.00056	.00130	.00130	.00234	.00234	55
.00013	.00013	.00057	.00057	.00131	.00131	.00236	.00236	56
.00014	.00014	.00058	.00058	.00133	.00133	.00238	.00238	57
.00014	.00014	.00059	.00059	.00134	.00134	.00240	.00240	58
.00015	.00015	.00060	.00060	.00136	.00136	.00242	.00242	59
.00015	.00015	.00061	.00061	.00137	.00137	.00244	.00244	60

Inserted by permission from Webb's Railroad Construction.

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS

	4°		5°		6°		7°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.00244	.00244	.00381	.00382	.00548	.00551	.00745	.00751	
1	.00246	.00246	.00383	.00385	.00551	.00554	.00749	.00755	
2	.00248	.00248	.00386	.00387	.00554	.00557	.00752	.00758	
3	.00250	.00250	.00388	.00390	.00557	.00560	.00756	.00762	
4	.00252	.00252	.00391	.00392	.00560	.00563	.00760	.00765	
5	.00254	.00254	.00393	.00395	.00563	.00566	.00763	.00769	
6	.00256	.00257	.00396	.00397	.00566	.00569	.00767	.00773	
7	.00258	.00259	.00398	.00400	.00569	.00573	.00770	.00776	
8	.00260	.00261	.00401	.00403	.00572	.00576	.00774	.00780	
9	.00262	.00263	.00404	.00405	.00576	.00579	.00778	.00784	
10	.00264	.00265	.00406	.00408	.00579	.00582	.00781	.00787	1
11	.00266	.00267	.00409	.00411	.00582	.00585	.00785	.00791	1
12	.00269	.00269	.00412	.00413	.00585	.00588	.00789	.00795	1
13	.00271	.00271	.00114	.00416	.00588	.00592	.00792	.00799	1
14	.00273	.00274	.00417	.00419	.00591	.00595	.00796	.00802	1
15	.00275	.00276	.00420	.00421	.00594	.00598	.00800	.00806	1
16	.00277	.00278	.00422	.00424	.00598	.00601	.00803	.00810	1
17	.00279	.00280	.00425	.00427	.00601	.00604	.00807	.00813	1
18	.00281	.00282	.00428	.00429	.00604	.00608	.00811	.00817	1
19	.00284	.00284	.00430	.00432	.00607	.00611	.00814	.00821	1
20	.00286	.00287	.00433	.00435	.00610	.00614	.00818	.00825	2
21	.00288	.00289	.00436	.00438	.00614	.00617	.00822	.00828	2
22	.00290	.00291	.00438	.00440	.00617	.00621	.00825	.00832	2
23	.00293	.00293	.00441	.00443	.00620	.00624	.00829	.00836	2
24	.00295	.00296	.00444	.00446	.00623	.00627	.00833	.00840	2
25	.00297	.00298	.00447	.00449	.00626	.00630	.00837	.00844	2
26	.00299	.00300	.00449	.00451	.00630	.00634	.00840	.00848	2
27	.00301	.00302	.00452	.00454	.00633	.00637	.00844	.00851	2
28	.00304	.00305	.00455	.00457	.00636	.00640	.00848	.00855	2
29	.00306	.00307	.00458	.00460	.00640	.00644	.00852	.00859	2
30	.00308	.00309	.00460	.00463	.00643	.00647	.00856	.00863	3
31	.00311	.00312	.00463	.00465	.00646	.00650	.00859	.00867	3
32	.00313	.00314	.00466	.00468	.00649	.00654	.00863	.00871	3
33	.00315	.00316	.00469	.00471	.00653	.00657	.00867	.00875	3
34	.00317	.00318	.00472	.00474	.00656	.00660	.00871	.00878	3
35	.00320	.00321	.00474	.00477	.00659	.00664	.00875	.00882	3
36	.00322	.00323	.00477	.00480	.00663	.00667	.00878	.00886	3
37	.00324	.00326	.00480	.00482	.00666	.00671	.00882	.00890	3
38	.00327	.00328	.00483	.00485	.00669	.00674	.00886	.00894	3
39	.00329	.00330	.00486	.00488	.00673	.00677	.00890	.00898	3
40	.00332	.00333	.00489	.00491	.00676	.00681	.00894	.00902	4
41	.00334	.00335	.00492	.00494	.00680	.00684	.00898	.00906	4
42	.00336	.00337	.00494	.00497	.00683	.00688	.00902	.00910	4
43	.00339	.00340	.00497	.00500	.00688	.00691	.00906	.00914	4
44	.00341	.00342	.00500	.00503	.00690	.00695	.00909	.00918	4
45	.00343	.00345	.00503	.00506	.00693	.00698	.00913	.00922	4
46	.00346	.00347	.00506	.00509	.00697	.00701	.00917	.00926	4
47	.00348	.00350	.00509	.00512	.00700	.00705	.00921	.00930	4
48	.00351	.00352	.00512	.00515	.00703	.00708	.00925	.00934	4
49	.00353	.00354	.00515	.00518	.00707	.00712	.00929	.00938	4
50	.00356	.00357	.00518	.00521	.00710	.00715	.00933	.00942	5
51	.00358	.00359	.00521	.00524	.00714	.00719	.00937	.00946	5
52	.00361	.00362	.00524	.00527	.00717	.00722	.00941	.00950	5
53	.00363	.00364	.00527	.00530	.00721	.00726	.00945	.00954	5
54	.00365	.00367	.00530	.00533	.00724	.00730	.00949	.00958	5
55	.00368	.00369	.00533	.00536	.00728	.00733	.00953	.00962	5
56	.00370	.00372	.00536	.00539	.00731	.00737	.00957	.00966	5
57	.00373	.00374	.00539	.00542	.00735	.00740	.00961	.00970	5
58	.00375	.00377	.00542	.00545	.00738	.00744	.00965	.00975	5
59	.00378	.00379	.00545	.00548	.00742	.00747	.00969	.00979	5
60	.00381	.00382	.00548	.00551	.00745	.00751	.00973	.00983	6

	8°		9°		10°		11°		
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.00973	.00983	.01231	.01247	.01519	.01543	.01837	.01872	0
0	.00977	.00987	.01236	.01251	.01524	.01548	.01843	.01877	1
0	.00981	.00991	.01240	.01256	.01529	.01553	.01848	.01883	2
0	.00985	.00995	.01245	.01261	.01534	.01558	.01854	.01889	3
0	.00989	.00999	.01249	.01265	.01540	.01564	.01860	.01895	4
0	.00994	.01004	.01254	.01270	.01545	.01569	.01865	.01901	5
0	.00998	.01008	.01259	.01275	.01550	.01574	.01871	.01906	6
0	.01002	.01012	.01263	.01279	.01555	.01579	.01876	.01912	7
0	.01006	.01016	.01268	.01284	.01560	.01585	.01882	.01918	8
0	.01010	.01020	.01272	.01289	.01565	.01590	.01888	.01924	9
0	.01014	.01024	.01277	.01294	.01570	.01595	.01893	.01930	10
0	.01018	.01029	.01282	.01298	.01575	.01601	.01899	.01936	11
0	.01022	.01033	.01286	.01303	.01580	.01606	.01904	.01941	12
0	.01027	.01037	.01291	.01308	.01586	.01611	.01910	.01947	13
0	.01031	.01041	.01296	.01313	.01591	.01616	.01916	.01953	14
0	.01035	.01046	.01300	.01318	.01596	.01622	.01921	.01959	15
0	.01039	.01050	.01305	.01322	.01601	.01627	.01927	.01965	16
0	.01043	.01054	.01310	.01327	.01606	.01633	.01933	.01971	17
0	.01047	.01059	.01314	.01332	.01612	.01638	.01939	.01977	18
0	.01052	.01063	.01319	.01337	.01617	.01643	.01944	.01983	19
0	.01056	.01067	.01324	.01342	.01622	.01649	.01950	.01989	20
0	.01060	.01071	.01329	.01346	.01627	.01654	.01956	.01995	21
0	.01064	.01076	.01333	.01351	.01632	.01659	.01961	.02001	22
0	.01069	.01080	.01338	.01356	.01638	.01665	.01967	.02007	23
0	.01073	.01084	.01343	.01361	.01643	.01670	.01973	.02013	24
0	.01077	.01089	.01348	.01366	.01648	.01676	.01979	.02019	25
0	.01081	.01093	.01352	.01371	.01653	.01681	.01984	.02025	26
0	.01086	.01097	.01357	.01376	.01659	.01687	.01990	.02031	27
0	.01090	.01102	.01362	.01381	.01664	.01692	.01996	.02037	28
0	.01094	.01106	.01367	.01386	.01669	.01698	.02002	.02043	29
0	.01098	.01111	.01371	.01391	.01675	.01703	.02008	.02049	30
0	.01103	.01115	.01376	.01395	.01680	.01709	.02013	.02055	31
0	.01107	.01119	.01381	.01400	.01685	.01714	.02019	.02061	32
0	.01111	.01124	.01386	.01405	.01690	.01720	.02025	.02067	33
0	.01116	.01128	.01391	.01410	.01696	.01725	.02031	.02073	34
0	.01120	.01133	.01396	.01415	.01701	.01731	.02037	.02079	35
0	.01124	.01137	.01400	.01420	.01706	.01736	.02042	.02085	36
0	.01129	.01142	.01405	.01425	.01712	.01742	.02048	.02091	37
0	.01133	.01146	.01410	.01430	.01717	.01747	.02054	.02097	38
0	.01137	.01151	.01415	.01435	.01723	.01753	.02060	.02103	39
0	.01142	.01155	.01420	.01440	.01728	.01758	.02066	.02110	40
0	.01146	.01160	.01425	.01445	.01733	.01764	.02072	.02116	41
0	.01151	.01164	.01430	.01450	.01739	.01769	.02078	.02122	42
0	.01155	.01169	.01435	.01455	.01744	.01775	.02084	.02128	43
0	.01159	.01173	.01439	.01461	.01750	.01781	.02090	.02134	44
0	.01164	.01178	.01444	.01466	.01755	.01786	.02095	.02140	45
0	.01168	.01182	.01449	.01471	.01760	.01792	.02101	.02146	46
0	.01173	.01187	.01454	.01476	.01766	.01793	.02107	.02153	47
0	.01177	.01191	.01459	.01481	.01771	.01803	.02113	.02159	48
0	.01182	.01196	.01464	.01486	.01777	.01809	.02119	.02165	49
0	.01186	.01200	.01469	.01491	.01782	.01815	.02125	.02171	50
0	.01191	.01205	.01474	.01496	.01788	.01820	.02131	.02178	51
0	.01195	.01209	.01479	.01501	.01793	.01826	.02137	.02184	52
0	.01200	.01214	.01484	.01506	.01799	.01832	.02143	.02190	53
0	.01204	.01219	.01489	.01512	.01804	.01837	.02149	.02196	54
0	.01209	.01223	.01494	.01517	.01810	.01843	.02155	.02203	55
0	.01213	.01228	.01499	.01522	.01815	.01849	.02161	.02209	56
0	.01218	.01233	.01504	.01527	.01821	.01854	.02167	.02215	57
0	.01222	.01237	.01509	.01532	.01826	.01860	.02173	.02221	58
0	.01227	.01242	.01514	.01537	.01832	.01866	.02179	.02228	59
0	.01231	.01247	.01519	.01543	.01837	.01872	.02185	.02234	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANT

	12°		13°		14°		15°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.02185	.02234	.02563	.02630	.02970	.03061	.03407	.03528	
1	.02191	.02240	.02570	.02637	.02977	.03069	.03415	.03536	
2	.02197	.02247	.02576	.02644	.02985	.03076	.03422	.03544	
3	.02203	.02253	.02583	.02651	.02992	.03084	.03430	.03552	
4	.02210	.02259	.02589	.02658	.02999	.03091	.03438	.03560	
5	.02216	.02266	.02596	.02665	.03006	.03099	.03445	.03568	
6	.02222	.02272	.02602	.02672	.03013	.03106	.03453	.03576	
7	.02228	.02279	.02609	.02679	.03020	.03114	.03460	.03584	
8	.02234	.02285	.02616	.02686	.03027	.03121	.03468	.03592	
9	.02240	.02291	.02622	.02693	.03034	.03129	.03476	.03601	
10	.02246	.02298	.02629	.02700	.03041	.03137	.03483	.03609	1
11	.02252	.02304	.02635	.02707	.03048	.03144	.03491	.03617	1
12	.02258	.02311	.02642	.02714	.03055	.03152	.03498	.03625	1
13	.02265	.02317	.02649	.02721	.03063	.03159	.03506	.03633	1
14	.02271	.02323	.02655	.02728	.03070	.03167	.03514	.03642	1
15	.02277	.02330	.02662	.02735	.03077	.03175	.03521	.03650	1
16	.02283	.02336	.02669	.02742	.03084	.03182	.03529	.03658	1
17	.02289	.02343	.02675	.02749	.03091	.03190	.03537	.03666	1
18	.02295	.02349	.02682	.02756	.03098	.03198	.03544	.03674	1
19	.02302	.02356	.02689	.02763	.03106	.03205	.03552	.03683	1
20	.02308	.02362	.02696	.02770	.03113	.03213	.03560	.03691	2
21	.02314	.02369	.02702	.02777	.03120	.03221	.03567	.03699	2
22	.02320	.02375	.02709	.02784	.03127	.03228	.03575	.03708	2
23	.02327	.02382	.02716	.02791	.03134	.03236	.03583	.03716	2
24	.02333	.02388	.02722	.02799	.03142	.03244	.03590	.03724	2
25	.02339	.02395	.02729	.02806	.03149	.03251	.03598	.03732	2
26	.02345	.02402	.02736	.02813	.03156	.03259	.03606	.03741	2
27	.02352	.02408	.02743	.02820	.03163	.03267	.03614	.03749	2
28	.02358	.02415	.02749	.02827	.03171	.03275	.03621	.03758	2
29	.02364	.02421	.02756	.02834	.03178	.03282	.03629	.03766	2
30	.02370	.02428	.02763	.02842	.03185	.03290	.03637	.03774	3
31	.02377	.02435	.02770	.02849	.03193	.03298	.03645	.03783	3
32	.02383	.02441	.02777	.02856	.03200	.03306	.03653	.03791	3
33	.02389	.02448	.02783	.02863	.03207	.03313	.03660	.03799	3
34	.02396	.02454	.02790	.02870	.03214	.03321	.03668	.03808	3
35	.02402	.02461	.02797	.02878	.03222	.03329	.03676	.03816	3
36	.02408	.02468	.02804	.02885	.03229	.03337	.03684	.03825	3
37	.02415	.02474	.02811	.02892	.03236	.03345	.03692	.03833	3
38	.02421	.02481	.02818	.02899	.03244	.03353	.03699	.03842	3
39	.02427	.02488	.02824	.02907	.03251	.03360	.03707	.03850	3
40	.02434	.02494	.02831	.02914	.03258	.03368	.03715	.03858	4
41	.02440	.02501	.02838	.02921	.03266	.03376	.03723	.03867	4
42	.02447	.02508	.02845	.02928	.03273	.03384	.03731	.03875	4
43	.02453	.02515	.02852	.02936	.03281	.03392	.03739	.03884	4
44	.02459	.02521	.02859	.02943	.03288	.03400	.03747	.03892	4
45	.02466	.02528	.02866	.02950	.03295	.03408	.03754	.03901	4
46	.02472	.02535	.02873	.02958	.03303	.03416	.03762	.03909	4
47	.02479	.02542	.02880	.02965	.03310	.03424	.03770	.03918	4
48	.02485	.02548	.02887	.02972	.03318	.03432	.03778	.03927	4
49	.02492	.02555	.02894	.02980	.03325	.03439	.03786	.03935	4
50	.02498	.02562	.02900	.02987	.03333	.03447	.03794	.03944	5
51	.02504	.02569	.02907	.02994	.03340	.03455	.03802	.03952	5
52	.02511	.02576	.02914	.03002	.03347	.03463	.03810	.03961	5
53	.02517	.02582	.02921	.03009	.03355	.03471	.03818	.03969	5
54	.02524	.02589	.02928	.03017	.03362	.03479	.03826	.03978	5
55	.02530	.02596	.02935	.03024	.03370	.03487	.03834	.03987	5
56	.02537	.02603	.02942	.03032	.03377	.03495	.03842	.03995	5
57	.02543	.02610	.02949	.03039	.03385	.03503	.03850	.04004	5
58	.02550	.02617	.02956	.03046	.03392	.03512	.03858	.04013	5
59	.02556	.02624	.02963	.03054	.03400	.03520	.03866	.04021	5
60	.02563	.02630	.02970	.03061	.03407	.03528	.03874	.04030	6

	16°		17°		18°		19°		
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec	′
	.03874	.04030	.04370	.04569	.04894	.05146	.05448	.05762	0
	.03882	.04039	.04378	.04578	.04903	.05156	.05458	.05773	1
	.03890	.04047	.04387	.04588	.04912	.05166	.05467	.05783	2
	.03898	.04056	.04395	.04597	.04921	.05176	.05477	.05794	3
	.03906	.04065	.04404	.04606	.04930	.05186	.05486	.05805	4
	.03914	.04073	.04412	.04616	.04939	.05196	.05496	.05815	5
	.03922	.04082	.04421	.04625	.04948	.05206	.05505	.05826	6
	.03930	.04091	.04429	.04635	.04957	.05216	.05515	.05836	7
	.03938	.04100	.04438	.04644	.04967	.05226	.05524	.05847	8
	.03946	.04108	.04446	.04653	.04976	.05236	.05534	.05858	9
	.03954	.04117	.04455	.04663	.04985	.05246	.05543	.05869	10
	.03963	.04126	.04464	.04672	.04994	.05256	.05553	.05879	11
	.03971	.04135	.04472	.04682	.05003	.05266	.05562	.05890	12
	.03979	.04144	.04481	.04691	.05012	.05276	.05572	.05901	13
	.03987	.04152	.04489	.04700	.05021	.05286	.05582	.05911	14
	.03995	.04161	.04498	.04710	.05030	.05297	.05591	.05922	15
	.04003	.04170	.04507	.04719	.05039	.05307	.05601	.05933	16
	.04011	.04179	.04515	.04729	.05048	.05317	.05610	.05944	17
	.04019	.04188	.04524	.04738	.05057	.05327	.05620	.05955	18
	.04028	.04197	.04533	.04748	.05067	.05337	.05630	.05965	19
	.04036	.04206	.04541	.04757	.05076	.05347	.05639	.05976	20
	.04044	.04214	.04550	.04767	.05085	.05357	.05649	.05987	21
	.04052	.04223	.04559	.04776	.05094	.05367	.05658	.05998	22
	.04060	.04232	.04567	.04786	.05103	.05378	.05668	.06009	23
	.04069	.04241	.04576	.04795	.05112	.05388	.05678	.06020	24
	.04077	.04250	.04585	.04805	.05122	.05398	.05687	.06030	25
	.04085	.04259	.04593	.04815	.05131	.05408	.05697	.06041	26
	.04093	.04268	.04602	.04824	.05140	.05418	.05707	.06052	27
	.04102	.04277	.04611	.04834	.05149	.05429	.05716	.06063	28
	.04110	.04286	.04620	.04843	.05158	.05439	.05726	.06074	29
	.04118	.04295	.04628	.04853	.05168	.05449	.05736	.06085	30
	.04126	.04304	.04637	.04863	.05177	.05460	.05746	.06096	31
	.04135	.04313	.04646	.04872	.05186	.05470	.05755	.06107	32
	.04143	.04322	.04655	.04882	.05195	.05480	.05765	.06118	33
	.04151	.04331	.04663	.04891	.05205	.05490	.05775	.06129	34
	.04159	.04340	.04672	.04901	.05214	.05501	.05785	.06140	35
	.04168	.04349	.04681	.04911	.05223	.05511	.05794	.06151	36
	.04176	.04358	.04690	.04920	.05232	.05521	.05804	.06162	37
	.04184	.04367	.04699	.04930	.05242	.05532	.05814	.06173	38
	.04193	.04376	.04707	.04940	.05251	.05542	.05824	.06184	39
	.04201	.04385	.04716	.04950	.05260	.05552	.05833	.06195	40
	.04209	.04394	.04725	.04959	.05270	.05563	.05843	.06206	41
	.04218	.04403	.04734	.04969	.05279	.05573	.05853	.06217	42
	.04226	.04413	.04743	.04979	.05288	.05584	.05863	.06228	43
	.04234	.04422	.04752	.04989	.05298	.05594	.05873	.06239	44
	.04243	.04431	.04760	.04998	.05307	.05604	.05882	.06250	45
	.04251	.04440	.04769	.05008	.05316	.05615	.05892	.06261	46
	.04260	.04449	.04778	.05018	.05326	.05625	.05902	.06272	47
	.04268	.04458	.04787	.05028	.05335	.05636	.05912	.06283	48
	.04276	.04468	.04796	.05038	.05344	.05646	.05922	.06295	49
	.04285	.04477	.04805	.05047	.05354	.05657	.05932	.06306	50
	.04293	.04486	.04814	.05057	.05363	.05667	.05942	.06317	51
	.04302	.04495	.04823	.05067	.05373	.05678	.05951	.06328	52
	.04310	.04504	.04832	.05077	.05382	.05688	.05961	.06339	53
	.04319	.04514	.04841	.05087	.05391	.05699	.05971	.06350	54
	.04327	.04523	.04850	.05097	.05401	.05709	.05981	.06362	55
	.04336	.04532	.04858	.05107	.05410	.05720	.05991	.06373	56
	.04344	.04541	.04867	.05116	.05420	.05730	.06001	.06384	57
	.04353	.04551	.04876	.05126	.05429	.05741	.06011	.06395	58
	.04361	.04560	.04885	.05136	.05439	.05751	.06021	.06407	59
	.04370	.04569	.04894	.05146	.05448	.05762	.06031	.06418	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECA

	20°		21°		22°		23°	
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.
0	.06031	.06418	.06642	.07115	.07282	.07853	.07950	.08656
1	.06041	.06429	.06652	.07126	.07293	.07866	.07961	.08649
2	.06051	.06440	.06663	.07138	.07303	.07879	.07972	.08663
3	.06061	.06452	.06673	.07150	.07314	.07892	.07984	.08676
4	.06071	.06463	.06684	.07162	.07325	.07904	.07995	.08690
5	.06081	.06474	.06694	.07174	.07336	.07917	.08006	.08703
6	.06091	.06486	.06705	.07186	.07347	.07930	.08018	.08717
7	.06101	.06497	.06715	.07199	.07358	.07943	.08029	.08730
8	.06111	.06508	.06726	.07211	.07369	.07955	.08041	.08744
9	.06121	.06520	.06736	.07223	.07380	.07968	.08052	.08757
10	.06131	.06531	.06747	.07235	.07391	.07981	.08064	.08771
11	.06141	.06542	.06757	.07247	.07402	.07994	.08075	.08784
12	.06151	.06554	.06768	.07259	.07413	.08006	.08086	.08798
13	.06161	.06565	.06778	.07271	.07424	.08019	.08098	.08811
14	.06171	.06577	.06789	.07283	.07435	.08032	.08109	.08825
15	.06181	.06588	.06799	.07295	.07446	.08045	.08121	.08839
16	.06191	.06600	.06810	.07307	.07457	.08058	.08132	.08852
17	.06201	.06611	.06820	.07320	.07468	.08071	.08144	.08866
18	.06211	.06622	.06831	.07332	.07479	.08084	.08155	.08880
19	.06221	.06634	.06841	.07344	.07490	.08097	.08167	.08893
20	.06231	.06645	.06852	.07356	.07501	.08109	.08178	.08907
21	.06241	.06657	.06863	.07368	.07512	.08122	.08190	.08921
22	.06252	.06668	.06873	.07380	.07523	.08135	.08201	.08934
23	.06262	.06680	.06884	.07393	.07534	.08148	.08213	.08948
24	.06272	.06691	.06894	.07405	.07545	.08161	.08225	.08962
25	.06282	.06703	.06905	.07417	.07556	.08174	.08236	.08975
26	.06292	.06715	.06916	.07429	.07568	.08087	.08248	.08989
27	.06302	.06726	.06926	.07442	.07579	.08200	.08259	.09003
28	.06312	.06738	.06937	.07454	.07590	.08213	.08271	.09017
29	.06323	.06749	.06948	.07466	.07601	.08226	.08282	.09030
30	.06333	.06761	.06958	.07479	.07612	.08239	.08294	.09044
31	.06343	.06773	.06969	.07491	.07623	.08252	.08306	.09058
32	.06353	.06784	.06980	.07503	.07634	.08265	.08317	.09072
33	.06363	.06796	.06990	.07516	.07645	.08278	.08329	.09086
34	.06374	.06807	.07001	.07528	.07657	.08291	.08340	.09099
35	.06384	.06819	.07012	.07540	.07668	.08305	.08352	.09113
36	.06394	.06831	.07022	.07553	.07679	.08318	.08364	.09127
37	.06404	.06843	.07033	.07565	.07690	.08331	.08375	.09141
38	.06415	.06854	.07044	.07578	.07701	.08344	.08387	.09155
39	.06425	.06866	.07055	.07590	.07713	.08357	.08399	.09169
40	.06435	.06878	.07065	.07602	.07724	.08370	.08410	.09183
41	.06445	.06889	.07076	.07615	.07735	.08383	.08422	.09197
42	.06456	.06901	.07087	.07627	.07746	.08397	.08434	.09211
43	.06466	.06913	.07098	.07640	.07757	.08410	.08445	.09224
44	.06476	.06925	.07108	.07652	.07769	.08423	.08457	.09238
45	.06486	.06936	.07119	.07665	.07780	.08436	.08469	.09252
46	.06497	.06948	.07130	.07677	.07791	.08449	.08481	.09266
47	.06507	.06960	.07141	.07690	.07802	.08463	.08492	.09280
48	.06517	.06972	.07151	.07702	.07814	.08476	.08504	.09294
49	.06528	.06984	.07162	.07715	.07825	.08485	.08516	.09308
50	.06538	.06995	.07173	.07727	.07836	.08503	.08528	.09323
51	.06548	.07007	.07184	.07740	.07848	.08516	.08539	.09337
52	.06559	.07019	.07195	.07752	.07859	.08529	.08551	.09351
53	.06569	.07031	.07206	.07765	.07870	.08542	.08563	.09365
54	.06580	.07043	.07216	.07778	.07881	.08556	.08575	.09379
55	.06590	.07055	.07227	.07790	.07893	.08569	.08586	.09393
56	.06600	.07067	.07238	.07803	.07904	.08582	.08598	.09407
57	.06611	.07079	.07249	.07816	.07915	.08596	.08610	.09421
58	.06621	.07091	.07260	.07828	.07927	.08069	.08622	.09435
59	.06632	.07103	.07271	.07841	.07938	.08623	.08634	.09449
60	.06642	.07115	.07282	.07853	.07950	.08636	.08645	.09464

24°		25°		26°		27°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
.08645	.09464	.09369	.10338	.10121	.11260	.10899	.12233	0
.08657	.09478	.09382	.10353	.10133	.11276	.10913	.12249	1
.08669	.09492	.09394	.10368	.10146	.11292	.10926	.12266	2
.08681	.09506	.09406	.10383	.10159	.11308	.10939	.12283	3
.08693	.09520	.09418	.10398	.10172	.11323	.10952	.12299	4
.08705	.09535	.09431	.10413	.10184	.11339	.10965	.12316	5
.08717	.09549	.09443	.10428	.10197	.11355	.10979	.12333	6
.08728	.09563	.09455	.10443	.10210	.11371	.10992	.12349	7
.08740	.09577	.09468	.10458	.10223	.11387	.11005	.12366	8
.08752	.09592	.09480	.10473	.10236	.11403	.11019	.12383	9
.08764	.09606	.09493	.10488	.10248	.11419	.11032	.12400	10
.08776	.09620	.09505	.10503	.10261	.11435	.11045	.12416	11
.08788	.09635	.09517	.10518	.10274	.11451	.11058	.12433	12
.08800	.09649	.09530	.10533	.10287	.11467	.11072	.12450	13
.08812	.09663	.09542	.10549	.10300	.11483	.11085	.12467	14
.08824	.09678	.09554	.10564	.10313	.11499	.11098	.12484	15
.08836	.09692	.09567	.10579	.10326	.11515	.11112	.12501	16
.08848	.09707	.09579	.10594	.10338	.11531	.11125	.12518	17
.08860	.09721	.09592	.10609	.10351	.11547	.11138	.12534	18
.08872	.09735	.09604	.10625	.10364	.11563	.11152	.12551	19
.08884	.09750	.09617	.10640	.10377	.11579	.11165	.12568	20
.08896	.09764	.09629	.10655	.10390	.11595	.11178	.12585	21
.08908	.09779	.09642	.10670	.10403	.11611	.11192	.12602	22
.08920	.09793	.09654	.10686	.10416	.11627	.11205	.12619	23
.08932	.09808	.09666	.10701	.10429	.11643	.11218	.12636	24
.08944	.09822	.09679	.10716	.10442	.11659	.11232	.12653	25
.08956	.09837	.09691	.10731	.10455	.11675	.11245	.12670	26
.08968	.09851	.09704	.10747	.10468	.11691	.11259	.12687	27
.08980	.09866	.09716	.10762	.10481	.11708	.11272	.12704	28
.08992	.09880	.09729	.10777	.10494	.11724	.11285	.12721	29
.09004	.09895	.09741	.10793	.10507	.11740	.11299	.12738	30
.09016	.09909	.09754	.10808	.10520	.11758	.11312	.12755	31
.09028	.09924	.09767	.10824	.10533	.11774	.11326	.12772	32
.09040	.09939	.09779	.10839	.10546	.11789	.11339	.12789	33
.09052	.09953	.09792	.10854	.10559	.11805	.11353	.12807	34
.09064	.09968	.09804	.10870	.10572	.11821	.11366	.12824	35
.09076	.09982	.09817	.10885	.10585	.11838	.11380	.12841	36
.09089	.09997	.09829	.10901	.10598	.11854	.11393	.12858	37
.09101	.10012	.09842	.10916	.10611	.11870	.11407	.12875	38
.09113	.10026	.09854	.10932	.10624	.11886	.11420	.12892	39
.09125	.10041	.09867	.10947	.10637	.11903	.11434	.12910	40
.09137	.10055	.09880	.10963	.10650	.11919	.11447	.12927	41
.09149	.10071	.09892	.10978	.10663	.11936	.11461	.12944	42
.09161	.10085	.09905	.10994	.10676	.11952	.11474	.12961	43
.09174	.10100	.09918	.11009	.10689	.11968	.11488	.12979	44
.09186	.10115	.09930	.11025	.10702	.11985	.11501	.12996	45
.09198	.10130	.09943	.11041	.10715	.12001	.11515	.13013	46
.09210	.10144	.09955	.11056	.10728	.12018	.11528	.13031	47
.09222	.10159	.09968	.11072	.10741	.12034	.11542	.13048	48
.09234	.10174	.09981	.11087	.10755	.12051	.11555	.13065	49
.09247	.10189	.09993	.11103	.10768	.12067	.11569	.13083	50
.09259	.10204	.10006	.11119	.10781	.12084	.11583	.13100	51
.09271	.10218	.10019	.11134	.10794	.12100	.11596	.13117	52
.09283	.10233	.10032	.11150	.10807	.12117	.11610	.13135	53
.09296	.10248	.10044	.11166	.10820	.12133	.11623	.13152	54
.09308	.10263	.10057	.11181	.10833	.12150	.11637	.13170	55
.09320	.10278	.10070	.11197	.10847	.12166	.11651	.13187	56
.09332	.10293	.10082	.11213	.10860	.12183	.11664	.13205	57
.09345	.10308	.10095	.11229	.10873	.12199	.11678	.13222	58
.09357	.10323	.10108	.11244	.10886	.12216	.11692	.13240	59
.09369	.10338	.10121	.11260	.10899	.12233	.11705	.13257	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS

	28°		29°		30°		31°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.11705	.13257	.12538	.14335	.13397	.15470	.14283	.16663	6
1	.11719	.13275	.12552	.14354	.13412	.15489	.14298	.16684	
2	.11733	.13292	.12566	.14372	.13427	.15509	.14313	.16704	
3	.11746	.13310	.12580	.14391	.13441	.15528	.14328	.16725	
4	.11760	.13327	.12595	.14409	.13456	.15548	.14343	.16745	
5	.11774	.13345	.12609	.14428	.13470	.15567	.14358	.16766	
6	.11787	.13362	.12623	.14446	.13485	.15587	.14373	.16786	
7	.11801	.13380	.12637	.14465	.13499	.15606	.14388	.16806	
8	.11815	.13398	.12651	.14483	.13514	.15626	.14403	.16827	
9	.11828	.13415	.12665	.14502	.13529	.15645	.14418	.16848	
10	.11842	.13433	.12679	.14521	.13543	.15665	.14433	.16868	1
11	.11856	.13451	.12694	.14539	.13558	.15684	.14449	.16889	1
12	.11870	.13468	.12708	.14558	.13573	.15704	.14464	.16909	1
13	.11883	.13486	.12722	.14576	.13587	.15724	.14479	.16930	1
14	.11897	.13504	.12736	.14595	.13602	.15743	.14494	.16950	1
15	.11911	.13521	.12750	.14614	.13616	.15763	.14509	.16971	1
16	.11925	.13539	.12765	.14632	.13631	.15782	.14524	.16992	1
17	.11938	.13557	.12779	.14651	.13646	.15802	.14539	.17012	1
18	.11952	.13575	.12793	.14670	.13660	.15822	.14554	.17033	1
19	.11966	.13593	.12807	.14689	.13675	.15841	.14569	.17054	1
20	.11980	.13610	.12822	.14707	.13690	.15861	.14584	.17075	2
21	.11994	.13628	.12836	.14726	.13705	.15881	.14599	.17095	2
22	.12007	.13646	.12850	.14745	.13719	.15901	.14615	.17116	2
23	.12021	.13664	.12864	.14764	.13734	.15920	.14630	.17137	2
24	.12035	.13682	.12879	.14782	.13749	.15940	.14645	.17158	2
25	.12049	.13700	.12893	.14801	.13763	.15960	.14660	.17178	2
26	.12063	.13718	.12907	.14820	.13778	.15980	.14675	.17199	2
27	.12077	.13735	.12921	.14839	.13793	.16000	.14690	.17220	2
28	.12091	.13753	.12936	.14858	.13808	.16019	.14706	.17241	2
29	.12104	.13771	.12950	.14877	.13822	.16039	.14721	.17262	2
30	.12118	.13789	.12964	.14896	.13837	.16059	.14736	.17283	3
31	.12132	.13807	.12979	.14914	.13852	.16079	.14751	.17304	3
32	.12146	.13825	.12993	.14933	.13867	.16099	.14766	.17325	3
33	.12160	.13843	.13007	.14952	.13881	.16119	.14782	.17346	3
34	.12174	.13861	.13022	.14971	.13896	.16139	.14797	.17367	3
35	.12188	.13879	.13036	.14990	.13911	.16159	.14812	.17388	3
36	.12202	.13897	.13051	.15009	.13926	.16179	.14827	.17409	3
37	.12216	.13916	.13065	.15028	.13941	.16199	.14843	.17430	3
38	.12230	.13934	.13079	.15047	.13955	.16219	.14858	.17451	3
39	.12244	.13952	.13094	.15066	.13970	.16239	.14873	.17472	3
40	.12257	.13970	.13108	.15085	.13985	.16259	.14888	.17493	4
41	.12271	.13988	.13122	.15104	.14000	.16279	.14904	.17514	4
42	.12285	.14006	.13137	.15124	.14015	.16299	.14919	.17535	4
43	.12299	.14024	.13151	.15143	.14030	.16319	.14934	.17556	4
44	.12313	.14042	.13166	.15162	.14044	.16339	.14949	.17577	4
45	.12327	.14061	.13180	.15181	.14059	.16359	.14965	.17598	4
46	.12341	.14079	.13195	.15200	.14074	.16380	.14980	.17620	4
47	.12355	.14097	.13209	.15219	.14089	.16400	.14995	.17641	4
48	.12369	.14115	.13223	.15239	.14104	.16420	.15011	.17662	4
49	.12383	.14134	.13238	.15258	.14119	.16440	.15026	.17683	4
50	.12397	.14152	.13252	.15277	.14134	.16460	.15041	.17704	5
51	.12411	.14170	.13267	.15296	.14149	.16481	.15057	.17726	5
52	.12425	.14188	.13281	.15315	.14164	.16501	.15072	.17747	5
53	.12439	.14207	.13296	.15335	.14179	.16521	.15087	.17768	5
54	.12454	.14225	.13310	.15354	.14194	.16541	.15103	.17790	5
55	.12468	.14243	.13325	.15373	.14208	.16562	.15118	.17811	5
56	.12482	.14262	.13339	.15393	.14223	.16582	.15134	.17832	5
57	.12496	.14280	.13354	.15412	.14238	.16602	.15149	.17854	5
58	.12510	.14299	.13368	.15431	.14253	.16623	.15164	.17875	5
59	.12524	.14317	.13383	.15451	.14268	.16643	.15180	.17896	5
60	.12538	.14335	.13397	.15470	.14283	.16663	.15195	.17918	6

	32°		33°		34°		35°		
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
	·15195	·17918	·16133	·19236	·17096	·20622	·18085	·22077	0
	·15211	·17939	·16149	·19259	·17113	·20645	·18101	·22102	1
	·15226	·17961	·16165	·19281	·17129	·20669	·18118	·22127	2
	·15241	·17982	·16181	·19304	·17145	·20693	·18135	·22152	3
	·15257	·18004	·16196	·19327	·17161	·20717	·18152	·22177	4
	·15272	·18025	·16212	·19349	·17178	·20740	·18168	·22202	5
	·15288	·18047	·16228	·19372	·17194	·20764	·18185	·22227	6
	·15303	·18068	·16244	·19394	·17210	·20788	·18202	·22252	7
	·15319	·18090	·16260	·19417	·17227	·20812	·18218	·22277	8
	·15334	·18111	·16276	·19440	·17243	·20836	·18235	·22302	9
	·15350	·18133	·16292	·19463	·17259	·20859	·18252	·22327	10
	·15365	·18155	·16308	·19485	·17276	·20883	·18269	·22352	11
	·15381	·18176	·16324	·19508	·17292	·20907	·18286	·22377	12
	·15396	·18198	·16340	·19531	·17308	·20931	·18302	·22402	13
	·15412	·18220	·16355	·19554	·17325	·20955	·18319	·22428	14
	·15427	·18241	·16371	·19576	·17341	·20979	·18336	·22453	15
	·15443	·18263	·16387	·19599	·17357	·21003	·18353	·22478	16
	·15458	·18285	·16403	·19622	·17374	·21027	·18369	·22503	17
	·15474	·18307	·16419	·19645	·17390	·21051	·18386	·22528	18
	·15489	·18328	·16435	·19668	·17407	·21075	·18403	·22554	19
	·15505	·18350	·16451	·19691	·17423	·21099	·18420	·22579	20
	·15520	·18372	·16467	·19713	·17439	·21123	·18437	·22604	21
	·15536	·18394	·16483	·19736	·17456	·21147	·18454	·22629	22
	·15552	·18416	·16499	·19759	·17472	·21171	·18470	·22655	23
	·15567	·18437	·16515	·19782	·17489	·21195	·18487	·22680	24
	·15583	·18459	·16531	·19805	·17505	·21220	·18504	·22706	25
	·15598	·18481	·16547	·19828	·17522	·21244	·18521	·22731	26
	·15614	·18503	·16563	·19851	·17538	·21268	·18538	·22756	27
	·15630	·18525	·16579	·19874	·17554	·21292	·18555	·22782	28
	·15645	·18547	·16595	·19897	·17571	·21316	·18572	·22807	29
	·15661	·18569	·16611	·19920	·17587	·21341	·18588	·22833	30
	·15676	·18591	·16627	·19944	·17604	·21365	·18605	·22858	31
	·15692	·18613	·16644	·19967	·17620	·21389	·18622	·22884	32
	·15708	·18635	·16660	·19990	·17637	·21414	·18639	·22909	33
	·15723	·18657	·16676	·20013	·17653	·21438	·18656	·22935	34
	·15739	·18679	·16692	·20036	·17670	·21462	·18673	·22960	35
	·15755	·18701	·16708	·20059	·17686	·21487	·18690	·22986	36
	·15770	·18723	·16724	·20083	·17703	·21511	·18707	·23012	37
	·15786	·18745	·16740	·20106	·17719	·21535	·18724	·23037	38
	·15802	·18767	·16756	·20129	·17736	·21560	·18741	·23063	39
	·15818	·18790	·16772	·20152	·17752	·21584	·18758	·23089	40
	·15833	·18812	·16788	·20176	·17769	·21609	·18775	·23114	41
	·15849	·18834	·16805	·20199	·17786	·21633	·18792	·23140	42
	·15865	·18856	·16821	·20222	·17802	·21658	·18809	·23166	43
	·15880	·18878	·16837	·20246	·17819	·21682	·18826	·23192	44
	·15896	·18901	·16853	·20269	·17835	·21707	·18843	·23217	45
	·15912	·18923	·16869	·20292	·17852	·21731	·18860	·23243	46
	·15928	·18945	·16885	·20316	·17868	·21756	·18877	·23269	47
	·15943	·18967	·16902	·20339	·17885	·21781	·18894	·23295	48
	·15959	·18990	·16918	·20363	·17902	·21805	·18911	·23321	49
	·15975	·19012	·16934	·20386	·17918	·21830	·18928	·23347	50
	·15991	·19034	·16950	·20410	·17935	·21855	·18945	·23373	51
	·16006	·19057	·16966	·20433	·17952	·21879	·18962	·23399	52
	·16022	·19079	·16983	·20457	·17968	·21904	·18979	·23424	53
	·16038	·19102	·16999	·20480	·17985	·21929	·18996	·23450	54
	·16054	·19124	·17015	·20504	·18001	·21953	·19013	·23476	55
	·16070	·19146	·17031	·20527	·18018	·21978	·19030	·23502	56
	·16085	·19169	·17047	·20551	·18035	·22003	·19047	·23529	57
	·16101	·19191	·17064	·20575	·18051	·22028	·19064	·23555	58
	·16117	·19214	·17080	·20598	·18068	·22053	·19081	·23581	59
	·16133	·19236	·17096	·20622	·18085	·22077	·19098	·23607	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANT

′	36° Vers.	36° Ex. sec.	37° Vers.	37° Ex. sec.	38° Vers.	38° Ex. sec.	39° Vers.	39° Ex. sec.	′
0	.19098	.23607	.20136	.25214	.21199	.26902	.22285	.28676	
1	.19115	.23633	.20154	.25241	.21217	.26931	.22304	.28706	
2	.19133	.23659	.20171	.25269	.21235	.26960	.22322	.28737	
3	.19150	.23685	.20189	.25298	.21253	.26988	.22340	.28767	
4	.19167	.23711	.20207	.25324	.21271	.27017	.22359	.28797	
5	.19184	.23738	.20224	.25351	.21289	.27046	.22377	.28828	
6	.19201	.23764	.20242	.25379	.21307	.27075	.22395	.28858	
7	.19218	.23790	.20259	.25406	.21324	.27104	.22414	.28889	
8	.19235	.23816	.20277	.25434	.21342	.27133	.22432	.28919	
9	.19252	.23843	.20294	.25462	.21360	.27162	.22450	.28950	
10	.19270	.23869	.20312	.25489	.21378	.27191	.22469	.28980	1
11	.19287	.23895	.20329	.25517	.21396	.27221	.22487	.29011	1
12	.19304	.23922	.20347	.25545	.21414	.27250	.22506	.29042	1
13	.19321	.23948	.20365	.25572	.21432	.27279	.22524	.29072	1
14	.19338	.23975	.20382	.25600	.21450	.27308	.22542	.29103	1
15	.19356	.24001	.20400	.25628	.21468	.27337	.22561	.29133	1
16	.19373	.24028	.20417	.25656	.21486	.27366	.22579	.29164	1
17	.19390	.24054	.20435	.25683	.21504	.27396	.22598	.29195	1
18	.19407	.24081	.20453	.25711	.21522	.27425	.22616	.29226	1
19	.19424	.24107	.20470	.25739	.21540	.27454	.22634	.29256	1
20	.19442	.24134	.20488	.25767	.21558	.27483	.22653	.29287	2
21	.19459	.24160	.20506	.25795	.21576	.27513	.22671	.29318	2
22	.19476	.24187	.20523	.25823	.21595	.27542	.22690	.29349	2
23	.19493	.24213	.20541	.25851	.21613	.27572	.22708	.29380	2
24	.19511	.24240	.20559	.25879	.21631	.27601	.22727	.29411	2
25	.19528	.24267	.20576	.25907	.21649	.27630	.22745	.29442	2
26	.19545	.24293	.20594	.25935	.21667	.27660	.22764	.29473	2
27	.19562	.24320	.20612	.25963	.21685	.27689	.22782	.29504	2
28	.19580	.24347	.20629	.25991	.21703	.27719	.22801	.29535	2
29	.19597	.24373	.20647	.26019	.21721	.27748	.22819	.29566	2
30	.19614	.24400	.20665	.26047	.21739	.27778	.22838	.29597	3
31	.19632	.24427	.20682	.26075	.21757	.27807	.22856	.29628	3
32	.19649	.24454	.20700	.26104	.21775	.27837	.22875	.29659	3
33	.19666	.24481	.20718	.26132	.21794	.27867	.22893	.29690	3
34	.19684	.24508	.20736	.26160	.21812	.27896	.22912	.29721	3
35	.19701	.24534	.20753	.26188	.21830	.27926	.22930	.29752	3
36	.19718	.24561	.20771	.26216	.21848	.27956	.22949	.29784	3
37	.19736	.24588	.20789	.26245	.21866	.27985	.22967	.29815	3
38	.19753	.24615	.20807	.26273	.21884	.28015	.22986	.29846	3
39	.19770	.24642	.20824	.26301	.21902	.28045	.23004	.29877	3
40	.19788	.24669	.20842	.26330	.21921	.28075	.23023	.29909	4
41	.19805	.24696	.20860	.26358	.21939	.28105	.23041	.29940	4
42	.19822	.24723	.20878	.26387	.21957	.28134	.23060	.29971	4
43	.19840	.24750	.20895	.26415	.21975	.28164	.23079	.30003	4
44	.19857	.24777	.20913	.26443	.21993	.28194	.23097	.30034	4
45	.19875	.24804	.20931	.26472	.22012	.28224	.23116	.30066	4
46	.19892	.24832	.20949	.26500	.22030	.28254	.23134	.30097	4
47	.19909	.24859	.20967	.26529	.22048	.28284	.23153	.30129	4
48	.19927	.24886	.20985	.26557	.22066	.28314	.23172	.30160	4
49	.19944	.24913	.21002	.26586	.22084	.28344	.23190	.30192	4
50	.19962	.24940	.21020	.26615	.22103	.28374	.23209	.30223	5
51	.19979	.24967	.21038	.26643	.22121	.28404	.23228	.30255	5
52	.19997	.24995	.21056	.26672	.22139	.28434	.23246	.30287	5
53	.20014	.25022	.21074	.26701	.22157	.28464	.23265	.30318	5
54	.20032	.25049	.21092	.26729	.22176	.28495	.23283	.30350	5
55	.20049	.25077	.21109	.26758	.22194	.28525	.23302	.30382	5
56	.20066	.25104	.21127	.26787	.22212	.28555	.23321	.30413	5
57	.20084	.25131	.21145	.26815	.22231	.28585	.23339	.30445	5
58	.20101	.25159	.21163	.26844	.22249	.28615	.23358	.30477	5
59	.20119	.25186	.21181	.26873	.22267	.28646	.23377	.30509	5
60	.20136	.25214	.21199	.26902	.22285	.28676	.23396	.30541	6

	40°		41°		42°		43°	
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
.23396	.30541	.24529	.32501	.25686	.34563	.26865	.36733	0
.23414	.30573	.24548	.32535	.25705	.34599	.26884	.36770	1
.23433	.30605	.24567	.32568	.25724	.34634	.26904	.36807	2
.23452	.30636	.24586	.32602	.25744	.34669	.26924	.36844	3
.23470	.30668	.24605	.32636	.25763	.34704	.26944	.36881	4
.23489	.30700	.24625	.32669	.25783	.34740	.26964	.36919	5
.23508	.30732	.24644	.32703	.25802	.34775	.26984	.36956	6
.23527	.30764	.24663	.32737	.25822	.34811	.27004	.36993	7
.23545	.30796	.24682	.32770	.25841	.34846	.27024	.37030	8
.23564	.30829	.24701	.32804	.25861	.34882	.27043	.37068	9
.23583	.30861	.24720	.32838	.25880	.34917	.27063	.37105	10
.23602	.30893	.24739	.32872	.25900	.34953	.27083	.37143	11
.23620	.30925	.24759	.32905	.25920	.34988	.27103	.37180	12
.23639	.30957	.24778	.32939	.25939	.35024	.27123	.37218	13
.23658	.30989	.24797	.32973	.25959	.35060	.27143	.37255	14
.23677	.31022	.24816	.33007	.25978	.35095	.27163	.37293	15
.23696	.31054	.24835	.33041	.25998	.35131	.27183	.37330	16
.23714	.31086	.24854	.33075	.26017	.35167	.27203	.37368	17
.23733	.31119	.24874	.33109	.26037	.35203	.27223	.37406	18
.23752	.31151	.24893	.33143	.26056	.35238	.27243	.37443	19
.23771	.31183	.24912	.33177	.26076	.35274	.27263	.37481	20
.23790	.31216	.24931	.33211	.26096	.35310	.27283	.37519	21
.23808	.31248	.24950	.33245	.26115	.35346	.27303	.37556	22
.23827	.31281	.24970	.33279	.26135	.35382	.27323	.37594	23
.23846	.31313	.24989	.33314	.26154	.35418	.27343	.37632	24
.23865	.31346	.25008	.33348	.26174	.35454	.27363	.37670	25
.23884	.31378	.25027	.33382	.26194	.35490	.27383	.37708	26
.23903	.31411	.25047	.33416	.26213	.35526	.27403	.37746	27
.23922	.31443	.25066	.33451	.26233	.35562	.27423	.37784	28
.23941	.31476	.25085	.33485	.26253	.35598	.27443	.37822	29
.23959	.31509	.25104	.33519	.26272	.35634	.27463	.37860	30
.23978	.31541	.25124	.33554	.26292	.35670	.27483	.37898	31
.23997	.31574	.25143	.33588	.26312	.35707	.27503	.37936	32
.24016	.31607	.25162	.33622	.26331	.35743	.27523	.37974	33
.24035	.31640	.25182	.33657	.26351	.35779	.27543	.38012	34
.24054	.31672	.25201	.33691	.26371	.35815	.27563	.38051	35
.24073	.31705	.25220	.33726	.26390	.35852	.27583	.38089	36
.24092	.31738	.25240	.33760	.26410	.35888	.27603	.38127	37
.24111	.31771	.25259	.33795	.26430	.35924	.27623	.38165	38
.24130	.31804	.25278	.33830	.26449	.35961	.27643	.38204	39
.24149	.31837	.25297	.33864	.26469	.35997	.27663	.38242	40
.24168	.31870	.25317	.33899	.26489	.36034	.27683	.38280	41
.24187	.31903	.25336	.33934	.26509	.36070	.27703	.38319	42
.24206	.31936	.25356	.33968	.26528	.36107	.27723	.38357	43
.24225	.31969	.25375	.34003	.26548	.36143	.27743	.38396	44
.24244	.32002	.25394	.34038	.26568	.36180	.27764	.38434	45
.24262	.32035	.25414	.34073	.26588	.36217	.27784	.38473	46
.24281	.32068	.25433	.34108	.26607	.36253	.27804	.38512	47
.24300	.32101	.25452	.34142	.26627	.36290	.27824	.38550	48
.24320	.32134	.25472	.34177	.26647	.36327	.27844	.38589	49
.24339	.32168	.25491	.34212	.26667	.36363	.27864	.38628	50
.24358	.32201	.25511	.34247	.26686	.36400	.27884	.38666	51
.24377	.32234	.25530	.34282	.26706	.36437	.27905	.38705	52
.24396	.32267	.25549	.34317	.26726	.36474	.27925	.38744	53
.24415	.32301	.25569	.34352	.26746	.36511	.27945	.38783	54
.24434	.32334	.25588	.34387	.26766	.36548	.27965	.38822	55
.24453	.32368	.25608	.34423	.26785	.36585	.27985	.38860	56
.24472	.32401	.25627	.34458	.26805	.36622	.28005	.38899	57
.24491	.32434	.25647	.34493	.26825	.36659	.28026	.38938	58
.24510	.32468	.25666	.34528	.26845	.36696	.28046	.38977	59
.24529	.32501	.25686	.34563	.26865	.36733	.28066	.39016	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	44°		45°		46°		47°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.28066	.39016	.29289	.41421	.30534	.43956	.31800	.46628	0
1	.28086	.39055	.29310	.41463	.30555	.43999	.31821	.46674	1
2	.28106	.39095	.29330	.41504	.30576	.44042	.31843	.46719	2
3	.28127	.39134	.29351	.41545	.30597	.44086	.31864	.46765	3
4	.28147	.39173	.29372	.41586	.30618	.44129	.31885	.46811	4
5	.28167	.39212	.29392	.41627	.30639	.44173	.31907	.46857	5
6	.28187	.39251	.29413	.41669	.30660	.44217	.31928	.46903	6
7	.28208	.39291	.29433	.41710	.30681	.44260	.31949	.46949	7
8	.28228	.39330	.29454	.41752	.30702	.44304	.31971	.46995	8
9	.28248	.39369	.29475	.41793	.30723	.44347	.31992	.47041	9
10	.28268	.39409	.29495	.41835	.30744	.44391	.32013	.47087	10
11	.28289	.39448	.29516	.41876	.30765	.44435	.32035	.47134	11
12	.28309	.39487	.29537	.41918	.30786	.44479	.32056	.47180	12
13	.28329	.39527	.29557	.41959	.30807	.44523	.32077	.47226	13
14	.28350	.39566	.29578	.42001	.30828	.44567	.32099	.47272	14
15	.28370	.39606	.29599	.42042	.30849	.44610	.32120	.47319	15
16	.28390	.39646	.29619	.42084	.30870	.44654	.32141	.47365	16
17	.28410	.39685	.29640	.42126	.30891	.44698	.32163	.47411	17
18	.28431	.39725	.29661	.42168	.30912	.44742	.32184	.47458	18
19	.28451	.39764	.29681	.42210	.30933	.44787	.32205	.47504	19
20	.28471	.39804	.29702	.42251	.30954	.44831	.32227	.47551	20
21	.28492	.39844	.29723	.42293	.30975	.44875	.32248	.47598	21
22	.28512	.39884	.29743	.42335	.30996	.44919	.32270	.47644	22
23	.28532	.39924	.29764	.42377	.31017	.44963	.32291	.47691	23
24	.28553	.39963	.29785	.42419	.31038	.45007	.32312	.47738	24
25	.28573	.40003	.29805	.42461	.31059	.45052	.32334	.47784	25
26	.28593	.40043	.29826	.42503	.31080	.45096	.32355	.47831	26
27	.28614	.40083	.29847	.42545	.31101	.45141	.32377	.47878	27
28	.28634	.40123	.29868	.42587	.31122	.45185	.32398	.47925	28
29	.28655	.40163	.29888	.42630	.31143	.45229	.32420	.47972	29
30	.28675	.40203	.29909	.42672	.31165	.45274	.32441	.48019	30
31	.28695	.40243	.29930	.42714	.31186	.45319	.32462	.48066	31
32	.28716	.40283	.29951	.42756	.31207	.45363	.32484	.48113	32
33	.28736	.40324	.29971	.42799	.31228	.45408	.32505	.48160	33
34	.28757	.40364	.29992	.42841	.31249	.45452	.32527	.48207	34
35	.28777	.40404	.30013	.42883	.31270	.45497	.32548	.48254	35
36	.28797	.40444	.30034	.42926	.31291	.45542	.32570	.48301	36
37	.28818	.40485	.30054	.42968	.31312	.45587	.32591	.48349	37
38	.28838	.40525	.30075	.43011	.31334	.45631	.32613	.48396	38
39	.28859	.40565	.30096	.43053	.31355	.45676	.32634	.48443	39
40	.28879	.40606	.30117	.43096	.31376	.45721	.32656	.48491	40
41	.28900	.40646	.30138	.43139	.31397	.45766	.32677	.48538	41
42	.28920	.40687	.30158	.43181	.31418	.45811	.32699	.48586	42
43	.28941	.40727	.30179	.43224	.31439	.45856	.32720	.48633	43
44	.28961	.40768	.30200	.43267	.31461	.45901	.32742	.48681	44
45	.28981	.40808	.30221	.43310	.31482	.45946	.32763	.48728	45
46	.29002	.40849	.30242	.43352	.31503	.45992	.32785	.48776	46
47	.29022	.40890	.30263	.43395	.31524	.46037	.32806	.48824	47
48	.29043	.40930	.30283	.43438	.31545	.46082	.32828	.48871	48
49	.29063	.40971	.30304	.43481	.31567	.46127	.32849	.48919	49
50	.29084	.41012	.30325	.43524	.31588	.46173	.32871	.48967	50
51	.29104	.41053	.30346	.43567	.31609	.46218	.32893	.49015	51
52	.29125	.41093	.30367	.43610	.31630	.46263	.32914	.49063	52
53	.29145	.41134	.30388	.43653	.31651	.46309	.32936	.49111	53
54	.29166	.41175	.30409	.43696	.31673	.46354	.32957	.49159	54
55	.29187	.41216	.30430	.43739	.31694	.46400	.32979	.49207	55
56	.29207	.41257	.30451	.43783	.31715	.46445	.33001	.49255	56
57	.29228	.41298	.30471	.43826	.31736	.46491	.33022	.49303	57
58	.29248	.41339	.30492	.43869	.31758	.46537	.33044	.49351	58
59	.29269	.41380	.30513	.43912	.31779	.46582	.33065	.49399	59
60	.29289	.41421	.30534	.43956	.31800	.46628	.33087	.49448	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	48°		49°		50°		51°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.33087	.49448	.34394	.52425	.35721	.55572	.37068	.58902	0
1	.33109	.49496	.34416	.52476	.35744	.55626	.37091	.58959	1
2	.33130	.49544	.34438	.52527	.35766	.55680	.37113	.59016	2
3	.33152	.49593	.34460	.52579	.35788	.55734	.37136	.59073	3
4	.33173	.49641	.34482	.52630	.35810	.55789	.37158	.59130	4
5	.33195	.49690	.34504	.52681	.35833	.55843	.37181	.59188	5
6	.33217	.49738	.34526	.52732	.35855	.55897	.37204	.59245	6
7	.33238	.49787	.34548	.52784	.35877	.55951	.37226	.59302	7
8	.33260	.49835	.34570	.52835	.35900	.56005	.37249	.59360	8
9	.33282	.49884	.34592	.52886	.35922	.56060	.37272	.59418	9
10	.33303	.49933	.34614	.52938	.35944	.56114	.37294	.59475	10
11	.33325	.49981	.34636	.52989	.35967	.56169	.37317	.59533	11
12	.33347	.50030	.34658	.53041	.35989	.56223	.37340	.59590	12
13	.33368	.50079	.34680	.53092	.36011	.56278	.37362	.59648	13
14	.33390	.50128	.34702	.53144	.36034	.56332	.37385	.59706	14
15	.33412	.50177	.34724	.53196	.36056	.56387	.37408	.59764	15
16	.33434	.50226	.34746	.53247	.36078	.56442	.37430	.59822	16
17	.33455	.50275	.34768	.53299	.36101	.56497	.37453	.59880	17
18	.33477	.50324	.34790	.53351	.36123	.56551	.37476	.59938	18
19	.33499	.50373	.34812	.53403	.36146	.56606	.37498	.59996	19
20	.33520	.50422	.34834	.53455	.36168	.56661	.37521	.60054	20
21	.33542	.50471	.34856	.53507	.36190	.56716	.37544	.60112	21
22	.33564	.50521	.34878	.53559	.36213	.56771	.37567	.60171	22
23	.33586	.50570	.34900	.53611	.36235	.56826	.37589	.60229	23
24	.33607	.50619	.34923	.53663	.36258	.56881	.37612	.60287	24
25	.33629	.50669	.34945	.53715	.36280	.56937	.37635	.60346	25
26	.33651	.50718	.34967	.53768	.36302	.56992	.37658	.60404	26
27	.33673	.50767	.34989	.53820	.36325	.57047	.37680	.60463	27
28	.33694	.50817	.35011	.53872	.36347	.57103	.37703	.60521	28
29	.33716	.50866	.35033	.53924	.36370	.57158	.37726	.60580	29
30	.33738	.50916	.35055	.53977	.36392	.57213	.37749	.60639	30
31	.33760	.50966	.35077	.54029	.36415	.57269	.37771	.60698	31
32	.33782	.51015	.35099	.54082	.36437	.57324	.37794	.60756	32
33	.33803	.51065	.35122	.54134	.36460	.57380	.37817	.60815	33
34	.33825	.51115	.35144	.54187	.36482	.57436	.37840	.60874	34
35	.33847	.51165	.35166	.54240	.36504	.57491	.37862	.60933	35
36	.33869	.51215	.35188	.54292	.36527	.57547	.37885	.60992	36
37	.33891	.51265	.35210	.54345	.36549	.57603	.37908	.61051	37
38	.33912	.51314	.35232	.54398	.36572	.57659	.37931	.61111	38
39	.33934	.51364	.35254	.54451	.36594	.57715	.37954	.61170	39
40	.33956	.51415	.35277	.54504	.36617	.57771	.37976	.61229	40
41	.33978	.51465	.35299	.54557	.36639	.57827	.37999	.61288	41
42	.34000	.51515	.35321	.54610	.36662	.57883	.38022	.61348	42
43	.34022	.51565	.35343	.54663	.36684	.57939	.38045	.61407	43
44	.34044	.51615	.35365	.54716	.36707	.57995	.38068	.61467	44
45	.34065	.51665	.35388	.54769	.36729	.58051	.38091	.61526	45
46	.34087	.51716	.35410	.54822	.36752	.58108	.38113	.61586	46
47	.34109	.51766	.35432	.54876	.36775	.58164	.38136	.61646	47
48	.34131	.51817	.35454	.54929	.36797	.58221	.38159	.61705	48
49	.34153	.51867	.35476	.54982	.36820	.58277	.38182	.61765	49
50	.34175	.51918	.35499	.55036	.36842	.58333	.38205	.61825	50
51	.34197	.51968	.35521	.55089	.36865	.58390	.38228	.61885	51
52	.34219	.52019	.35543	.55143	.36887	.58447	.38251	.61945	52
53	.34241	.52069	.35565	.55196	.36910	.58503	.38274	.62005	53
54	.34262	.52120	.35588	.55250	.36932	.58560	.38296	.62065	54
55	.34284	.52171	.35610	.55303	.36955	.58617	.38319	.62125	55
56	.34306	.52222	.35632	.55357	.36978	.58674	.38342	.62185	56
57	.34328	.52273	.35654	.55411	.37000	.58731	.38365	.62246	57
58	.34350	.52323	.35677	.55465	.37023	.58788	.38388	.62306	58
59	.34372	.52374	.35699	.55518	.37045	.58845	.38411	.62366	59
60	.34394	.52425	.35721	.55572	.37068	.58902	.38434	.62427	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	52° Vers.	52° Ex. sec.	53° Vers.	53° Ex. sec.	54° Vers.	54° Ex. sec.	55° Vers.	55° Ex. sec.	′
0	.38434	.62427	.39819	.66164	.41221	.70130	.42642	.74345	0
1	.38457	.62487	.39842	.66228	.41245	.70198	.42666	.74417	1
2	.38480	.62548	.39865	.66292	.41269	.70267	.42690	.74490	2
3	.38503	.62609	.39888	.66357	.41292	.70335	.42714	.74562	3
4	.38526	.62669	.39911	.66421	.41316	.70403	.42738	.74635	4
5	.38549	.62730	.39935	.66486	.41339	.70472	.42762	.74708	5
6	.38571	.62791	.39958	.66550	.41363	.70540	.42785	.74781	6
7	.38594	.62852	.39981	.66615	.41386	.70609	.42809	.74854	7
8	.38617	.62913	.40005	.66679	.41410	.70677	.42833	.74927	8
9	.38640	.62974	.40028	.66744	.41433	.70746	.42857	.75000	9
10	.38663	.63035	.40051	.66809	.41457	.70815	.42881	.75073	10
11	.38686	.63096	.40074	.66873	.41481	.70884	.42905	.75146	11
12	.38709	.63157	.40098	.66938	.41504	.70953	.42929	.75219	12
13	.38732	.63218	.40121	.67003	.41528	.71022	.42953	.75293	13
14	.38755	.63279	.40144	.67068	.41551	.71091	.42976	.75366	14
15	.38778	.63341	.40168	.67133	.41575	.71160	.43000	.75440	15
16	.38801	.63402	.40191	.67199	.41599	.71229	.43024	.75513	16
17	.38824	.63464	.40214	.67264	.41622	.71298	.43048	.75587	17
18	.38847	.63525	.40237	.67329	.41646	.71368	.43072	.75661	18
19	.38870	.63587	.40261	.67394	.41670	.71437	.43096	.75734	19
20	.38893	.63648	.40284	.67460	.41693	.71506	.43120	.75808	20
21	.38916	.63710	.40307	.67525	.41717	.71576	.43144	.75882	21
22	.38939	.63772	.40331	.67591	.41740	.71646	.43168	.75956	22
23	.38962	.63834	.40354	.67656	.41764	.71715	.43192	.76031	23
24	.38985	.63895	.40378	.67722	.41788	.71785	.43216	.76105	24
25	.39009	.63957	.40401	.67788	.41811	.71855	.43240	.76179	25
26	.39032	.64019	.40424	.67853	.41835	.71925	.43264	.76253	26
27	.39055	.64081	.40448	.67919	.41859	.71995	.43287	.76328	27
28	.39078	.64144	.40471	.67985	.41882	.72065	.43311	.76402	28
29	.39101	.64206	.40494	.68051	.41906	.72135	.43335	.76477	29
30	.39124	.64268	.40518	.68117	.41930	.72205	.43359	.76552	30
31	.39147	.64330	.40541	.68183	.41953	.72275	.43383	.76628	31
32	.39170	.64393	.40565	.68250	.41977	.72346	.43407	.76701	32
33	.39193	.64455	.40588	.68316	.42001	.72416	.43431	.76776	33
34	.39216	.64518	.40611	.68382	.42024	.72487	.43455	.76851	34
35	.39239	.64580	.40635	.68449	.42048	.72557	.43479	.76926	35
36	.39262	.64643	.40658	.68515	.42072	.72628	.43503	.77001	36
37	.39286	.64705	.40682	.68582	.42096	.72698	.43527	.77077	37
38	.39309	.64768	.40705	.68648	.42119	.72769	.43551	.77152	38
39	.39332	.64831	.40728	.68715	.42143	.72840	.43575	.77227	39
40	.39355	.64894	.40752	.68782	.42167	.72911	.43599	.77303	40
41	.39378	.64957	.40775	.68848	.42191	.72982	.43623	.77378	41
42	.39401	.65020	.40799	.68915	.42214	.73053	.43647	.77454	42
43	.39424	.65083	.40822	.68982	.42238	.73124	.43671	.77530	43
44	.39447	.65146	.40846	.69049	.42262	.73195	.43695	.77606	44
45	.39471	.65209	.40869	.69116	.42285	.73267	.43720	.77681	45
46	.39494	.65272	.40893	.69183	.42309	.73338	.43744	.77757	46
47	.39517	.65336	.40916	.69250	.42333	.73409	.43768	.77833	47
48	.39540	.65399	.40939	.69318	.42357	.73481	.43792	.77910	48
49	.39563	.65462	.40963	.69385	.42381	.73552	.43816	.77986	49
50	.39586	.65526	.40986	.69452	.42404	.73624	.43840	.78062	50
51	.39610	.65589	.41010	.69520	.42428	.73696	.43864	.78138	51
52	.39633	.65653	.41033	.69587	.42452	.73768	.43888	.78215	52
53	.39656	.65717	.41057	.69655	.42476	.73840	.43912	.78291	53
54	.39679	.65780	.41080	.69723	.42499	.73911	.43936	.78368	54
55	.39702	.65844	.41104	.69790	.42523	.73983	.43960	.78445	55
56	.39726	.65908	.41127	.69858	.42547	.74056	.43984	.78521	56
57	.39749	.65972	.41151	.69926	.42571	.74128	.44008	.78598	57
58	.39772	.66036	.41174	.69994	.42595	.74200	.44032	.78675	58
59	.39795	.66100	.41198	.70062	.42619	.74272	.44057	.78752	59
60	.39819	.66164	.41221	.70130	.42642	.74345	.44081	.78829	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	56°		57°		58°		59°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.44081	.78829	.45536	.83608	.47008	.88708	.48496	.94160	0
1	.44105	.78908	.45560	.83690	.47033	.88796	.48521	.94254	1
2	.44129	.78984	.45585	.83773	.47057	.88884	.48546	.94349	2
3	.44153	.79061	.45609	.83855	.47082	.88972	.48571	.94443	3
4	.44177	.79138	.45634	.83938	.47107	.89060	.48596	.94537	4
5	.44201	.79216	.45658	.84020	.47131	.89148	.48621	.94632	5
6	.44225	.79293	.45683	.84103	.47156	.89237	.48646	.94726	6
7	.44250	.79371	.45707	.84186	.47181	.89325	.48671	.94821	7
8	.44274	.79449	.45731	.84269	.47206	.89414	.48696	.94916	8
9	.44298	.79527	.45756	.84352	.47230	.89503	.48721	.95011	9
10	.44322	.79604	.45780	.84435	.47255	.89591	.48746	.95106	10
11	.44346	.79682	.45805	.84518	.47280	.89680	.48771	.95201	11
12	.44370	.79761	.45829	.84601	.47304	.89769	.48796	.95296	12
13	.44395	.79839	.45854	.84685	.47329	.89858	.48821	.95392	13
14	.44419	.79917	.45878	.84768	.47354	.89948	.48846	.95487	14
15	.44443	.79995	.45903	.84852	.47379	.90037	.48871	.95583	15
16	.44467	.80074	.45927	.84935	.47403	.90126	.48896	.95678	16
17	.44491	.80152	.45951	.85019	.47428	.90216	.48921	.95774	17
18	.44516	.80231	.45976	.85103	.47453	.90305	.48946	.95870	18
19	.44540	.80309	.46000	.85187	.47478	.90395	.48971	.95966	19
20	.44564	.80388	.46025	.85271	.47502	.90485	.48996	.96062	20
21	.44588	.80467	.46049	.85355	.47527	.90575	.49021	.96158	21
22	.44612	.80546	.46074	.85439	.47552	.90665	.49046	.96255	22
23	.44637	.80625	.46098	.85523	.47577	.90755	.49071	.96351	23
24	.44661	.80704	.46123	.85608	.47601	.90845	.49096	.96448	24
25	.44685	.80783	.46147	.85692	.47626	.90935	.49121	.96544	25
26	.44709	.80862	.46172	.85777	.47651	.91026	.49146	.96641	26
27	.44734	.80942	.46196	.85861	.47676	.91116	.49171	.96738	27
28	.44758	.81021	.46221	.85946	.47701	.91207	.49196	.96835	28
29	.44782	.81101	.46246	.86031	.47725	.91297	.49221	.96932	29
30	.44806	.81180	.46270	.86116	.47750	.91388	.49246	.97029	30
31	.44831	.81260	.46295	.86201	.47775	.91479	.49271	.97127	31
32	.44855	.81340	.46319	.86286	.47800	.91570	.49296	.97224	32
33	.44879	.81419	.46344	.86371	.47825	.91661	.49321	.97322	33
34	.44903	.81499	.46368	.86457	.47849	.91752	.49346	.97420	34
35	.44928	.81579	.46393	.86542	.47874	.91844	.49372	.97517	35
36	.44952	.81659	.46417	.86627	.47899	.91935	.49397	.97615	36
37	.44976	.81740	.46442	.86713	.47924	.92027	.49422	.97713	37
38	.45001	.81820	.46466	.86799	.47949	.92118	.49447	.97811	38
39	.45025	.81900	.46491	.86885	.47974	.92210	.49472	.97910	39
40	.45049	.81981	.46516	.86970	.47998	.92302	.49497	.98008	40
41	.45073	.82061	.46540	.87056	.48023	.92394	.49522	.98107	41
42	.45098	.82142	.46565	.87142	.48048	.92486	.49547	.98205	42
43	.45122	.82222	.46589	.87229	.48073	.92578	.49572	.98304	43
44	.45146	.82303	.46614	.87315	.48098	.92670	.49597	.98403	44
45	.45171	.82384	.46639	.87401	.48123	.92762	.49623	.98502	45
46	.45195	.82465	.46663	.87488	.48148	.92855	.49648	.98601	46
47	.45219	.82546	.46688	.87574	.48172	.92947	.49673	.98700	47
48	.45244	.82627	.46712	.87661	.48197	.93040	.49698	.98799	48
49	.45268	.82709	.46737	.87748	.48222	.93133	.49723	.98899	49
50	.45292	.82790	.46762	.87834	.48247	.93226	.49748	.98998	50
51	.45317	.82871	.46786	.87921	.48272	.93319	.49773	.99098	51
52	.45341	.82953	.46811	.88008	.48297	.93412	.49799	.99198	52
53	.45365	.83034	.46836	.88095	.48322	.93505	.49824	.99298	53
54	.45390	.83116	.46860	.88183	.48347	.93598	.49849	.99398	54
55	.45414	.83198	.46885	.88270	.48372	.93692	.49874	.99498	55
56	.45439	.83280	.46909	.88357	.48396	.93785	.49899	.99598	56
57	.45463	.83362	.46934	.88445	.48421	.93879	.49924	.99698	57
58	.45487	.83444	.46959	.88532	.48446	.93973	.49950	.99799	58
59	.45512	.83526	.46983	.88620	.48471	.94068	.49975	.99899	59
60	.45536	.83608	.47008	.88708	.48496	.94160	.50000	1.00000	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	60°		61°		62°		63°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	·50000	1·00000	·51519	1 06267	·53053	1·13005	·54601	1·20269	0
1	·50025	1·00101	·51544	1·06375	·53079	1·13122	·54627	1·20395	1
2	·50050	1·00202	·51570	1·06483	·53104	1·13239	·54653	1·20521	2
3	·50076	1·00303	·51595	1·06592	·53130	1·13356	·54679	1·20647	3
4	·50101	1·00404	·51621	1·06701	·53156	1·13473	·54705	1·20773	4
5	·50126	1·00505	·51646	1·06809	·53181	1·13590	·54731	1·20900	5
6	·50151	1·00607	·51672	1·06918	·53207	1·13707	·54757	1·21026	6
7	·50176	1·00708	·51697	1·07027	·53233	1·13825	·54782	1·21153	7
8	·50202	1·00810	·51723	1·07137	·53258	1·13942	·54808	1·21280	8
9	·50227	1·00912	·51748	1·07246	·53284	1·14060	·54834	1·21407	9
10	·50252	1·01014	·51774	1·07356	·53310	1·14178	·54860	1·21535	10
11	·50277	1·01116	·51799	1·07465	·53336	1·14296	·54886	1·21662	11
12	·50303	1·01218	·51825	1·07575	·53361	1·14414	·54912	1·21790	12
13	·50328	1·01320	·51850	1·07685	·53387	1·14533	·54938	1·21918	13
14	·50353	1·01422	·51876	1·07795	·53413	1·14651	·54964	1·22045	14
15	·50378	1·01525	·51901	1·07905	·53439	1·14770	·54990	1·22174	15
16	·50404	1·01628	·51927	1·08015	·53464	1·14889	·55016	1·22302	16
17	·50429	1·01730	·51952	1·08126	·53490	1·15008	·55042	1·22430	17
18	·50454	1·01833	·51978	1·08236	·53516	1·15127	·55068	1·22559	18
19	·50479	1·01936	·52003	1·08347	·53542	1·15246	·55094	1·22688	19
20	·50505	1·02039	·52029	1·08458	·53567	1·15366	·55120	1·22817	20
21	·50530	1·02143	·52054	1·08569	·53593	1·15485	·55146	1·22946	21
22	·50555	1·02246	·52080	1·08680	·53619	1·15605	·55172	1·23075	22
23	·50581	1·02349	·52105	1·08791	·53645	1·15725	·55198	1·23205	23
24	·50606	1·02453	·52131	1·08903	·53670	1·15845	·55224	1·23334	24
25	·50631	1·02557	·52156	1·09014	·53696	1·15965	·55250	1·23464	25
26	·50656	1·02661	·52182	1·09126	·53722	1·16085	·55278	1·23594	26
27	·50682	1·02765	·52207	1·09238	·53748	1·16206	·55302	1·23724	27
28	·50707	1·02869	·52233	1·09350	·53774	1·16326	·55328	1·23855	28
29	·50732	1·02973	·52259	1·09462	·53799	1·16447	·55354	1·23985	29
30	·50758	1·03077	·52284	1·09574	·53825	1·16568	·55380	1·24116	30
31	·50783	1·03182	·52310	1·09686	·53851	1·16689	·55406	1·24247	31
32	·50808	1·03286	·52335	1·09799	·53877	1·16810	·55432	1·24378	32
33	·50834	1·03391	·52361	1·09911	·53903	1·16932	·55458	1·24509	33
34	·50859	1·03496	·52386	1·10024	·53928	1·17053	·55484	1·24640	34
35	·50884	1·03601	·52412	1·10137	·53954	1·17175	·55510	1·24772	35
36	·50910	1·03706	·52438	1·10250	·53980	1·17297	·55536	1·24903	36
37	·50935	1·03811	·52463	1·10363	·54006	1·17419	·55563	1·25035	37
38	·50960	1·03916	·52489	1·10477	·54032	1·17541	·55589	1·25167	38
39	·50986	1·04022	·52514	1·10590	·54058	1·17663	·55615	1·25300	39
40	·51011	1·04128	·52540	1·10704	·54083	1·17786	·55641	1·25432	40
41	·51036	1·04233	·52566	1·10817	·54109	1·17909	·55667	1·25565	41
42	·51062	1·04339	·52591	1·10931	·54135	1·18031	·55693	1·25697	42
43	·51087	1·04445	·52617	1·11045	·54161	1·18154	·55719	1·25830	43
44	·51113	1·04551	·52642	1·11159	·54187	1·18277	·55745	1·25963	44
45	·51138	1·04658	·52668	1·11274	·54213	1·18401	·55771	1·26097	45
46	·51163	1·04764	·52694	1·11388	·54238	1·18524	·55797	1·26230	46
47	·51189	1·04870	·52719	1·11503	·54264	1·18648	·55823	1·26364	47
48	·51214	1·04977	·52745	1·11617	·54290	1·18772	·55849	1·26498	48
49	·51239	1·05084	·52771	1·11732	·54316	1·18895	·55878	1·26632	49
50	·51265	1·05191	·52796	1·11847	·54342	1·19019	·55902	1·26766	50
51	·51290	1·05298	·52822	1·11963	·54368	1·19144	·55928	1·26900	51
52	·51316	1·05405	·52848	1·12078	·54394	1·19268	·55954	1·27035	52
53	·51341	1·05512	·52873	1·12193	·54420	1·19393	·55980	1·27169	53
54	·51366	1·05619	·52899	1·12309	·54446	1·19517	·56006	1·27304	54
55	·51392	1·05727	·52924	1·12425	·54471	1·19642	·56032	1·27439	55
56	·51417	1·05835	·52950	1·12540	·54497	1·19767	·56058	1·27574	56
57	·51443	1·05942	·52976	1·12657	·54523	1·19892	·56084	1·27710	57
58	·51468	1·06050	·53001	1·12773	·54549	1·20018	·56111	1·27845	58
59	·51494	1·06158	·53027	1·12889	·54575	1·20143	·56137	1·27981	59
60	·51519	1·06267	·53053	1·13005	·54601	1·20269	·56163	1·28117	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	64°		65°		66°		67°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.56163	1.28117	.57738	1.36820	.59326	1.45859	.60927	1.55930	0
1	.56189	1.28253	.57765	1.36768	.59353	1.46020	.60954	1.56106	1
2	.56215	1.28390	.57791	1.36916	.59379	1.46181	.60980	1.56282	2
3	.56241	1.28526	.57817	1.37064	.59406	1.46342	.61007	1.56458	3
4	.56267	1.28663	.57844	1.37212	.59433	1.46504	.61034	1.56634	4
5	.56294	1.28800	.57870	1.37361	.59459	1.46665	.61061	1.56811	5
6	.56320	1.28937	.57896	1.37509	.59486	1.46827	.61088	1.56988	6
7	.56346	1.29074	.57923	1.37658	.59512	1.46989	.61114	1.57165	7
8	.56372	1.29211	.57949	1.37808	.59539	1.47152	.61141	1.57342	8
9	.56398	1.29349	.57976	1.37957	.59566	1.47314	.61168	1.57520	9
10	.56425	1.29487	.58002	1.38107	.59592	1.47477	.61195	1.57698	10
11	.56451	1.29625	.58028	1.38258	.59619	1.47640	.61222	1.57876	11
12	.56477	1.29763	.58055	1.38408	.59645	1.47804	.61248	1.58054	12
13	.56503	1.29901	.58081	1.38558	.59672	1.47967	.61275	1.58233	13
14	.56529	1.30040	.58108	1.38707	.59699	1.48131	.61302	1.58412	14
15	.56555	1.30179	.58134	1.38857	.59725	1.48295	.61329	1.58591	15
16	.56582	1.30318	.58160	1.39008	.59752	1.48459	.61356	1.58771	16
17	.56608	1.30457	.58187	1.39159	.59779	1.48624	.61383	1.58950	17
18	.56634	1.30596	.58213	1.39311	.59805	1.48789	.61409	1.59130	18
19	.56660	1.30735	.58240	1.39462	.59832	1.48954	.61436	1.59311	19
20	.56687	1.30875	.58266	1.39614	.59859	1.49119	.61463	1.59491	20
21	.56713	1.31015	.58293	1.39766	.59885	1.49284	.61490	1.59672	21
22	.56739	1.31155	.58319	1.39918	.59912	1.49450	.61517	1.59853	22
23	.56765	1.31295	.58345	1.40070	.59938	1.49616	.61544	1.60035	23
24	.56791	1.31436	.58372	1.40222	.59965	1.49782	.61570	1.60217	24
25	.56818	1.31576	.58398	1.40375	.59992	1.49948	.61597	1.60399	25
26	.56844	1.31717	.58425	1.40528	.60018	1.50115	.61624	1.60581	26
27	.56870	1.31858	.58451	1.40681	.60045	1.50282	.61651	1.60763	27
28	.56896	1.31999	.58478	1.40835	.60072	1.50449	.61678	1.60946	28
29	.56923	1.32140	.58504	1.40988	.60098	1.50617	.61705	1.61129	29
30	.56949	1.32282	.58531	1.41142	.60125	1.50784	.61732	1.61313	30
31	.56975	1.32424	.58557	1.41296	.60152	1.50952	.61759	1.61496	31
32	.57001	1.32566	.58584	1.41450	.60178	1.51120	.61785	1.61680	32
33	.57028	1.32708	.58610	1.41605	.60205	1.51289	.61812	1.61864	33
34	.57054	1.32850	.58637	1.41760	.60232	1.51457	.61839	1.62049	34
35	.57080	1.32993	.58663	1.41914	.60259	1.51626	.61866	1.62234	35
36	.57106	1.33135	.58690	1.42070	.60285	1.51795	.61893	1.62419	36
37	.57133	1.33278	.58716	1.42225	.60312	1.51965	.61920	1.62604	37
38	.57159	1.33422	.58743	1.42380	.60339	1.52134	.61947	1.62790	38
39	.57185	1.33565	.58769	1.42536	.60365	1.52304	.61974	1.62976	39
40	.57212	1.33708	.58796	1.42692	.60392	1.52474	.62001	1.63162	40
41	.57238	1.33852	.58822	1.42848	.60419	1.52645	.62027	1.63348	41
42	.57264	1.33996	.58849	1.43005	.60445	1.52815	.62054	1.63535	42
43	.57291	1.34140	.58875	1.43162	.60472	1.52986	.62081	1.63722	43
44	.57317	1.34284	.58902	1.43318	.60499	1.53157	.62108	1.63909	44
45	.57343	1.34429	.58928	1.43476	.60526	1.53329	.62135	1.64097	45
46	.57369	1.34573	.58955	1.43633	.60552	1.53500	.62162	1.64285	46
47	.57396	1.34718	.58981	1.43790	.60579	1.53672	.62189	1.64473	47
48	.57422	1.34863	.59008	1.43948	.60606	1.53845	.62216	1.64662	48
49	.57448	1.35009	.59034	1.44106	.60633	1.54017	.62243	1.64851	49
50	.57475	1.35154	.59061	1.44264	.60659	1.54190	.62270	1.65040	50
51	.57501	1.35300	.59087	1.44423	.60686	1.54363	.62297	1.65229	51
52	.57527	1.35446	.59114	1.44582	.60713	1.54536	.62324	1.65419	52
53	.57554	1.35592	.59140	1.44741	.60740	1.54709	.62351	1.65609	53
54	.57580	1.35738	.59167	1.44900	.60766	1.54883	.62378	1.65799	54
55	.57606	1.35885	.59194	1.45059	.60793	1.55057	.62405	1.65989	55
56	.57633	1.36031	.59220	1.45219	.60820	1.55231	.62431	1.66180	56
57	.57659	1.36178	.59247	1.45378	.60847	1.55405	.62458	1.66371	57
58	.57685	1.36325	.59273	1.45539	.60873	1.55580	.62485	1.66563	58
59	.57712	1.36473	.59300	1.45699	.60900	1.55755	.62512	1.66755	59
60	.57738	1.36620	.59326	1.45859	.60927	1.55930	.62539	1.66947	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	68°		69°		70°		71°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.62539	1.66947	.64163	1.79043	.65798	1.92380	.67443	2.07155	0
1	.62566	1.67139	.64190	1.79254	.65825	1.92614	.67471	2.07415	1
2	.62593	1.67332	.64218	1.79466	.65853	1.92849	.67498	2.07675	2
3	.62620	1.67525	.64245	1.79679	.65880	1.93083	.67526	2.07936	3
4	.62647	1.67718	.64272	1.79891	.65907	1.93318	.67553	2.08197	4
5	.62674	1.67911	.64299	1.80104	.65935	1.93554	.67581	2.08459	5
6	.62701	1.68105	.64326	1.80318	.65962	1.93790	.67608	2.08721	6
7	.62728	1.68299	.64353	1.80531	.65989	1.94026	.67636	2.08983	7
8	.62755	1.68494	.64381	1.80746	.66017	1.94263	.67663	2.09246	8
9	.62782	1.68689	.64408	1.80960	.66044	1.94500	.67691	2.09510	9
10	.62809	1.68884	.64435	1.81175	.66071	1.94737	.67718	2.09774	10
11	.62836	1.69079	.64462	1.81390	.66099	1.94975	.67746	2.10038	11
12	.62863	1.69275	.64489	1.81605	.66126	1.95213	.67773	2.10303	12
13	.62890	1.69471	.64517	1.81821	.66154	1.95452	.67801	2.10568	13
14	.62917	1.69667	.64544	1.82037	.66181	1.95691	.67829	2.10834	14
15	.62944	1.69864	.64571	1.82254	.66208	1.95931	.67856	2.11101	15
16	.62971	1.70061	.64598	1.82471	.66236	1.96171	.67884	2.11367	16
17	.62998	1.70258	.64625	1.82688	.66263	1.96411	.67911	2.11635	17
18	.63025	1.70455	.64653	1.82906	.66290	1.96652	.67939	2.11903	18
19	.63052	1.70653	.64680	1.83124	.66318	1.96893	.67966	2.12171	19
20	.63079	1.70851	.64707	1.83342	.66345	1.97135	.67994	2.12440	20
21	.63106	1.71050	.64734	1.83561	.66373	1.97377	.68021	2.12709	21
22	.63133	1.71249	.64761	1.83780	.66400	1.97619	.68049	2.12979	22
23	.63161	1.71448	.64789	1.83999	.66427	1.97862	.68077	2.13249	23
24	.63188	1.71647	.64816	1.84219	.66455	1.98106	.68104	2.13520	24
25	.63215	1.71847	.64843	1.84439	.66482	1.98349	.68132	2.13791	25
26	.63242	1.72047	.64870	1.84659	.66510	1.98594	.68159	2.14063	26
27	.63269	1.72247	.64898	1.84880	.66537	1.98838	.68187	2.14335	27
28	.63296	1.72448	.64925	1.85102	.66564	1.99083	.68214	2.14608	28
29	.63323	1.72649	.64952	1.85323	.66592	1.99329	.68242	2.14881	29
30	.63350	1.72850	.64979	1.85545	.66619	1.99574	.68270	2.15155	30
31	.63377	1.73052	.65007	1.85767	.66647	1.99821	.68297	2.15429	31
32	.63404	1.73254	.65034	1.85990	.66674	2.00067	.68325	2.15704	32
33	.63431	1.73456	.65061	1.86213	.66702	2.00315	.68352	2.15979	33
34	.63458	1.73659	.65088	1.86437	.66729	2.00562	.68380	2.16255	34
35	.63485	1.73862	.65116	1.86661	.66756	2.00810	.68408	2.16531	35
36	.63512	1.74065	.65143	1.86885	.66784	2.01059	.68435	2.16808	36
37	.63539	1.74269	.65170	1.87109	.66811	2.01308	.68463	2.17085	37
38	.63566	1.74473	.65197	1.87334	.66839	2.01557	.68490	2.17363	38
39	.63594	1.74677	.65225	1.87560	.66866	2.01807	.68518	2.17641	39
40	.63621	1.74881	.65252	1.87785	.66894	2.02057	.68546	2.17920	40
41	.63648	1.75086	.65279	1.88011	.66921	2.02308	.68573	2.18199	41
42	.63675	1.75292	.65306	1.88238	.66949	2.02559	.68601	2.18479	42
43	.63702	1.75497	.65334	1.88465	.66976	2.02810	.68628	2.18759	43
44	.63729	1.75703	.65361	1.88692	.67003	2.03062	.68656	2.19040	44
45	.63756	1.75909	.65388	1.88920	.67031	2.03315	.68684	2.19322	45
46	.63783	1.76116	.65416	1.89148	.67058	2.03568	.68711	2.19604	46
47	.63810	1.76323	.65443	1.89376	.67086	2.03821	.68739	2.19886	47
48	.63838	1.76530	.65470	1.89605	.67113	2.04075	.68767	2.20169	48
49	.63865	1.76737	.65497	1.89834	.67141	2.04329	.68794	2.20453	49
50	.63892	1.76945	.65525	1.90063	.67168	2.04584	.68822	2.20737	50
51	.63919	1.77154	.65552	1.90293	.67196	2.04839	.68849	2.21021	51
52	.63946	1.77362	.65579	1.90524	.67223	2.05094	.68877	2.21306	52
53	.63973	1.77571	.65607	1.90754	.67251	2.05350	.68905	2.21592	53
54	.64000	1.77780	.65634	1.90986	.67278	2.05607	.68932	2.21878	54
55	.64027	1.77990	.65661	1.91217	.67306	2.05864	.68960	2.22165	55
56	.64055	1.78200	.65689	1.91449	.67333	2.06121	.68988	2.22452	56
57	.64082	1.78410	.65716	1.91681	.67361	2.06379	.69015	2.22740	57
58	.64109	1.78621	.65743	1.91914	.67388	2.06637	.69043	2.23028	58
59	.64136	1.78832	.65771	1.92147	.67416	2.06896	.69071	2.23317	59
60	.64163	1.79043	.65798	1.92380	.67443	2.07155	.69098	2.23607	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	72°		73°		74°		75°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	·69098	2·23607	·70763	2·42030	·72436	2·62796	·74118	2·86370	0
1	·69126	2·23897	·70791	2·42356	·72464	2·63164	·74146	2·86790	1
2	·69154	2·24187	·70818	2·42683	·72492	2·63533	·74174	2·87211	2
3	·69181	2·24478	·70846	2·43010	·72520	2·63903	·74202	2·87633	3
4	·69209	2·24770	·70874	2·43337	·72548	2·64274	·74231	2·88056	4
5	·69237	2·25062	·70902	2·43666	·72576	2·64645	·74259	2·88479	5
6	·69264	2·25355	·70930	2·43995	·72604	2·65018	·74287	2·88904	6
7	·69292	2·25648	·70958	2·44324	·72632	2·65391	·74315	2·89330	7
8	·69320	2·25942	·70985	2·44655	·72660	2·65765	·74343	2·89756	8
9	·69347	2·26237	·71013	2·44986	·72688	2·66140	·74371	2·90184	9
10	·69375	2·26531	·71041	2·45317	·72716	2·66515	·74399	2·90613	10
11	·69403	2·26827	·71069	2·45650	·72744	2·66892	·74427	2·91042	11
12	·69430	2·27123	·71097	2·45983	·72772	2·67269	·74455	2·91473	12
13	·69458	2·27420	·71125	2·46316	·72800	2·67647	·74484	2·91904	13
14	·69486	2·27717	·71153	2·46651	·72828	2·68025	·74512	2·92337	14
15	·69514	2·28015	·71180	2·46986	·72856	2·68405	·74540	2·92770	15
16	·69541	2·28313	·71208	2·47321	·72884	2·68785	·74568	2·93204	16
17	·69569	2·28612	·71236	2·47658	·72912	2·69167	·74596	2·93640	17
18	·69597	2·28912	·71264	2·47995	·72940	2·69549	·74624	2·94076	18
19	·69624	2·29212	·71292	2·48333	·72968	2·69931	·74652	2·94514	19
20	·69652	2·29512	·71320	2·48671	·72996	2·70315	·74680	2·94952	20
21	·69680	2·29814	·71348	2·49010	·73024	2·70700	·74709	2·95392	21
22	·69708	2·30115	·71375	2·49350	·73052	2·71085	·74737	2·95832	22
23	·69735	2·30418	·71403	2·49691	·73080	2·71471	·74765	2·96274	23
24	·69763	2·30721	·71431	2·50032	·73108	2·71858	·74793	2·96716	24
25	·69791	2·31024	·71459	2·50374	·73136	2·72246	·74821	2·97160	25
26	·69818	2·31328	·71487	2·50716	·73164	2·72635	·74849	2·97604	26
27	·69846	2·31633	·71515	2·51060	·73192	2·73024	·74878	2·98050	27
28	·69874	2·31939	·71543	2·51404	·73220	2·73414	·74906	2·98497	28
29	·69902	2·32244	·71571	2·51748	·73248	2·73806	·74934	2·98944	29
30	·69929	2·32551	·71598	2·52094	·73276	2·74198	·74962	2·99393	30
31	·69957	2·32858	·71626	2·52440	·73304	2·74591	·74990	2·99843	31
32	·69985	2·33166	·71654	2·52787	·73332	2·74984	·75018	3·00293	32
33	·70013	2·33474	·71682	2·53134	·73360	2·75379	·75047	3·00745	33
34	·70040	2·33783	·71710	2·53482	·73388	2·75775	·75075	3·01198	34
35	·70068	2·34092	·71738	2·53831	·73416	2·76171	·75103	3·01652	35
36	·70096	2·34403	·71766	2·54181	·73444	2·76568	·75131	3·02107	36
37	·70124	2·34713	·71794	2·54531	·73472	2·76966	·75159	3·02563	37
38	·70151	2·35025	·71822	2·54883	·73500	2·77365	·75187	3·03020	38
39	·70179	2·35336	·71850	2·55235	·73529	2·77765	·75216	3·03479	39
40	·70207	2·35649	·71877	2·55587	·73557	2·78166	·75244	3·03938	40
41	·70235	2·35962	·71905	2·55940	·73585	2·78568	·75272	3·04398	41
42	·70263	2·36276	·71933	2·56294	·73613	2·78970	·75300	3·04860	42
43	·70290	2·36590	·71961	2·56649	·73641	2·79374	·75328	3·05322	43
44	·70318	2·36905	·71989	2·57005	·73669	2·79778	·75356	3·05786	44
45	·70346	2·37221	·72017	2·57361	·73697	2·80183	·75385	3·06251	45
46	·70374	2·37537	·72045	2·57718	·73725	2·80589	·75413	3·06717	46
47	·70401	2·37854	·72073	2·58076	·73753	2·80996	·75441	3·07184	47
48	·70429	2·38171	·72101	2·58434	·73781	2·81404	·75469	3·07652	48
49	·70457	2·38489	·72129	2·58794	·73809	2·81813	·75497	3·08121	49
50	·70485	2·38808	·72157	2·59154	·73837	2·82223	·75526	3·08591	50
51	·70513	2·39128	·72185	2·59514	·73865	2·82633	·75554	3·09063	51
52	·70540	2·39448	·72213	2·59876	·73893	2·83045	·75582	3·09535	52
53	·70568	2·39768	·72241	2·60238	·73921	2·83457	·75610	3·10009	53
54	·70596	2·40089	·72269	2·60601	·73950	2·83871	·75639	3·10484	54
55	·70624	2·40411	·72296	2·60965	·73978	2·84285	·75667	3·10960	55
56	·70652	2·40734	·72324	2·61330	·74006	2·84700	·75695	3·11437	56
57	·70679	2·41057	·72352	2·61695	·74034	2·85116	·75723	3·11915	57
58	·70707	2·41381	·72380	2·62061	·74062	2·85533	·75751	3·12394	58
59	·70735	2·41705	·72408	2·62428	·74090	2·85951	·75780	3·12875	59
60	·70763	2·42030	·72436	2·62796	·74118	2·86370	·75808	3·13357	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	76°		77°		78°		79°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.75808	3.13357	.77505	3.44541	.79209	3.80973	.80919	4.24084	0
1	.75836	3.13839	.77533	3.45102	.79237	3.81633	.80948	4.24870	1
2	.75864	3.14323	.77562	3.45664	.79266	3.82294	.80976	4.25658	2
3	.75892	3.14809	.77590	3.46228	.79294	3.82956	.81005	4.26448	3
4	.75921	3.15295	.77618	3.46793	.79323	3.83621	.81033	4.27241	4
5	.75949	3.15782	.77647	3.47360	.79351	3.84288	.81062	4.28036	5
6	.75977	3.16271	.77675	3.47928	.79380	3.84956	.81090	4.28833	6
7	.76005	3.16761	.77703	3.48498	.79408	3.85627	.81119	4.29634	7
8	.76034	3.17252	.77732	3.49069	.79437	3.86299	.81148	4.30436	8
9	.76062	3.17744	.77760	3.49642	.79465	3.86973	.81176	4.31241	9
10	.76090	3.18238	.77788	3.50216	.79493	3.87649	.81205	4.32049	10
11	.76118	3.18733	.77817	3.50791	.79522	3.88327	.81233	4.32859	11
12	.76147	3.19228	.77845	3.51368	.79550	3.89007	.81262	4.33671	12
13	.76175	3.19725	.77874	3.51947	.79579	3.89689	.81290	4.34486	13
14	.76203	3.20224	.77902	3.52527	.79607	3.90373	.81319	4.35304	14
15	.76231	3.20723	.77930	3.53109	.79636	3.91058	.81348	4.36124	15
16	.76260	3.21224	.77959	3.53692	.79664	3.91746	.81376	4.36947	16
17	.76288	3.21726	.77987	3.54277	.79693	3.92436	.81405	4.37772	17
18	.76316	3.22229	.78015	3.54863	.79721	3.93128	.81433	4.38600	18
19	.76344	3.22734	.78044	3.55451	.79750	3.93821	.81462	4.39430	19
20	.76373	3.23239	.78072	3.56041	.79778	3.94517	.81491	4.40263	20
21	.76401	3.23746	.78101	3.56632	.79807	3.95215	.81519	4.41099	2.
22	.76429	3.24255	.78129	3.57224	.79835	3.95914	.81548	4.41937	2.
23	.76458	3.24764	.78157	3.57819	.79864	3.96616	.81576	4.42778	23
24	.76486	3.25275	.78186	3.58414	.79892	3.97320	.81605	4.43622	24
25	.76514	3.25787	.78214	3.59012	.79921	3.98025	.81633	4.44468	25
26	.76542	3.26300	.78242	3.59611	.79949	3.98733	.81662	4.45317	26
27	.76571	3.26814	.78271	3.60211	.79978	3.99443	.81691	4.46169	27
28	.76599	3.27330	.78299	3.60813	.80006	4.00155	.81719	4.47023	28
29	.76627	3.27847	.78328	3.61417	.80035	4.00869	.81748	4.47881	29
30	.76655	3.28366	.78356	3.62023	.80063	4.01585	.81776	4.48740	30
31	.76684	3.28885	.78384	3.62630	.80092	4.02303	.81805	4.49603	31
32	.76712	3.29406	.78413	3.63238	.80120	4.03024	.81834	4.50468	32
33	.76740	3.29929	.78441	3.63849	.80149	4.03746	.81862	4.51337	33
34	.76769	3.30452	.78470	3.64461	.80177	4.04471	.81891	4.52208	34
35	.76797	3.30977	.78498	3.65074	.80206	4.05197	.81919	4.53081	35
36	.76825	3.31503	.78526	3.65690	.80234	4.05926	.81948	4.53958	36
37	.76854	3.32031	.78555	3.66307	.80263	4.06657	.81977	4.54837	37
38	.76882	3.32560	.78583	3.66925	.80291	4.07390	.82005	4.55720	38
39	.76910	3.33090	.78612	3.67545	.80320	4.08125	.82034	4.56605	39
40	.76938	3.33622	.78640	3.68167	.80348	4.08863	.82063	4.57493	40
41	.76967	3.34154	.78669	3.68791	.80377	4.09602	.82091	4.58383	41
42	.76995	3.34689	.78697	3.69417	.80405	4.10344	.82120	4.59277	42
43	.77023	3.35224	.78725	3.70044	.80434	4.11088	.82148	4.60174	43
44	.77052	3.35761	.78754	3.70673	.80462	4.11835	.82177	4.61073	44
45	.77080	3.36299	.78782	3.71303	.80491	4.12583	.82206	4.61976	45
46	.77108	3.36839	.78811	3.71935	.80520	4.13334	.82234	4.62881	46
47	.77137	3.37380	.78839	3.72569	.80548	4.14087	.82263	4.63790	47
48	.77165	3.37923	.78868	3.73205	.80577	4.14842	.82292	4.64701	48
49	.77193	3.38466	.78896	3.73843	.80605	4.15599	.82320	4.65616	49
50	.77222	3.39012	.78924	3.74482	.80634	4.16359	.82349	4.66533	50
51	.77250	3.39558	.78953	3.75123	.80662	4.17121	.82377	4.67454	51
52	.77278	3.40106	.78981	3.75766	.80691	4.17886	.82406	4.68377	52
53	.77307	3.40656	.79010	3.76411	.80719	4.18652	.82435	4.69304	53
54	.77335	3.41206	.79038	3.77057	.80748	4.19421	.82463	4.70234	54
55	.77363	3.41759	.79067	3.77705	.80776	4.20193	.82492	4.71166	55
56	.77392	3.42312	.79095	3.78355	.80805	4.20966	.82521	4.72102	56
57	.77420	3.42867	.79123	3.79007	.80833	4.21742	.82549	4.73041	57
58	.77448	3.43424	.79152	3.79661	.80862	4.22521	.82578	4.73983	58
59	.77477	3.43982	.79180	3.80316	.80891	4.23301	.82607	4.74929	59
60	.77505	3.44541	.79209	3.80973	.80919	4.24084	.82635	4.75877	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	80°		81°		82°		83°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.82635	4.75877	.84357	5.39245	.86083	6.18530	.87813	7.20551	0
1	.82664	4.76829	.84385	5.40422	.86112	6.20020	.87842	7.22500	1
2	.82692	4.77784	.84414	5.41602	.86140	6.21517	.87871	7.24457	2
3	.82721	4.78742	.84443	5.42787	.86169	6.23019	.87900	7.26425	3
4	.82750	4.79703	.84471	5.43977	.86198	6.24529	.87929	7.28402	4
5	.82778	4.80667	.84500	5.45171	.86227	6.26044	.87957	7.30388	5
6	.82807	4.81635	.84529	5.46369	.86256	6.27566	.87986	7.32384	6
7	.82836	4.82606	.84558	5.47572	.86284	6.29095	.88015	7.34390	7
8	.82864	4.83581	.84586	5.48779	.86313	6.30630	.88044	7.36405	8
9	.82893	4.84558	.84615	5.49991	.86342	6.32171	.88073	7.38431	9
10	.82922	4.85539	.84644	5.51208	.86371	6.33719	.88102	7.40466	10
11	.82950	4.86524	.84673	5.52429	.86400	6.35274	.88131	7.42511	11
12	.82979	4.87511	.84701	5.53655	.86428	6.36835	.88160	7.44566	12
13	.83008	4.88502	.84730	5.54886	.86457	6.38403	.88188	7.46632	13
14	.83036	4.89497	.84759	5.56121	.86486	6.39978	.88217	7.48707	14
15	.83065	4.90495	.84788	5.57361	.86515	6.41560	.88246	7.50793	15
16	.83094	4.91496	.84816	5.58606	.86544	6.43148	.88275	7.52889	16
17	.83122	4.92501	.84845	5.59855	.86573	6.44743	.88304	7.54996	17
18	.83151	4.93509	.84874	5.61110	.86601	6.46346	.88333	7.57113	18
19	.83180	4.94521	.84903	5.62369	.86630	6.47955	.88362	7.59241	19
20	.83208	4.95536	.84931	5.63633	.86659	6.49571	.88391	7.61379	20
21	.83237	4.96555	.84960	5.64902	.86688	6.51194	.88420	7.63528	21
22	.83266	4.97577	.84989	5.66176	.86717	6.52825	.88448	7.65688	22
23	.83294	4.98603	.85018	5.67454	.86746	6.54462	.88477	7.67859	23
24	.83323	4.99633	.85046	5.68738	.86774	6.56107	.88506	7.70041	24
25	.83352	5.00666	.85075	5.70027	.86803	6.57759	.88535	7.72234	25
26	.83380	5.01703	.85104	5.71321	.86832	6.59418	.88564	7.74438	26
27	.83409	5.02743	.85133	5.72620	.86861	6.61085	.88593	7.76653	27
28	.83438	5.03787	.85162	5.73924	.86890	6.62759	.88622	7.78880	28
29	.83467	5.04834	.85190	5.75233	.86919	6.64441	.88651	7.81118	29
30	.83495	5.05886	.85219	5.76547	.86947	6.66130	.88680	7.83367	30
31	.83524	5.06941	.85248	5.77866	.86976	6.67826	.88709	7.85628	31
32	.83553	5.08000	.85277	5.79191	.87005	6.69530	.88737	7.87901	32
33	.83581	5.09062	.85305	5.80521	.87034	6.71242	.88766	7.90186	33
34	.83610	5.10129	.85334	5.81856	.87063	6.72962	.88795	7.92482	34
35	.83639	5.11199	.85363	5.83196	.87092	6.74689	.88824	7.94791	35
36	.83667	5.12273	.85392	5.84542	.87120	6.76424	.88853	7.97111	36
37	.83696	5.13350	.85420	5.85893	.87149	6.78167	.88882	7.99444	37
38	.83725	5.14432	.85449	5.87250	.87178	6.79918	.88911	8.01788	38
39	.83754	5.15517	.85478	5.88612	.87207	6.81677	.88940	8.04148	39
40	.83782	5.16607	.85507	5.89979	.87236	6.83443	.88969	8.06515	40
41	.83811	5.17700	.85536	5.91352	.87265	6.85218	.88998	8.08897	41
42	.83840	5.18797	.85564	5.92731	.87294	6.87001	.89027	8.11292	42
43	.83868	5.19898	.85593	5.94115	.87322	6.88792	.89055	8.13699	43
44	.83897	5.21004	.85622	5.95505	.87351	6.90592	.89084	8.16120	44
45	.83926	5.22113	.85651	5.96900	.87380	6.92400	.89113	8.18553	45
46	.83954	5.23226	.85680	5.98301	.87409	6.94216	.89142	8.20999	46
47	.83983	5.24343	.85708	5.99708	.87438	6.96040	.89171	8.23459	47
48	.84012	5.25464	.85737	6.01120	.87467	6.97873	.89200	8.25931	48
49	.84041	5.26590	.85766	6.02538	.87496	6.99714	.89229	8.28417	49
50	.84069	5.27719	.85795	6.03962	.87524	7.01565	.89258	8.30917	50
51	.84098	5.28853	.85823	6.05392	.87553	7.03423	.89287	8.33430	51
52	.84127	5.29991	.85852	6.06828	.87582	7.05291	.89316	8.35957	52
53	.84155	5.31133	.85881	6.08269	.87611	7.07167	.89345	8.38497	53
54	.84184	5.32279	.85910	6.09717	.87640	7.09052	.89374	8.41052	54
55	.84213	5.33429	.85939	6.11171	.87669	7.10946	.89403	8.43620	55
56	.84242	5.34584	.85967	6.12630	.87698	7.12849	.89431	8.46203	56
57	.84270	5.35743	.85996	6.14096	.87726	7.14760	.89460	8.48800	57
58	.84299	5.36906	.86025	6.15568	.87755	7.16681	.89489	8.51411	58
59	.84328	5.38073	.86054	6.17046	.87784	7.18612	.89518	8.54037	59
60	.84357	5.39245	.86083	6.18530	.87813	7.20551	.89547	8.56677	60

TABLE 18.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	84° Vers.	84° Ex. sec.	85° Vers.	85° Ex. sec.	86° Vers.	86° Ex. sec.	′
0	.89547	8.56677	.91284	10.47371	.93024	13.33559	0
1	.89576	8.59332	.91313	10.51199	.93053	13.39547	1
2	.89605	8.62002	.91342	10.55052	.93082	13.45586	2
3	.89634	8.64687	.91371	10.58932	.93111	13.51676	3
4	.89663	8.67387	.91400	10.62837	.93140	13.57817	4
5	.89692	8.70103	.91429	10.66769	.93169	13.64011	5
6	.89721	8.72833	.91458	10.70728	.93198	13.70258	6
7	.89750	8.75579	.91487	10.74714	.93227	13.76558	7
8	.89779	8.78341	.91516	10.78727	.93257	13.82913	8
9	.89808	8.81119	.91545	10.82768	.93286	13.89323	9
10	.89836	8.83912	.91574	10.86837	.93315	13.95788	10
11	.89865	8.86722	.91603	10.90934	.93344	14.02310	11
12	.89894	8.89547	.91632	10.95060	.93373	14.08890	12
13	.89923	8.92389	.91661	10.99214	.93402	14.15527	13
14	.89952	8.95248	.91690	11.03397	.93431	14.22223	14
15	.89981	8.98123	.91719	11.07610	.93460	14.28979	15
16	.90010	9.01015	.91748	11.11852	.93489	14.35795	16
17	.90039	9.03923	.91777	11.16125	.93518	14.42672	17
18	.90068	9.06849	.91806	11.20427	.93547	14.49611	18
19	.90097	9.09792	.91835	11.24761	.93576	14.56614	19
20	.90126	9.12752	.91864	11.29125	.93605	14.63679	20
21	.90155	9.15730	.91893	11.33521	.93634	14.70810	21
22	.90184	9.18725	.91922	11.37948	.93663	14.78005	22
23	.90213	9.21739	.91951	11.42408	.93692	14.85268	23
24	.90242	9.24770	.91980	11.46900	.93721	14.92597	24
25	.90271	9.27819	.92009	11.51424	.93750	14.99995	25
26	.90300	9.30887	.92038	11.55982	.93779	15.07462	26
27	.90329	9.33973	.92067	11.60572	.93808	15.14999	27
28	.90358	9.37077	.92096	11.65197	.93837	15.22607	28
29	.90388	9.40201	.92125	11.69856	.93866	15.30287	29
30	.90415	9.43343	.92154	11.74550	.93895	15.38041	30
31	.90444	9.46505	.92183	11.79278	.93924	15.45869	31
32	.90473	9.49685	.92212	11.84042	.93953	15.53772	32
33	.90502	9.52886	.92241	11.88841	.93982	15.61751	33
34	.90531	9.56106	.92270	11.93677	.94011	15.69808	34
35	.90560	9.59346	.92299	11.98549	.94040	15.77944	35
36	.90589	9.62605	.92328	12.03458	.94069	15.86159	36
37	.90618	9.65885	.92357	12.08404	.94098	15.94456	37
38	.90647	9.69186	.92386	12.13388	.94127	16.02835	38
39	.90676	9.72507	.92415	12.18411	.94156	16.11297	39
40	.90705	9.75849	.92444	12.23472	.94186	16.19843	40
41	.90734	9.79212	.92473	12.28572	.94215	16.28476	41
42	.90763	9.82596	.92502	12.33712	.94244	16.37196	42
43	.90792	9.86001	.92531	12.38891	.94273	16.46005	43
44	.90821	9.89428	.92560	12.44112	.94302	16.54903	44
45	.90850	9.92877	.92589	12.49373	.94331	16.63893	45
46	.90879	9.96348	.92618	12.54676	.94360	16.72975	46
47	.90908	9.99841	.92647	12.60021	.94389	16.82152	47
48	.90937	10.03356	.92676	12.65408	.94418	16.91424	48
49	.90966	10.06894	.92705	12.70838	.94447	17.00794	49
50	.90995	10.10455	.92734	12.76312	.94476	17.10262	50
51	.91024	10.14039	.92763	12.81829	.94505	17.19830	51
52	.91053	10.17646	.92792	12.87391	.94534	17.29501	52
53	.91082	10.21277	.92821	12.92999	.94563	17.39274	53
54	.91111	10.24932	.92850	12.98651	.94592	17.49153	54
55	.91140	10.28610	.92879	13.04350	.94621	17.59139	55
56	.91169	10.32313	.92908	13.10096	.94650	17.69233	56
57	.91197	10.36040	.92937	13.15889	.94679	17.79438	57
58	.91226	10.39792	.92966	13.21730	.94708	17.89755	58
59	.91255	10.43569	.92995	13.27620	.94737	18.00185	59
60	.91284	10.47371	.93024	13.33559	.94766	18.10732	60

TABLE 18.– NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	87° Vers.	87° Ex. sec.	88° Vers.	88° Ex. sec.	89° Vers.	89° Ex. sec.	′
0	.94768	18.10732	.96510	27.65371	.98255	56 29869	0
1	.94795	18.21397	.96539	27.89440	.98284	57.26976	1
2	.94825	18.32182	.96568	28.13917	.98313	58.27431	2
3	.94854	18.43088	.96597	28.38812	.98342	59.31411	3
4	.94883	18.54119	.96626	28.64137	.98371	60.39105	4
5	.94912	18.65275	.96655	28.89903	.98400	61.50715	5
6	.94941	18.76560	.96684	29.16120	.98429	62.66460	6
7	.94970	18.87976	.96714	29.42802	.98458	63.86572	7
8	.94999	18.99524	.96743	29.69960	.98487	65.11304	8
9	.95028	19.11208	.96772	29.97607	.98517	66.40927	9
10	.95057	19.23028	.96801	30.25758	.98546	67.75736	10
11	.95086	19.34989	.96830	30.54425	.98575	69.16047	11
12	.95115	19.47093	.96859	30.83623	.98604	70.62285	12
13	.95144	19.59341	.96888	31.13366	.98633	72.14583	13
14	.95173	19.71737	.96917	31.43671	.98662	73.73586	14
15	.95202	19.84283	.96946	31.74554	.98691	75.39655	15
16	.95231	19.96982	.96975	32.06030	.98720	77.13274	16
17	.95260	20.09838	.97004	32.38118	.98749	78.94988	17
18	.95289	20.22852	.97033	32.70835	.98778	80.85315	18
19	.95318	20.36027	.97062	33.04199	.98807	82.84947	19
20	.95347	20.49368	.97092	33.38232	.98836	84.94561	20
21	.95377	20.62876	.97121	33.72952	.98866	87.14924	21
22	.95406	20.76555	.97150	34.08380	.98895	89.46886	22
23	.95435	20.90409	.97179	34.44539	.98924	91.91387	23
24	.95464	21.04440	.97208	34.81452	.98953	94.49471	24
25	.95493	21.18653	.97237	35.19141	.98982	97.22303	25
26	.95522	21.33050	.97266	35.57633	.99011	100.1119	26
27	.95551	21.47635	.97295	35.96953	.99040	103.1757	27
28	.95580	21.62413	.97324	36.37127	.99069	106.4311	28
29	.95609	21.77386	.97353	36.78185	.99098	109.8966	29
30	.95638	21.92559	.97382	37.20155	.99127	113.5930	30
31	.95667	22.07935	.97411	37.63068	.99156	117.5444	31
32	.95696	22.23520	.97440	38.06957	.99186	121.7780	32
33	.95725	22.39316	.97470	38.51855	.99215	126.3253	33
34	.95754	22.55328	.97499	38.97797	.99244	131.2223	34
35	.95783	22.71563	.97528	39.44820	.99278	136.5111	35
36	.95812	22.88022	.97557	39.92963	.99302	142.2406	36
37	.95842	23.04712	.97586	40.42266	.99331	148.4684	37
38	.95871	23.21637	.97615	40.92772	.99360	155.2623	38
39	.95900	23.38802	.97644	41.44525	.99389	162.7033	39
40	.95929	23.56212	.97673	41.97571	.99418	170.8883	40
41	.95958	23.73873	.97702	42.51961	.99447	179.9350	41
42	.95987	23.91790	.97731	43.07746	.99476	189.9868	42
43	.96016	24.09969	.97760	43.64980	.99505	201.2212	43
44	.96045	24.28414	.97789	44.23720	.99535	213.8600	44
45	.96074	24.47134	.97819	44.84026	.99564	228.1839	45
46	.96103	24.66132	.97848	45.45963	.99593	244.5540	46
47	.96132	24.85417	.97877	46.09596	.99622	263.4427	47
48	.96161	25.04994	.97906	46.74997	.99651	285.4795	48
49	.96190	25.24869	.97935	47.42241	.99680	311.5230	49
50	.96219	25.45051	.97964	48.11406	.99709	342.7752	50
51	.96248	25.65546	.97993	48.82576	.99738	380.9723	51
52	.96277	25.86360	.98022	49.55840	.99767	428.7187	52
53	.96307	26.07503	.98051	50.31290	.99796	490.1070	53
54	.96336	26.28981	.98080	51.09027	.99825	571.9581	54
55	.96365	26.50804	.98109	51.89156	.99855	686.5496	55
56	.96394	26.72978	.98138	52.71790	.99884	858.4369	56
57	.96423	26.95513	.98168	53.57046	.99913	1144.916	57
58	.96452	27.18417	.98197	54.45053	.99942	1717.874	58
59	.96481	27.41700	.98226	55.35946	.99971	3436.747	59
60	.96510	27.65371	.98255	56.29869	1.00000	Infinite	60

Depth	Base 12	Base 14	Base 16	Base 18	Base 22	Base 24	Base 26	Base 28
1	45	53	60	68	82	90	97	105
2	93	107	122	137	167	181	196	211
3	142	163	186	208	253	275	297	319
4	193	222	252	281	341	370	400	430
5	245	282	319	356	431	468	505	542
6	300	344	389	433	522	567	611	656
7	356	408	460	512	616	668	719	771
8	415	474	533	593	711	770	830	889
9	475	542	608	675	808	875	942	1008
10	537	611	685	759	907	981	1056	1130
11	601	682	764	845	1008	1090	1171	1253
12	667	756	844	933	1111	1200	1289	1378
13	734	831	926	1023	1216	1312	1408	1505
14	804	907	1010	1115	1322	1426	1530	1633
15	875	986	1096	1208	1431	1542	1653	1764
16	948	1067	1184	1304	1541	1659	1778	1896
17	1023	1149	1274	1401	1653	1779	1905	2031
18	1100	1233	1366	1500	1767	1900	2033	2167
19	1179	1319	1460	1601	1882	2023	2164	2305
20	1259	1407	1555	1704	2000	2148	2296	2444
21	1342	1497	1653	1808	2119	2275	2431	2586
22	1426	1589	1752	1915	2241	2404	2567	2730
23	1512	1682	1853	2023	2364	2534	2705	2875
24	1600	1778	1955	2133	2489	2667	2844	3022
25	1690	1875	2060	2245	2616	2801	2986	3171
26	1781	1974	2166	2359	2744	2937	3130	3322
27	1875	2075	2274	2475	2875	3075	3275	3475
28	1970	2178	2384	2593	3007	3215	3422	3630
29	2068	2282	2496	2713	3142	3356	3571	3786
30	2167	2389	2610	2833	3278	3500	3722	3944
31	2268	2497	2726	2956	3416	3645	3875	4105
32	2370	2607	2844	3081	3556	3793	4030	4267
33	2475	2719	2964	3208	3697	3942	4186	4431
34	2581	2833	3085	3337	3841	4093	4344	4596
35	2690	2949	3208	3468	3986	4245	4505	4764
36	2800	3067	3333	3600	4133	4400	4667	4933
37	2912	3186	3460	3734	4282	4556	4831	5105
38	3026	3307	3589	3870	4433	4715	4996	5278
39	3142	3431	3719	4008	4586	4875	5164	5453
40	3259	3556	3852	4148	4741	5037	5333	5630
41	3379	3682	3986	4290	4897	5201	5505	5808
42	3500	3811	4122	4433	5056	5367	5678	5989
43	3623	3942	4260	4579	5216	5534	5853	6171
44	3748	4074	4400	4726	5378	5704	6030	6356
45	3875	4208	4541	4875	5542	5875	6208	6542
46	4004	4344	4684	5026	5707	6048	6389	6730
47	4134	4482	4830	5179	5875	6223	6571	6919
48	4267	4622	4978	5333	6044	6400	6756	7111
49	4401	4764	5127	5490	6216	6579	6942	7305
50	4537	4907	5278	5648	6389	6759	7130	7500
51	4675	5053	5430	5808	6564	6942	7319	7697
52	4815	5200	5584	5970	6741	7126	7511	7896
53	4956	5349	5741	6134	6919	7312	7705	8097
54	5100	5500	5900	6300	7100	7500	7900	8300
55	5245	5653	6060	6468	7282	7690	8097	8505
56	5393	5807	6222	6637	7467	7881	8296	8711
57	5542	5964	6386	6808	7653	8075	8497	8919
58	5693	6122	6552	6981	7841	8270	8700	9130
59	5845	6282	6719	7156	8031	8468	8905	9342
60	6000	6444	6889	7333	8222	8667	9111	9556

Depth	Base 12	Base 14	Base 16	Base 18	Base 22	Base 24	Base 26	Base 28
1	46	54	61	69	83	91	98	106
2	96	111	126	141	170	185	200	215
3	150	172	194	217	261	283	306	328
4	207	237	267	296	356	385	415	444
5	269	306	343	380	454	491	528	565
6	333	378	422	467	556	600	644	689
7	402	454	506	557	661	713	765	817
8	474	533	593	652	770	830	889	948
9	550	617	683	750	883	950	1017	1083
10	630	704	778	852	1000	1074	1148	1222
11	713	794	876	957	1120	1202	1283	1365
12	800	889	978	1067	1244	1333	1422	1511
13	891	987	1083	1180	1372	1469	1565	1661
14	985	1089	1193	1296	1504	1607	1711	1815
15	1083	1194	1306	1417	1639	1750	1861	1972
16	1185	1304	1422	1541	1779	1896	2015	2133
17	1291	1417	1543	1669	1920	2046	2172	2298
18	1400	1533	1667	1800	2067	2200	2333	2467
19	1513	1654	1794	1935	2217	2357	2498	2639
20	1630	1778	1926	2074	2370	2519	2667	2815
21	1750	1906	2061	2217	2528	2683	2839	2994
22	1874	2037	2200	2363	2689	2852	3015	3178
23	2002	2172	2343	2513	2854	3024	3194	3365
24	2133	2311	2489	2667	3022	3200	3378	3556
25	2269	2454	2639	2824	3194	3380	3565	3750
26	2407	2600	2793	2985	3370	3563	3756	3948
27	2550	2750	2950	3150	3550	3750	3950	4151
28	2696	2904	3111	3319	3733	3941	4148	4356
29	2846	3061	3276	3491	3920	4135	4350	4565
30	3000	3222	3444	3667	4111	4333	4556	4778
31	3157	3387	3617	3846	4306	4535	4765	4994
32	3319	3556	3793	4030	4504	4741	4978	5215
33	3483	3728	3972	4217	4706	4950	5194	5439
34	3652	3904	4156	4407	4911	5163	5415	5667
35	3824	4083	4343	4602	5120	5380	5639	5898
36	4000	4267	4533	4800	5333	5600	5867	6133
37	4180	4454	4728	5002	5550	5824	6098	6372
38	4363	4644	4926	5207	5770	6052	6333	6615
39	4550	4839	5128	5417	5994	6283	6572	6861
40	4741	5037	5333	5630	6222	6519	6815	7111
41	4935	5239	5543	5846	6454	6757	7061	7365
42	5133	5444	5756	6067	6689	7000	7311	7622
43	5335	5654	5972	6291	6928	7246	7565	7883
44	5541	5867	6193	6519	7170	7496	7822	8148
45	5750	6083	6417	6750	7417	7750	8083	8417
46	5963	6304	6644	6985	7667	8007	8348	8689
47	6180	6528	6876	7224	7920	8269	8617	8965
48	6400	6756	7111	7467	8178	8533	8889	9244
49	6624	6987	7350	7713	8439	8802	9165	9528
50	6852	7222	7593	7963	8704	9074	9444	9815
51	7083	7461	7839	8217	8972	9350	9728	10106
52	7319	7704	8089	8474	9244	9630	10015	10400
53	7557	7950	8343	8735	9520	9913	10306	10698
54	7800	8200	8600	9000	9800	10200	10600	11000
55	8046	8454	8861	9269	10083	10491	10898	11306
56	8296	8711	9126	9541	10370	10785	11200	11615
57	8550	8972	9394	9817	10661	11083	11506	11928
58	8807	9237	9667	10096	10956	11385	11815	12244
59	9069	9506	9943	10380	11254	11691	12128	12565
60	9333	9778	10222	10667	11556	12000	12444	12889

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	48	56	63	70	78	107	115	122
2	104	119	133	148	163	222	237	252
3	167	189	211	233	256	344	367	389
4	237	267	296	326	356	474	504	533
5	315	352	389	426	463	611	648	685
6	400	444	489	533	578	756	800	844
7	493	544	596	648	700	907	959	1011
8	593	652	711	770	830	1067	1126	1185
9	700	767	833	900	967	1233	1300	1367
10	815	889	963	1037	1111	1407	1481	1556
11	937	1019	1100	1181	1263	1589	1670	1752
12	1067	1156	1244	1333	1422	1778	1867	1956
13	1204	1300	1396	1493	1589	1974	2070	2167
14	1348	1452	1556	1659	1763	2178	2281	2385
15	1500	1611	1722	1833	1944	2389	2500	2611
16	1659	1778	1896	2015	2133	2607	2726	2844
17	1826	1952	2078	2204	2330	2833	2959	3085
18	2000	2133	2267	2400	2533	3067	3200	3333
19	2181	2322	2463	2604	2744	3307	3448	3589
20	2370	2519	2667	2815	2963	3556	3704	3852
21	2567	2722	2878	3033	3189	3811	3967	4122
22	2770	2933	3096	3259	3422	4074	4237	4444
23	2981	3152	3322	3493	3663	4344	4515	4685
24	3200	3378	3556	3733	3911	4622	4800	4978
25	3426	3611	3796	3981	4167	4907	5093	5278
26	3659	3852	4044	4237	4430	5200	5393	5585
27	3900	4100	4300	4500	4700	5500	5700	5900
28	4148	4356	4563	4770	4978	5807	6015	6222
29	4404	4619	4833	5048	5263	6122	6337	6552
30	4667	4889	5111	5333	5556	6444	6667	6889
31	4937	5167	5396	5626	5856	6774	7004	7233
32	5215	5452	5689	5926	6163	7111	7348	7585
33	5500	5744	5989	6233	6478	7456	7700	7944
34	5793	6044	6296	6548	6800	7807	8059	8311
35	6093	6352	6611	6870	7130	8167	8426	8685
36	6400	6667	6933	7200	7467	8533	8800	9067
37	6715	6989	7263	7537	7811	8907	9181	9456
38	7037	7319	7600	7881	8163	9289	9570	9852
39	7367	7656	7944	8233	8522	9678	9967	10256
40	7704	8000	8296	8593	8889	10074	10370	10667
41	8048	8352	8656	8959	9263	10478	10781	11085
42	8400	8711	9022	9333	9644	10889	11200	11511
43	8759	9078	9396	9715	10033	11307	11626	11944
44	9126	9452	9778	10104	10430	11733	12059	12385
45	9500	9833	10167	10500	10833	12167	12500	12833
46	9881	10222	10563	10904	11244	12607	12948	13289
47	10270	10619	10967	11315	11663	13056	13404	13752
48	10667	11022	11378	11733	12089	13511	13867	14222
49	11070	11433	11796	12159	12522	13974	14337	14700
50	11481	11852	12222	12593	12963	14444	14815	15185
51	11900	12278	12656	13033	13411	14922	15300	15678
52	12326	12711	13096	13481	13867	15407	15793	16178
53	12759	13152	13544	13937	14330	15900	16293	16685
54	13200	13600	14000	14400	14800	16400	16800	17200
55	13648	14056	14463	14870	15278	16907	17315	17722
56	14104	14519	14933	15348	15763	17422	17837	18252
57	14567	14989	15411	15833	16256	17944	18367	18789
58	15037	15467	15896	16326	16756	18474	18904	19333
59	15515	15952	16389	16826	17263	19011	19448	19885
60	16000	16444	16889	17333	17778	19556	20000	20444

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	50	57	65	72	80	109	117	124
2	111	126	141	156	170	230	244	259
3	183	206	228	250	272	361	383	406
4	267	296	326	356	385	504	533	563
5	361	398	435	472	509	657	694	731
6	467	511	556	600	644	822	867	911
7	583	635	687	739	791	998	1050	1102
8	711	770	830	889	948	1185	1244	1304
9	850	917	983	1050	1116	1383	1450	1517
10	1000	1074	1148	1222	1296	1593	1667	1741
11	1161	1243	1324	1406	1487	1813	1894	1976
12	1333	1422	1511	1600	1689	2044	2133	2222
13	1517	1613	1709	1806	1902	2287	2383	2480
14	1711	1815	1919	2022	2126	2541	2644	2748
15	1917	2028	2139	2250	2361	2806	2917	3028
16	2133	2252	2370	2489	2607	3081	3200	3319
17	2361	2487	2613	2739	2865	3369	3494	3620
18	2600	2733	2867	3000	3133	3667	3800	3933
19	2850	2991	3131	3272	3413	3976	4117	4257
20	3111	3259	3407	3556	3704	4296	4444	4592
21	3383	3539	3694	3850	4005	4628	4783	4939
22	3667	3830	3993	4156	4318	4970	5133	5296
23	3961	4131	4302	4472	4642	5324	5494	5665
24	4267	4444	4622	4800	4978	5689	5867	6044
25	4583	4769	4954	5139	5324	6065	6250	6435
26	4911	5104	5296	5489	5681	6452	6644	6837
27	5250	5450	5650	5850	6050	6850	7050	7250
28	5600	5807	6015	6222	6430	7259	7467	7674
29	5961	6176	6391	6606	6820	7680	7894	8109
30	6333	6556	6778	7000	7222	8111	8333	8555
31	6717	6946	7176	7406	7635	8554	8783	9013
32	7111	7348	7585	7822	8059	9007	9244	9482
33	7517	7761	8006	8250	8494	9472	9717	9962
34	7933	8185	8437	8689	8941	9948	10200	10452
35	8361	8620	8880	9139	9398	10435	10694	10954
36	8800	9067	9333	9600	9867	10933	11200	11467
37	9250	9524	9798	10072	10346	11443	11717	11991
38	9711	9993	10274	10556	10837	11963	12244	12526
39	10183	10472	10761	11050	11339	12494	12783	13072
40	10667	10963	11259	11556	11852	13037	13333	13630
41	11161	11465	11769	12072	12376	13591	13894	14198
42	11667	11978	12289	12600	12911	14156	14467	14778
43	12183	12502	12820	13139	13457	14731	15050	15369
44	12711	13037	13363	13689	14015	15319	15644	15970
45	13250	13583	13917	14250	14583	15917	16250	16583
46	13800	14141	14481	14822	15163	16526	16867	17207
47	14361	14709	15057	15406	15754	17146	17494	17843
48	14933	15289	15644	16000	16356	17778	18133	18489
49	15517	15880	16243	16606	16968	18420	18783	19146
50	16111	16481	16852	17222	17592	19074	19444	19815
51	16717	17094	17472	17850	18228	19739	20117	20494
52	17333	17719	18104	18489	18874	20415	20800	21185
53	17961	18354	18746	19139	19531	21102	21494	21887
54	18600	19000	19400	19800	20200	21800	22200	22600
55	19250	19657	20065	20472	20880	22509	22917	23324
56	19911	20326	20741	21156	21570	23230	23644	24059
57	20583	21006	21428	21850	22272	23961	24383	24805
58	21267	21696	22126	22556	22985	24704	25133	25563
59	21961	22398	22835	23272	23709	25457	25894	26332
60	22667	23111	23556	24000	24444	26222	26667	27111

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	52	59	67	74	81	111	119	126
2	119	133	148	163	178	237	252	267
3	200	222	244	267	289	378	400	422
4	296	326	356	385	415	533	563	593
5	407	444	481	519	556	704	741	778
6	533	578	622	667	711	889	933	978
7	674	726	778	830	881	1089	1141	1193
8	830	889	948	1007	1067	1304	1363	1422
9	1000	1067	1133	1200	1267	1533	1600	1667
10	1185	1259	1333	1407	1481	1778	1852	1926
11	1385	1467	1548	1630	1711	2037	2119	2200
12	1600	1689	1778	1867	1956	2311	2400	2489
13	1830	1926	2022	2119	2215	2600	2696	2793
14	2074	2178	2281	2385	2489	2904	3007	3111
15	2333	2444	2556	2667	2778	3222	3333	3444
16	2607	2726	2844	2963	3081	3556	3674	3793
17	2896	3022	3148	3274	3400	3904	4030	4156
18	3200	3333	3467	3600	3733	4267	4400	4533
19	3519	3659	3800	3941	4081	4644	4785	4926
20	3852	4000	4148	4296	4444	5037	5185	5333
21	4200	4356	4511	4667	4822	5444	5600	5756
22	4563	4730	4889	5052	5215	5867	6030	6193
23	4941	5111	5281	5452	5622	6304	6474	6644
24	5333	5511	5689	5867	6044	6756	6933	7111
25	5741	5926	6111	6296	6481	7222	7407	7593
26	6163	6356	6548	6741	6933	7704	7896	8089
27	6600	6800	7000	7200	7400	8200	8400	8600
28	7052	7259	7467	7674	7881	8711	8919	9126
29	7519	7733	7948	8163	8378	9237	9452	9667
30	8000	8222	8444	8667	8889	9778	10000	10222
31	8496	8726	8956	9185	9415	10333	10563	10793
32	9007	9244	9481	9719	9956	10904	11141	11378
33	9533	9778	10022	10267	10511	11489	11733	11978
34	10074	10326	10578	10830	11081	12089	12341	12593
35	10630	10889	11148	11407	11667	12704	12963	13222
36	11200	11467	11733	12000	12267	13333	13600	13867
37	11785	12059	12333	12607	12881	13978	14252	14526
38	12385	12667	12948	13230	13511	14637	14919	15200
39	13000	13289	13578	13867	14156	15311	15600	15889
40	13630	13926	14222	14519	14815	16000	16296	16593
41	14274	14578	14881	15185	15489	16704	17007	17311
42	14933	15244	15556	15867	16178	17422	17733	18044
43	15607	15926	16224	16563	16881	18156	18474	18793
44	16296	16622	16948	17274	17600	18904	19230	19556
45	17000	17333	17667	18000	18333	19667	20000	20333
46	17719	18059	18400	18741	19081	20444	20785	21126
47	18452	18800	19148	19496	19844	21237	21585	21933
48	19200	19556	19911	20267	20622	22044	22400	22756
49	19963	20326	20689	21052	21415	22867	23230	23593
50	20741	20711	21481	21852	22222	23704	24074	24444
51	21533	21911	22289	22667	23044	24556	24933	25311
52	22341	22726	23111	23496	23881	25422	25807	26193
53	23163	23556	23948	24341	24733	26304	26696	27089
54	24000	24400	24800	25200	25600	27200	27600	28000
55	24852	25259	25667	26074	26481	28111	28519	28926
56	25719	26133	26548	26963	27378	29037	29452	29867
57	26600	27022	27444	27867	28289	29978	30400	30822
58	27496	27926	28356	28785	29215	30933	31363	31793
59	28407	28844	29281	29719	30156	31904	32341	32778
60	29333	29778	30222	30667	31111	32889	33333	33778

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	56	63	70	78	85	115	122	130
2	133	148	163	178	193	252	267	281
3	233	256	278	300	322	411	433	456
4	356	385	415	444	474	593	622	652
5	500	537	574	611	648	796	833	870
6	667	711	756	800	844	1022	1067	1111
7	856	907	959	1011	1063	1270	1322	1374
8	1067	1126	1185	1244	1304	1541	1600	1659
9	1300	1367	1433	1500	1567	1833	1900	1967
10	1556	1630	1704	1778	1852	2148	2222	2296
11	1833	1915	1996	2078	2159	2485	2567	2648
12	2133	2222	2311	2400	2489	2844	2933	3022
13	2456	2552	2648	2744	2841	3226	3322	3419
14	2800	2904	3007	3111	3215	3630	3733	3837
15	3167	3278	3389	3500	3611	4056	4167	4278
16	3556	3674	3793	3911	4030	4504	4622	4741
17	3967	4093	4219	4344	4470	4974	5100	5226
18	4400	4533	4667	4800	4933	5467	5600	5733
19	4856	4996	5137	5278	5419	5981	6122	6263
20	5333	5481	5630	5778	5926	6519	6667	6815
21	5833	5989	6144	6300	6456	7078	7233	7389
22	6356	6519	6681	6844	7007	7659	7822	7985
23	6900	7070	7241	7411	7581	8263	8433	8504
24	7467	7644	7822	8000	8178	8889	9067	9144
25	8056	8241	8426	8611	8796	9537	9722	9807
26	8667	8859	9052	9244	9437	10207	10400	10593
27	9300	9500	9700	9900	10100	10900	11100	11300
28	9956	10163	10370	10578	10785	11615	11822	12030
29	10633	10848	11063	11278	11493	12352	12567	12781
30	11333	11556	11778	12000	12222	13111	13333	13556
31	12056	12285	12515	12744	12974	13893	14122	14352
32	12800	13037	13274	13511	13748	14696	14933	15170
33	13567	13811	14056	14300	14544	15522	15767	16011
34	14356	14607	14859	15111	15363	16370	16622	16874
35	15167	15426	15685	15944	16204	17241	17500	17759
36	16000	16267	16533	16800	17067	18133	18400	18667
37	16856	17130	17404	17678	17952	19048	19322	19596
38	17733	18015	18296	18578	18859	19985	20267	20548
39	18633	18922	19211	19500	19789	20944	21233	21522
40	19556	19852	20148	20444	20741	21926	22222	22516
41	20500	20804	21107	21411	21715	22930	23233	23537
42	21467	21778	22089	22400	22711	23956	24267	24578
43	22456	22774	23093	23411	23730	25004	25322	25641
44	23467	23793	24119	24444	24770	26074	26400	26726
45	24500	24833	25167	25500	25833	27167	27500	27833
46	25556	25896	26237	26578	26919	28281	28622	28963
47	26633	26981	27330	27678	28026	29419	29767	30115
48	27733	28089	28444	28800	29156	30578	30933	31289
49	28856	29219	29581	29944	30307	31759	32122	32485
50	30000	30370	30741	31111	31481	32963	33333	33704
51	31167	31544	31922	32300	32678	34189	34567	34944
52	32356	32741	33126	33511	33896	35437	35822	36207
53	33567	33959	34352	34744	35137	36707	37100	37493
54	34800	35200	35600	36000	36400	38000	38400	38800
55	36056	36463	36870	37278	37685	39315	39722	40130
56	37333	37748	38163	38578	38993	40652	41067	41481
57	38633	39056	39478	39900	40322	42011	42433	42856
58	39956	40385	40815	41244	41674	43393	43822	44252
59	41300	41737	42174	42611	43048	44796	45233	45670
60	42667	43111	43556	44000	44444	46222	46667	47111

Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.
1	3.7	51	188.9	101	374.1	151	559.3	[illegible]	744.4
2	7.4	52	192.6	102	377.8	152	563.0	202	748.2
3	11.1	53	196.3	103	381.5	153	566.7	203	751.9
4	14.8	54	200.0	104	385.2	154	570 4	204	755.6
5	18.5	55	203.7	105	388.9	155	574.1	205	759.3
6	22.2	56	207.4	106	392.6	156	577.8	206	763.0
7	25.9	57	211.1	107	396.3	157	581.5	207	766.7
8	29.6	58	214.8	108	400 0	158	585.2	208	770.4
9	33.3	59	218.5	109	403.7	159	588.9	209	774.1
10	37.0	60	222.2	110	407.4	160	592.6	210	777.8
11	40.7	61	225.9	111	411.1	161	596.3	211	781.5
12	44.4	62	229.6	112	414.8	162	600.0	212	785.2
13	48.1	63	233.3	113	418.5	163	603.7	213	788.9
14	51.9	64	237.0	114	422.2	164	607.4	214	792.6
15	55.6	65	240.7	115	425.9	165	611.1	215	796.3
16	59.3	66	244.4	116	429.6	166	614 8	216	800.0
17	63.0	67	248.2	117	433.3	167	618.5	217	803.7
18	66.7	68	251.9	118	437.0	168	622.2	218	807.4
19	70.4	69	255.6	119	440.7	169	625.9	219	[illegible]11.1
20	74.1	70	259.3	120	444.4	170	629.6	220	[illegible]14.8
21	77.8	71	263.0	121	448.2	171	633.3	221	[illegible]18.5
22	81.5	72	266.7	122	451.9	172	637.0	222	822.2
23	85.2	73	270.4	123	455 6	173	640.7	223	825.9
24	88.9	74	274.1	124	459.3	174	644.4	224	829.6
25	92 6	75	277.8	125	463.0	175	648.2	225	833.3
26	96 3	76	281.5	126	466.7	176	651.9	226	837.0
27	100.0	77	285 2	127	470.4	177	655.6	227	840.7
28	103.7	78	288.9	128	474 1	178	659.3	228	844.4
29	107.4	79	292.6	129	477.8	179	663.0	229	848.2
30	111.1	80	296.3	130	481.5	180	666.7	230	851.9
31	114 8	81	300.0	131	485.2	181	670.4	231	855.6
32	118.5	82	303.7	132	488 9	182	674.1	232	859.3
33	122.2	83	307.4	133	492.6	183	[illegible]77.8	233	863.0
34	125 9	84	311.1	134	496 3	184	681.5	234	866.7
35	129 6	85	314.8	135	500.0	185	685 2	235	870.4
36	133.3	86	318.5	136	503.7	186	688.9	236	874.1
37	137.0	87	322.2	137	507.4	187	692.6	237	877.8
38	140.7	88	325.9	138	511.1	188	696 3	238	881.5
39	144 4	89	329.6	139	514.8	189	700.0	239	885.2
40	148.2	90	333.3	140	518.5	190	703.7	240	888.9
41	151.9	91	337.0	141	522.2	191	707.4	241	[illegible]92.6
42	155.6	92	340.7	142	525.9	192	711.1	242	896.3
43	159 3	93	344.4	143	529.6	193	714.8	243	900.0
44	163.0	94	348.2	144	533.3	194	718.5	244	903.7
45	166.7	95	351.9	145	537.0	195	722.2	245	907.4
46	170.4	96	355.6	146	540.7	196	725.9	246	[illegible]11.1
47	174.1	97	359.3	147	544.4	197	729.6	247	[illegible]14.8
48	177.8	98	363.0	148	548.2	198	733.3	248	[illegible]18.5
49	181.5	99	366.7	149	551.9	199	737.0	249	922.2
50	185.2	100	370.4	150	555.6	200	740.7	250	925.9

Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.
251	929.6	301	1114.8	351	1300.0	401	1485.2	451	1670.4
252	933.3	302	1118.5	352	1303.7	402	1488.9	452	1674.1
253	937.0	303	1122.2	353	1307.4	403	1492.6	453	1677.8
254	940.7	304	1125.9	354	1311.1	404	1496.3	454	1681.5
255	944.4	305	1129.6	355	1314.8	405	1500.0	455	1685.2
256	948.2	306	1133.3	356	1318.5	406	1503.7	456	1688.9
257	951.9	307	1137.0	357	1322.2	407	1507.4	457	1692.6
258	955.6	308	1140.7	358	1325.9	408	1511.1	458	1696.3
259	959.3	309	1144.4	359	1329.6	409	1514.8	459	1700.0
260	963.0	310	1148.2	360	1333.3	410	1518.5	460	1703.7
261	966.7	311	1151.9	361	1337.0	411	1522.2	461	1707.4
262	970.4	312	1155.6	362	1340.7	412	1525.9	462	1711.1
263	974.1	313	1159.3	363	1344.4	413	1529.6	463	1714.8
264	977.8	314	1163.0	364	1348.2	414	1533.3	464	1718.5
265	981.5	315	1166.7	365	1351.9	415	1537.0	465	1722.2
266	985.2	316	1170.4	366	1355.6	416	1540.7	466	1725.9
267	988.9	317	1174.1	367	1359.3	417	1544.4	467	1729.6
268	992.6	318	1177.8	368	1363.0	418	1548.2	468	1733.3
269	996.3	319	1181.5	369	1366.7	419	1551.9	469	1737.0
270	1000.0	320	1185.2	370	1370.4	420	1555.6	470	1740.7
271	1003.7	321	1188.9	371	1374.1	421	1559.3	471	1744.4
272	1007.4	322	1192.6	372	1377.8	422	1563.0	472	1748.2
273	1011.1	323	1196.3	373	1381.5	423	1566.7	473	1751.9
274	1014.8	324	1200.0	374	1385.2	424	1570.4	474	1755.6
275	1018.5	325	1203.7	375	1388.9	425	1574.1	475	1759.3
276	1022.2	326	1207.4	376	1392.6	426	1577.8	476	1763.0
277	1025.9	327	1211.1	377	1396.3	427	1581.5	477	1766.7
278	1029.6	328	1214.8	378	1400.0	428	1585.2	478	1770.4
279	1033.3	329	1218.5	379	1403.7	429	1588.9	479	1774.1
280	1037.0	330	1222.2	380	1407.4	430	1592.6	480	1777.8
281	1040.7	331	1225.9	381	1411.1	431	1596.3	481	1781.5
282	1044.4	332	1229.6	382	1414.8	432	1600.0	482	1785.2
283	1048.2	333	1233.3	383	1418.5	433	1603.7	483	1788.9
284	1051.9	334	1237.0	384	1422.2	434	1607.4	484	1792.6
285	1055.6	335	1240.7	385	1425.9	435	1611.1	485	1796.3
286	1059.3	336	1244.4	386	1429.6	436	1614.8	486	1800.0
287	1063.0	337	1248.2	387	1433.3	437	1618.5	487	1803.7
288	1066.7	338	1251.9	388	1437.0	438	1622.2	488	1807.4
289	1070.4	339	1255.6	389	1440.7	439	1625.9	489	1811.1
290	1074.1	340	1259.3	390	1444.4	440	1629.6	490	1814.8
291	1077.8	341	1263.0	391	1448.2	441	1633.3	491	1818.5
292	1081.5	342	1266.7	392	1451.9	442	1637.0	492	1822.2
293	1085.2	343	1270.4	393	1455.6	443	1640.7	493	1825.9
294	1088.9	344	1274.1	394	1459.3	444	1644.4	494	1829.6
295	1092.6	345	1277.8	395	1463.0	445	1648.2	495	1833.3
296	1096.3	346	1281.5	396	1466.7	446	1651.9	496	1837.0
297	1100.0	347	1285.2	397	1470.4	447	1655.6	497	1840.7
298	1103.7	348	1288.9	398	1474.1	448	1659.3	498	1844.4
299	1107.4	349	1292.6	399	1477.8	449	1663.0	499	1848.2
300	1111.1	350	1296.3	400	1481.5	450	1666.7	500	1851.9

Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.
501	1855.6	551	2040.7	601	2225.9	651	2411.1	701	2596.3
502	1859.3	552	2044.4	602	2229.6	652	2414.8	702	2600.0
503	1863.0	553	2048.2	603	2233.3	653	2418.5	703	2603.7
504	1866.7	554	2051.9	604	2237.0	654	2422.2	704	2607.4
505	1870.4	555	2055.6	605	2240.7	655	2425.9	705	2611.1
506	1874.1	556	2059.3	606	2244.4	656	2429.6	706	2614.8
507	1877.8	557	2063.0	607	2248.2	657	2433.3	707	2618.5
508	1881.5	558	2066.7	608	2251.9	658	2437.0	708	2622.2
509	1885.2	559	2070.4	609	2255.6	659	2440.7	709	2625.9
510	1888.9	560	2074.1	610	2259.3	660	2444.4	710	2629.6
511	1892.6	561	2077.8	611	2263.0	661	2448.2	711	2633.3
512	1896.3	562	2081.5	612	2266.7	662	2451.9	712	2637.0
513	1900.0	563	2085.2	613	2270.4	663	2455.6	713	2640.7
514	1903.7	564	2088.9	614	2274.1	664	2459.3	714	2644.4
515	1907.4	565	2092.6	615	2277.8	665	2463.0	715	2648.2
516	1911.1	566	2096.3	616	2281.5	666	2466.7	716	2651.9
517	1914.8	567	2100.0	617	2285.2	667	2470.4	717	2655.6
518	1918.5	568	2103.7	618	2288.9	668	2474.1	718	2659.3
519	1922.2	569	2107.4	619	2292.6	669	2477.8	719	2663.0
520	1925.9	570	2111.1	620	2296.3	670	2481.5	720	2666.7
521	1929 6	571	2114.8	621	2300.0	671	2485.2	721	2670.4
522	1933.3	572	2118.5	622	2303.7	672	2488.9	722	2674.1
523	1937.0	573	2122.2	623	2307.4	673	2492 6	723	2677.8
524	1940.7	574	2125.9	624	2311.1	674	2496.3	724	2681.5
525	1944.4	575	2129.6	625	2314.8	675	2500.0	725	2685.2
526	1948.2	576	2133.3	626	2318.5	676	2503.7	726	2688.9
527	1951.9	577	2137.0	627	2322.2	677	2507.4	727	2692.6
528	1955.6	578	2140.7	628	2325.9	678	2511.1	728	2696.3
529	1959.3	579	2144.4	629	2329.6	679	2514.8	729	2700.0
530	1963.0	580	2148.2	630	2333.3	680	2518.5	730	2703.7
531	1966.7	581	2151.9	631	2337.0	681	2522.2	731	2707.4
532	1970.4	582	2155.6	632	2340.7	682	2525.9	732	2711.1
533	1974.1	583	2159.3	633	2344.4	683	2529.6	733	2714.8
534	1977.8	584	2163.0	634	2348.2	684	2533.3	734	2718.5
535	1981.5	585	2166.7	635	2351.9	685	2537.0	735	2722.2
536	1985.2	586	2170.4	636	2355.6	686	2540 7	736	2725.9
537	1988.9	587	2174.1	637	2359.3	687	2544.4	737	2729.6
538	1992.6	588	2177.8	638	2363.0	688	2548.2	738	2733.3
539	1996.3	589	2181.5	639	2366.7	689	2551.9	739	2737.0
540	2000.0	590	2185.2	640	2370.4	690	2555.6	740	2740.7
541	2003.7	591	2188.9	641	2374.1	691	2559.3	741	2744.4
542	2007.4	592	2192.6	642	2377.8	692	2563.0	742	2748.2
543	2011.1	593	2196.3	643	2381.5	693	2566.7	743	2751.9
544	2014.8	594	2200.0	644	2385.2	694	2570.4	744	2755.6
545	2018.5	595	2203.7	645	2388.9	695	2574.1	745	2759.3
546	2022.2	596	2207.4	646	2392.6	696	2577.8	746	2763.0
547	2025.9	597	2211.1	647	2396.3	697	2581.5	747	2766.7
548	2029.6	598	2214.8	648	2400.0	698	2585.2	748	2770.4
549	2033.3	599	2218.5	649	2403.7	699	2588.9	749	2774.1
550	2037.0	600	2222.2	650	2407.4	700	2592.6	750	2777.8

Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.	Area. Sq. Ft.	Cubic Yards.
751	2781.5	801	2966.7	851	3151.9	901	3337.0	951	3522.2
752	2785.2	802	2970.4	852	3155.6	902	3340.7	952	3525.9
753	2788.9	803	2974.1	853	3159.3	903	3344.4	953	3529.6
754	2792.6	804	2977.8	854	3163.0	904	3348.2	954	3533.3
755	2796.3	805	2981.5	855	3166.7	905	3351.9	955	3537.0
756	2800.0	806	2985.2	856	3170.4	906	3355.6	956	3540.7
757	2803.7	807	2988.9	857	3174.1	907	3359.3	957	3544.4
758	2807.4	808	2992.6	858	3177.8	908	3363.0	958	3548.2
759	2811.1	809	2996.3	859	3181.5	909	3366.7	959	3551.9
760	2814.8	810	3000.0	860	3185.2	910	3370.4	960	3555.6
761	2818.5	811	3003.7	861	3188.9	911	3374.1	961	3559.3
762	2822.2	812	3007.4	862	3192.6	912	3377.8	962	3563.0
763	2825.9	813	3011.1	863	3196.3	913	3381.5	963	3566.7
764	2829.6	814	3014.8	864	3200.0	914	3385.2	964	3570.4
765	2833.3	815	3018.5	865	3203.7	915	3388.9	965	3574.1
766	2837.0	816	3022.2	866	3207.4	916	3392.6	966	3577.8
767	2840.7	817	3025.9	867	3211.1	917	3396.3	967	3581.5
768	2844.4	818	3029.6	868	3214.8	918	3400.0	968	3585.2
769	2848.2	819	3033.3	869	3218.5	919	3403.7	969	3588.9
770	2851.9	820	3037.0	870	3222.2	920	3407.4	970	3592.6
771	2855.6	821	3040.7	871	3225.9	921	3411.1	971	3596.3
772	2859.3	822	3044.4	872	3229.6	922	3414.8	972	3600.0
773	2863.0	823	3048.2	873	3233.3	923	3418.5	973	3603.7
774	2866.7	824	3051.9	874	3237.0	924	3422.2	974	3607.4
775	2870.4	825	3055.6	875	3240.7	925	3425.9	975	3611.1
776	2874.1	826	3059.3	876	3244.4	926	3429.6	976	3614.8
777	2877.8	827	3063.0	877	3248.2	927	3433.3	977	3618.5
778	2881.5	828	3066.7	878	3251.9	928	3437.0	978	3622.2
779	2885.2	829	3070.4	879	3255.6	929	3440.7	979	3625.9
780	2888.9	830	3074.1	880	3259.3	930	3444.4	980	3629.6
781	2892.6	831	3077.8	881	3263.0	931	3448.2	981	3633.3
782	2896.3	832	3081.5	882	3266.7	932	3451.9	982	3637.0
783	2900.0	833	3085.2	883	3270.4	933	3455.6	983	3640.7
784	2903.7	834	3088.9	884	3274.1	934	3459.3	984	3644.4
785	2907.4	835	3092.6	885	3277.8	935	3463.0	985	3648.2
786	2911.1	836	3096.3	886	3281.5	936	3466.7	986	3651.9
787	2914.8	837	3100.0	887	3285.2	937	3470.4	987	3655.6
788	2918.5	838	3103.7	888	3288.9	938	3474.1	988	3659.3
789	2922.2	839	3107.4	889	3292.6	939	3477.8	989	3663.0
790	2925.9	840	3111.1	890	3296.3	940	3481.5	990	3666.7
791	2929.6	841	3114.8	891	3300.0	941	3485.2	991	3670.4
792	2933.3	842	3118.5	892	3303.7	942	3488.9	992	3674.1
793	2937.0	843	3122.2	893	3307.4	943	3492.6	993	3677.8
794	2940.7	844	3125.9	894	3311.1	944	3496.3	994	3681.5
795	2944.4	845	3129.6	895	3314.8	945	3500.0	995	3685.2
796	2948.2	846	3133.3	896	3318.5	946	3503.7	996	3688.9
797	2951.9	847	3137.0	897	3322.2	947	3507.4	997	3692.6
798	2955.6	848	3140.7	898	3325.9	948	3511.1	998	3696.3
799	2959.3	849	3144.4	899	3329.6	949	3514.8	999	3700.0
800	2963.0	850	3148.2	900	3333.3	950	3518.5	1000	3703.7

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$= \frac{xy}{2} \times 8 = 12705$

$12x =$

$1050 \times (n-2)$

$8 \overline{)1260}$

157^{50}

46

78^{45}

$1=3$

$x^2+y^2=z^2$

$y' = \frac{1}{2}z$

$\frac{xy}{2} \times 12 = 190,575$

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